On injective attributed characterization of 2-way deterministic finite state transducers

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Definitions and notation

A 2DFT starting from the left (right) is a 7-tuple $\mathbf{T}=(Q, X, L, R, Y, \delta, q_0)$, where

(i) Q is a finite, nonempty set of states;
(ii) X is a finite, nonempty input alphabet;
(iii) L (left endmarker) and R (right endmarker) are distinguished symbols not in X;
(iv) Y is a finite output alphabet;
(v) $\delta: Q\times(X\cup\{L, R\})\to Q\times Y\times\{\mathit{left}, \mathit{right}\}$ is a partial function;
(vi) $q_0\in Q$ is the initial state.

Informally $\mathbf{T}$ functions as follows. The input word is surrounded by the two endmarkers, and $\mathbf{T}$ starts from state q_0 with its tape head reading the left (right) endmarker. The moves of $\mathbf{T}$ are described by the transition function δ in the usual way (cf. [1]). The transduction terminates successfully when $\mathbf{T}$ moves right of R or left of L. It is obvious that the left or right start of $\mathbf{T}$ is only a technical question. $\mathbf{T}$ is called an 1DFT if δ allows it moving in only one direction.

Let A be a finite, nonempty set such that $A=A_s\cup A_i$ and $A_s\cap A_i=\emptyset$. The elements of A_s and A_i are called synthesized attributes (s-attributes) and inherited attributes (i-attributes), respectively. Define the monoid $\mathbf{M}(A, Y)$ (Y is a finite alphabet) as follows. $M(A, Y)$ consists of all partial functions of A into $A\times Y^*$. Disjoining $\xi\in M(A, Y)$ into four parts we can represent it by the following diagram,

$$\xi:\quad \begin{tikzcd} A_s \arrow[r, "\bar{\xi}_s"] \arrow[d, "\bar{\xi}_i"'] & A_s\times Y^* \\ A_i\times Y^* & A_i \arrow[l, "\underline{\xi}_i"] \arrow[u, "\underline{\xi}_s"'] \end{tikzcd}$$

where $\xi=\bar{\xi}_s\cup\bar{\xi}_i\cup\underline{\xi}_s\cup\underline{\xi}_i$ and $\bar{\xi}_s, \bar{\xi}_i, \underline{\xi}_s, \underline{\xi}_i$ have pairwise disjoint domains. To make this kind of diagrams composable we rather consider ξ as a partial function $\xi': A\times Y^*\to A\times Y^*$, where $\xi'(a, w)$ $(a\in A, w\in Y^*)$ can be obtained from $\xi(a)$

by prefixing its second component with w. For simplicity we use the abusing notation $\xi'=\xi$, and do not indicate the factor Y^* in the diagrams. For $a\in A$, the first and the second component of $\xi(a)$ will be denoted by $\operatorname{attr}(\xi, a)$ and $\operatorname{out}(\xi, a)$, respectively. $\operatorname{attr}(\xi)$ will denote the partial function $\{(a, \operatorname{attr}(\xi, a))|a\in A\}$. If $Y=\emptyset$, then we identify ξ with $\operatorname{attr}(\xi)$. ξ is called injective if such is $\operatorname{attr}(\xi)$. Now if $\xi, \eta\in M(A, Y)$, then $\xi\circ\eta=\zeta$ can be constructed as follows.

$$\zeta:\quad \begin{tikzcd} A_s \arrow[r, "\bar{\xi}_s"] \arrow[d, "\bar{\xi}_i"'] & A_s \arrow[r, "\bar{\eta}_s"] \arrow[d, shift left, "\bar{\eta}_i"] & A_s \\ A_i & A_i \arrow[l, "\underline{\xi}_i"] \arrow[u, shift left, "\underline{\xi}_s"] & A_i \arrow[l, "\underline{\eta}_i"] \arrow[u, "\underline{\eta}_s"'] \end{tikzcd}$$

$$\bar{\zeta}_s = \bigcup_{n\geqq 0} (\bar{\xi}_s\circ(\bar{\eta}_i\circ\underline{\xi}_s)^n\circ\bar{\eta}_s);\quad \bar{\zeta}_i = \bar{\xi}_i\cup\Big(\bigcup_{n\geqq 0} (\bar{\xi}_s\circ(\bar{\eta}_i\circ\underline{\xi}_s)^n\circ\bar{\eta}_i\circ\underline{\xi}_i)\Big).$$

$$\underline{\zeta}_i = \bigcup_{n\geqq 0} (\underline{\eta}_i\circ(\underline{\xi}_s\circ\bar{\eta}_i)^n\circ\underline{\xi}_i);\quad \underline{\zeta}_s = \underline{\eta}_s\cup\Big(\bigcup_{n\geqq 0} (\underline{\eta}_i\circ(\underline{\xi}_s\circ\bar{\eta}_i)^n\circ\underline{\xi}_s\circ\bar{\eta}_s)\Big).$$

ζ is well defined, since in each case those partial mappings, the union of which must be taken have pairwise disjoint domains. It is easy to verify that this composition is associative, preserves injectivity, and the unit element of $\mathbf{M}(A, Y)$ corresponds to the identity map of $A\times Y^*$. For $a\in A$, $\operatorname{path}(\xi\circ\eta, a)$ will denote the sequence of attributes reached in the above composite diagram during the computation of $\xi\circ\eta(a)$.

