COMPUTER PROGRAM BASED ON FINITE ELEMENT METHOD FOR STATIC ANALYSIS OF PLANAR STRUCTURES OF ARTICULATED WOODEN BARS

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ABSTRACT

Today modern design must meet several requirements related in particular to be determined by the precision of solutions for various types of structures.
A major task is to determine the behavior of mechanical structures or structural elements in effect of external actions. By applying the finite element method, physical systems governed by partial differential equations with having an infinite number of degrees of freedom are reduced to discrete physical systems with a finite number of degrees of freedom governed by algebraic equations.

Specifically, the essential question is: what is the answer structure when subjected to external actions (variations of strength, temperature, etc.).

Program designed by the authors using the finite element tool engineer put in hand work necessary to optimize the design, with positive effects on the complete analysis of stress and tensions in planar structures of articulated bars.

1. INTRODUCTION

In this paper, program designed using finite element calculation was adopted by the authors following simplifying assumption: the flat structure of articulated bars made of wood will not take into account material anisotropy, considering that by its geometry and external forces acting on the nodes of the structure, the structure is similar to the response of isotropic materials.

By adopting this hypothesis, computer program developed by the authors can be adapted to any type of material used to make the structure. It is only necessary to replace in the program only the geometric, the physico-mechanical and material characteristics [6], [7], [8]..

2. MATERIALS AND METHODS

This type of wooden structures studied and presented in the paper is requested to stretching and compression. Structure is composed of bars whit 2 nodes and 2 degrees of freedom on each node. The two degrees of freedom per node are the horizontally and vertically displacements [3], [9], [10]. It aims to determine the nodals elastic equilibrium equations using the displacements method [4], [11], [14], [15]. The analysis requires two reference systems one local that is attached to each element of the bar and a global for the analysis of the entire structure of bars.

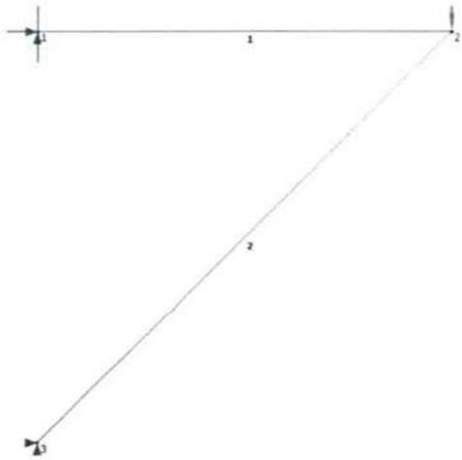

Figure 1.

It presents the structure calculation algorithm, which is based program developed by the authors.

By removing a bar element node structure and introduction of nodal forces expressed in the local reference system to obtain bar elongation or shortening (1).

$$\Delta l = \frac{f_i \cdot l}{E \cdot A}. \tag{1}$$

Where:

f_i – nodal force in "i" node.

l – length of the bar

$E \cdot A$ – tensile and compressive stiffness of the bar.

Length of each bar (2) is determinate with the relation

$$l(i) = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \tag{2}$$

Nodal forces acting on nodes at the ends of each element (3), (4), are equal and opposite [12], [13].

Matrix of nodal forces (5) in local reference system is

$$f_i = \frac{E \cdot A}{l}(u_i - u_j), \tag{3}$$

$$f_j = \frac{E \cdot A}{l}(u_j - u_i), \tag{4}$$

$$\{f\} = \begin{Bmatrix} f_i \\ f_j \end{Bmatrix} = \frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = [k] \cdot \{d\}.$$

(5)

In the global reference system, each node bar element has a horizontal and vertical displacement [3], [7], [8]. Designing nodal displacements in local reference system in the direction bar elements obtaining the expressions of them depending on global displacements (10).

$$u_i = U_i \cdot \cos\alpha + V_i \cdot \sin\alpha, \tag{6}$$

$$u_j = U_j \cdot \cos\alpha + V_j \cdot \sin\alpha, \tag{7}$$

$$l = \cos\alpha, \tag{8}$$

$$m = \sin\alpha, \tag{9}$$

$$\{d\}=\begin{Bmatrix} u_i \\ u_j \end{Bmatrix}=\begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}\begin{Bmatrix} U_i \\ V_i \\ U_j \\ V_j \end{Bmatrix}=[k]\cdot\{D\}. \tag{10}$$

Where:

$\{d\}$ – nodal displacements vector in local system;

$L=\begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}$ – directors cosine vectors of element;

$\{D\}$ – nodal displacements vector in global system;

$[k]$ – stiffness matrix of element.