Definition. A simple deterministic attributed string transducer (SDAST) starting from the left (right) is a 7-tuple $\mathbf{A}=(A, X, L, R, Y, h, a_0)$, where

(i) $A=A_s\cup A_i$ is the finite, nonempty set of attributes, $A_s\cap A_i=\emptyset$;

(ii) X, L, R and Y are as in the case of a 2DFT;

(iii) h is a mapping of $X(L, R)=X\cup\{L, R\}$ into $M(A, Y)$;

(iv) if $\mathbf{A}$ starts from the left, then $a_0\in A_s$, else $a_0\in A_i$.

Denote the extension of h to a homomorphism of $X(L, R)^*$ into $\mathbf{M}(A, Y)$ also by h. Then the transform of $w\in X^*$ by $\mathbf{A}$ is $\operatorname{out}(h(LwR), a_0)$. $\mathbf{A}$ is called injective if $h(x)$ is injective for every $x\in X(L, R)$.

Lemma 1. 2DFT and SDAST are equivalent, i.e. they define the same class of mappings.

Proof. Let $\mathbf{T}=(Q, X, L, R, Y, \delta, q_0)$ be a 2DFT, and define the SDAST $\mathbf{A}=(2Q, X, L, R, Y, h, a_0)$ as follows. A_s and A_i are two (disjoint) isomorphic copies of Q. Let q_s and q_i denote the corresponding s-attribute and i-attribute of a state $q\in Q$, respectively. Then for $x\in X(L, R)$ and $q\in Q$, $h(x)(q_s)$ and $h(x)(q_i)$ are defined iff $\delta(q, x)$ is defined, and in this case

$$h(x)(q_s) = h(x)(q_i) = \left(\begin{Bmatrix} q'_s \\ q'_i \end{Bmatrix}, w\right) \quad\text{if}\quad \delta(q, x) = \left(q', w, \begin{Bmatrix} \mathit{right} \\ \mathit{left} \end{Bmatrix}\right).$$

$a_0=(q_0)_s$ if $\mathbf{T}$ starts from the left, otherwise $a_0=(q_0)_i$. It is easy to see that $\mathbf{T}$ and $\mathbf{A}$ are equivalent.

Let $\mathbf{A}=(A, X, L, R, Y, h, a_0)$ be an SDAST and define the 2DFT $\mathbf{T}=(A, X, L, R, Y, \delta, a_0)$ as follows. For $x\in X(L, R)$ and $a\in A$, $\delta(a, x)$ is defined iff $h(x)(a)$ is defined, and in this case

$$\delta(a, x) = \left(b,\ w,\ \begin{Bmatrix} right \\ left \end{Bmatrix}\right) \quad \text{if} \quad h(x)(a) = (b, w) \quad \text{with} \quad b\in\begin{Bmatrix} A_s \\ A_i \end{Bmatrix}.$$

The equivalence of $\mathbf{T}$ and $\mathbf{A}$ is again evident. Now we prove a lemma similar to Lemma 1 in [2].

Lemma 2. Every SDAST mapping is the composition of two 1DFT mappings and an injective SDAST mapping.

Proof. Let $\mathbf{A}=(A, X, L, R, Y, h, a_0)$ be an SDAST starting from the left, $w\in X^*$, and suppose that $LwR=w_1xw_2$ for some $x\in X(L, R)$, $w_i\in X(L, R)^*$ $(i=1,2)$. The triple $\alpha=(w_1, x, w_2)$ indicates an x-labelled node in LwR. Let $\xi=\operatorname{attr}(h(w_1))$, $\eta=\operatorname{attr}(h(xw_2))$, called the left and right dependency graphs of α, respectively, and define the subsets $A_s^{(u)}$ and $A_i^{(u)}$ of A as:

(i) if $h(LwR)(a_0)$ is undefined, then $A_s^{(u)}=A_i^{(u)}=\emptyset$;

(ii) else $A_s^{(u)}=\bigcup\limits_{n\geqq 0} (\xi_s\circ(\bar{\eta}_i\circ\xi_s)^n(a_0))$,

$$A_i^{(u)} = \bigcup_{n\geqq 0} (\xi_s\circ(\bar{\eta}_i\circ\xi_s)^n\circ\bar{\eta}_i(a_0)).$$

$A_s^{(u)}(A_i^{(u)})$ is the set of useful s-attributes (i-attributes) at node α, i.e. only these attributes of α take part in the transduction of w. Our goal is to mark each node of LwR with a set $A_u\subseteq A$ which consists of the useful s-attributes of the node and the useful i-attributes of its right neighbour. (Take $A_i^{(u)}=\emptyset$ at the "right neighbour of (Lw, R, λ)".) This can be achieved by the successive application of two 1DFT as follows. The first 1DFT $\mathbf{T}_1$ starts from the right and marks each node with a pair consisting of the right dependency graph of the node and that of its right neighbour. The set of possible right dependency graphs is finite, so it can be used as the set of states for $\mathbf{T}_1$. The second 1DFT $\mathbf{T}_2$ starts from the left, and at each node first computes the left dependency graph of the node and that of its right neighbour, then from the mark put by $\mathbf{T}_1$ it is able to compute A_u and write it out as a new mark.

Let $X'\subseteq X(L, R)\times P(A)$ denote the alphabet of those marked symbols that can be achieved by the above marking process, and let $\mathbf{A}'=(A, X', L, R, Y, h', a_0)$ be the following SDAST (starting from the left).

(i) $h'(L)$ and $h'(R)$ are equal to the unit element of $\mathbf{M}(A, Y)$;

(ii) if $(x, A_u)\in X'$, then $h'((x, A_u))$ is the restriction of $h(x)$ to A_u.

$\mathbf{A}'$ is injective, because any duplication would imply a circular dependence among the useful attributes, which is impossible. (Note that if $(x, A_u)\in X'$, then there exist $w_1, w_2\in X(L, R)^*$ such that $w_1xw_2=LwR$ for some $w\in X^*$, and the set of useful attributes at the node (w_1, x, w_2) and its right neighbour is A_u.) It is also clear that the composite application of $\mathbf{T}_1, \mathbf{T}_2$ and $\mathbf{A}'$ defines the same mapping as $\mathbf{A}$. The case of a right start can be treated symmetrically.

Simulation of 1DFT by injective SDAST

Let $\mathbf{T}=(Q, X, L, R, Y, \delta, \bar{q})$ be an 1DFT starting e.g. from the left, $Q=\{q_1, ..., q_n\}$. It can be supposed without loss of generality that δ is completely defined on $Q\times X$. We shall use the following attributes to simulate $\mathbf{T}$.

$$A_s^{(n)} = \{\mathrm{s}(i,j)|1 \leqq i \neq j \leqq n\} \cup \{\mathrm{s}(i)|i\in[n]\}$$

as synthesized attributes, and

$$A_i^{(n)} = \{\mathrm{i}(i,j)|1 \leqq i \neq j \leqq n\}$$

as inherited ones.

For $A_n=A_s^{(n)}\cup A_i^{(n)}$ let $H\subseteq M(A_n, \emptyset)$ be defined as follows. $\xi\in H$ iff it satisfies the following three conditions.

(i) for every $1\leqq j<k\leqq n$ $\{\xi(\mathrm{i}(j,k)), \xi(\mathrm{i}(k,j))\}=\{\mathrm{s}(j,k), \mathrm{s}(k,j)\}$;

(ii) there exists an $i\in[n]$ such that

a) $\xi(\mathrm{s}(1))=\mathrm{s}(i)$, and

b) $\xi(\mathrm{i}(i,j))=\mathrm{s}(\min(i,j), \max(i,j))$ for every $j\neq i$;

(iii) for every $1\leqq j\neq k\leqq n$ and $i\neq 1$, $\xi(\mathrm{s}(j,k))$ and $\xi(\mathrm{s}(i))$ are undefined.

It is easy to check that the elements of H are injective. We can define an equivalence relation on H as follows. $\xi\equiv\eta$ iff $\xi(\mathrm{s}(1))=\eta(\mathrm{s}(1))$. Let η_i $(i\in[n])$ be an arbitrary representant of the equivalence class characterized by $\eta_i(\mathrm{s}(1))=\mathrm{s}(i)$.

Lemma 3. For any mapping $f\colon Q\to Q$ there exists an injective $\xi_f\in M(A_n, \emptyset)$ such that

$$f(q_i) = q_j (i, j\in[n]) \quad \text{implies} \quad \eta_i\circ\xi_f \equiv \eta_j. \tag{1}$$

Proof. We follow an induction on n to construct ξ_f. The case $n=1$ is trivial. Let $n=p+1$ for some $p\geqq 1$, and suppose first that f is injective. Then take

$$\xi_f(\mathrm{s}(i)) = \mathrm{s}(j) \quad \text{if} \quad f(q_i) = q_j,$$

$$\xi_f(\mathrm{s}(i,j)) = \mathrm{s}(i',j') \quad \text{if} \quad (f(q_i), f(q_j)) = (q_{i'}, q_{j'}),$$

$$\xi_f(\mathrm{i}(i,j)) = \mathrm{i}(i',j') \quad \text{if} \quad (q_i, q_j) = (f(q_{i'}), f(q_{j'})).$$