The vectors of nodal forces in local system (11) expressed according to nodal forces in global reference system is

$$\{f\}=\begin{Bmatrix} f_i \\ f_j \end{Bmatrix}=\begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}\begin{Bmatrix} F_{xi} \\ F_{yi} \\ F_{xj} \\ F_{yj} \end{Bmatrix}. \tag{11}$$

Given the relationships shown are obtained elastic nodal equation in local system (12) and global system (13).

$$\{f\}=[k]\cdot\{d\}; \tag{12}$$

$$\{F\}=[L]^T\cdot[k]\cdot[L]\cdot\{d\}.$$

(13)

Where:

$[K]=[L]^T\cdot[k]\cdot[L]$ – element stiffness matrix [8], [11] in global reference system (14).

$$[K]=[L]^T\cdot[k]\cdot[L]=\begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{bmatrix}\frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}. \tag{14}$$

Elastic nodal equation in local system and global system [9], [12], becomes (15), (16)

$$\begin{Bmatrix} F_{xi} \\ F_{yi} \\ F_{xj} \\ F_{yj} \end{Bmatrix}=\frac{EA}{l}\cdot\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ ml & m^2 & -ml & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -ml & -m^2 & ml & m^2 \end{bmatrix}\begin{Bmatrix} U_i \\ V_i \\ U_j \\ V_j \end{Bmatrix}, \tag{15}$$

$$\{F\}=[K]\{D\}. \tag{16}$$

By assembling the stiffness matrices of elements obtaining the stiffness matrix of the entire structure.

Solving the system of nodal equations of equilibrium [11] leads to the determination of nodal displacements (17).

$$\{D\}=[K]^{-1}\{F\}. \tag{17}$$

Calculation of tensile or compressive effort [1], [2], [5], of each bar element ("i") is determined by the relationship (18)

$$N(i) = \frac{EA}{l} \cdot \Delta l . \tag{18}$$

Normal tension for each element [2], [5] is determined using the relationship (19)

$$\sigma_x(i) = \frac{N(i)}{A} . \tag{19}$$

Calculation algorithm presented is theoretical support necessary to design computer program using finite element method.

Initial data structure considered are the following:

$F = 1000\,[N]$;

Young's modulus $E = 0{,}12 \cdot 10^6 [N/mm^2]$(Mpa);

$A = 100[mm^2]$;

```
clear;clc;clf;
%Cartesian coordinates of the nodes expressed in [mm]
noduri=[ 0    0
         300  0
         0   -300]
%Finite element matrix (including the Young's modulus and cross section areas in[mm^2])
    node  node Young's modulus   areas section
elem=[ 1    2         200000          100
       2    3         200000          100]
% Forces applied to the beam
    %node   fx    fy
forte=[ 2       0 -1000]
% Boundary conditions applied
    % node bx  by
cond=[   1    1    1
         3    1    1  ]
%Number of nodes structure
nnd=length(noduri(:,2))
% Number of elements structure
nel=length(elem(:,4))
% Determine the number of forces and boundary conditions applied to the structure
nnf=length(forte(:,1))
ncond=length(cond(:,1))
%Vector of nodal coordinates on x and y axis
cx=noduri(:,1)
cy=noduri(:,2)
%Number of degrees of freedom per node (ngn),element (nel) and the total number of degrees
of freedom (nec)
ngn=2
ngel=2*ngn
nec=nnd*ngn
% Initialization to zero for MR (stiffness matrix), F (Vector of nodal forces) and index
MR=zeros(nec,nec)
F=zeros(nec)
index=zeros(2*ngn)
for i=1:nel
nod1=elem(i,1)
```

```
nod2=elem(i,2)
E=elem(i,3)
A=elem(i,4)
% Length of beam finite elements and the value of matrix stiffness
le=sqrt((cx(nod2)-cx(nod1))^2+(cy(nod2)-cy(nod1))^2)
ka=E*A/le
% Cosines directors of each beam elements.
c=(cx(nod2)-cx(nod1))/le
s=(cy(nod2)-cy(nod1))/le
length(i)=le'
% Vectors cosine directors of each beam elements
vc(i)=c
vs(i)=s
%Position of the element stiffness matrix terms in the global stiffness matrix.
index(1)=ngn*nod1-1
index(2)=ngn*nod1
index(3)=ngn*nod2-1
index(4)=ngn*nod2
% Element stiffness matrix of the horizontal bar.
mrelp=[ c*c c*s
        c*s s*s]
% Element stiffness matrix inclined at an angle bar.
mrel=ka*[ mrelp -mrelp
          -mrelp  mrelp]
% Assembling the stiffness matrix of each element in the global stiffness matrix.
for i1=1:ngel
j1=index(i1)
for i2=1:ngel
j2=index(i2)
MR(j1,j2)=MR(j1,j2)+mrel(i1,i2)
end
end
end
% Addition of concentrated forces on the structure.
for i=1:nnf
n=forte(i,1) % forces acting node
if forte(i,2)~=0
% Force on the x direction in the global reference system
f=forte(i,2)
F(ngn*(n-1)+1)=F(ngn*(n-1)+1)+f
end
if forte(i,3)~=0
% Force on the y direction in the global reference system
f=forte(i,3)
F(ngn*(n-1)+2)=F(ngn*(n-1)+2)+f
end
end
% Applying boundary conditions.
for i=1:ncond
n=cond(i,1) % node where displacement is zero.
```

% Displacement zero on the x axes in the global reference system.

```
if cond(i,2)==1
MR(ngn*(n-1)+1,:)=zeros(1,nec)
MR(:,ngn*(n-1)+1)=zeros(nec,1)
MR(ngn*(n-1)+1,ngn*(n-1)+1)=1
F(ngn*(n-1)+1)=0
end
```

% Displacement zero on the y axes in the global reference system.

```
if cond(i,3)==1
MR(ngn*(n-1)+2,:)=zeros(1,nec)
MR(:,ngn*(n-1)+2)=zeros(nec,1)
MR(ngn*(n-1)+2,ngn*(n-1)+2)=1
F(ngn*(n-1)+2)=0
end
end
```

% Calculation of nodal displacements

```
depl=MR\F
for i=1:nnd
u(i)=depl(ngn*(i-1)+1)
v(i)=depl(ngn*(i-1)+2)
end
```

% Display unknowns displacements.

```
fprintf('nodul u(mm) v(mm)\n')
for i=1:nnd
fprintf('%3.f %3.9f %3.9f\n',i,u(i),v(i))
end
fprintf('\n')
```

% Determination of normal stress and sectional efforts

```
for i=1:nel
nod1=elem(i,1)
nod2=elem(i,2)
E=elem(i,3)
A=elem(i,4)
```

% Length of each bar element

```
le=sqrt((cx(nod2)-cx(nod1))^2+(cy(nod2)-cy(nod1))^2)
```

% Cosines directors of each beam elements.

```
c=(cx(nod2)-cx(nod1))/le
s=(cy(nod2)-cy(nod1))/le
```

% dn1 and dn2, vectors of nodal displacements at the ends of the bar element

```
dn1=[ u(nod1) v(nod1)]
dn2=[ u(nod2) v(nod2)]
```

% Tensile and compressive stiffness and directors cosine vector vd

```
ka=E*A/le
vd=[ c s]
```

% Elongation or shortening expressed as the difference between nodal displacements of % the bar in the local reference system of finite element

```
dl=dot(dn2,vd)-dot(dn1,vd)
```

% Displacements of "nod2" in local reference system is dot (dn2, vd) (projection of NOD2 global displacement in the direction bar).

% Displacements of "nod1" in local reference system is dot1 (dn1, vd) (global displacement

projection on the direction nod1 bar), dot represents the scalar product.
% Determination of tensile or compression sectional effort and tensions tx for each bar "i" of the structure.

```
N(i)=ka*dl
tx(i)=N(i)/A
end
```

% Display unknowns represented by sectional efforts

```
fprintf('elementul  efortul sectional Fx(N)\n')
for i=1:nel
fprintf(' %3.f %6.2f\n',i,N(i))
end
fprintf('\n')
```

% Display unknowns tensions

```
fprintf('elementul tensiune tx(MPa)\n')
for i=1:nel
fprintf(' %3.f %3.2f\n',i,tx(i))
end
```

3. CONCLUSIONS

Some of the data obtained by running the program: stiffness of bars, length of bars, sectional effort and normal stresses in each bar is shown below:

length (element 1) = 300.0000
length (element 2) = 424.2641
ka = 40000
ka =2.8284e+004
node1 node 2 node 3
u = 0 0.0250 0
v = 0 -0.0957 0
node u(mm) v(mm)
1 0.000000000 0.000000000
2 0.025000000 -0.095710678
3 0.000000000 0.000000000
N1=1.0e+003 *1.0000 -1.4142
N2=1.0e+003 *(-1.4142)
tx(1) = 10.0000
tx(2) =-14.1421
element sectional effort
1 1000.00
2 -1414.21
element normal stress tx(MPa)
1 10.00
2 -14.14

Numerical method has the advantage that the computer program developed by the author, leads to solutions of the problem that converge to the "exact" solution. The paper presented, is a novelty in terms of adapting to a full calculation of structures regardless of physical-mechanical properties of materials they are made.

The main steps that were followed in this program by the author are:
-stiffness matrices-writing of the elements composing the structure of the structure;
-calculation of the cosine directors and transformation matrices;
-matrix assembly of each beam in the global stiffness matrix of the structure;
-establishment of nodal forces for the entire structure;
-application related conditions;
-determining the nodal equilibrium equations system;
-determining the efforts and the tension at each beam ends.

Analytical solving of any type of structure with geometric and physical-mechanical characteristics specific require more time and precision of results is not so great.

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