It is clear that (1) is satisfied this way. If f is not injective, then interchange the subscripts of the states so that $f^{-1}(q_{p+1})=\emptyset$ should hold. Let $g=f|Q\setminus\{q_{p+1}\}$, and construct $\xi_g\in M(A_p, \emptyset)$ to satisfy (1). This goes together with a reordering of $Q\setminus\{q_{p+1}\}$ that we fix from now on. Let $f(q_{p+1})=q_m$ and $g^{-1}(q_m)=\{q_{m_1}, ..., q_{m_k}\}$, where $m_i<m_j$ if $1\leqq i<j\leqq k$. We construct ξ_f in two steps.

Step 1. (i) for each $j\in[k]$

a) $\xi_f(\mathrm{s}(m_j))=\mathrm{i}(m_j, p+1)$, $\xi_f(\mathrm{s}(m_j, p+1))=\xi_g(\mathrm{s}(m_j))$,

b) $\xi_f(\mathrm{s}(p+1, m_j))$ is undefined;

(ii) $\xi_f(\mathrm{s}(p+1))=$ *if* $k=0$ *then* $\mathrm{s}(m)$ *else* $\mathrm{i}(p+1, m_k)$;

(iii) for each $j\in[k-1]$, $\xi_f(\mathrm{s}(m_k, m_j))=\mathrm{i}(p+1, m_j)$;

(iv) for any other $a\in\{\mathrm{s}(i)|i\in[p]\}\cup\{\mathrm{s}(i,j)|1\leqq i\neq j\leqq p$ and $f(q_i)=f(q_j)\}$, $\xi_f(a)=\xi_g(a)$.

It is easy to see that (iii) is in fact not a real modification of ξ_g, because $\xi_g(\mathrm{s}(m_k, m_j))$ is undefined. (i)/b assures the same situation for ξ_f. It is also clear that the segment of ξ_f defined so far is injective. After describing the first step of the construction we can prove that if $f(q_i)=q_j$, then

$$\eta_i \circ \xi_f(\mathbf{s}(1)) = \mathbf{s}(j). \tag{2}$$

If $j \neq m$, then we only have to observe that path $(\eta_i \circ \xi_f, \mathbf{s}(1)) = $ path $(\eta_i \circ \xi_g, \mathbf{s}(1))$. The abusing notation η_i can be used on both sides of this equation provided $\eta_i \in M(A_p, \emptyset)$ on the right hand side is the restriction of $\eta_i \in M(A_{p+1}, \emptyset)$ on the left hand side. Let $j=m$, $q_i \in g^{-1}(q_m)$, path $(\eta_i \circ \xi_g, \mathbf{s}(1)) = \mathbf{s}(i) \times \alpha$, where α is an appropriate sequence of attributes ending with $\mathbf{s}(m)$. Then, using (i)/a, (2) follows from the equality

$$\text{path}\,(\eta_i \circ \xi_f, \mathbf{s}(1)) = (\mathbf{s}(i), \mathbf{i}(i, p+1), \mathbf{s}(i, p+1)) \times \alpha.$$

Finally, if $i=p+1$, then for the first sight it seems possible that the last attribute of path$(\eta_i \circ \xi_f, \mathbf{s}(1))$ is $\mathbf{s}(p+1, r)$, where $q_r \in g^{-1}(q_m)$. (By (i)/b ξ_f is undefined on these attributes.) However, this would imply that the tail of this path should be $(\mathbf{s}(r), \mathbf{i}(r, p+1), \mathbf{s}(p+1, r))$, which is impossible. Thus, the last attribute of the path must be $\mathbf{s}(m)$, which is the only way out of the circle it has entered (i.e. of the set $\{\mathbf{s}(r), \mathbf{s}(r, s), \mathbf{i}(r, s) \mid r \neq s, \{q_r, q_s\} \subseteq f^{-1}(q_m)\}$).

Step 2. (i) for each $i \in [p]$

$$(\xi_f(\mathbf{i}(i, p+1)), \xi_f(\mathbf{i}(p+1, i))) = (\mathbf{s}(i, p+1), \mathbf{s}(p+1, i));$$

(ii) if $f^{-1}(q_i)=\emptyset$ for some $i \in [p+1]$, then

$$(\xi_f(\mathbf{i}(m, i)), \xi_f(\mathbf{i}(i, m))) = (\mathbf{s}(\min(m, i), \max(m, i)), \mathbf{s}(\max(m, i), \min(m, i)));$$

(iii) if $i \in [p]$, $i \neq m$ and $f^{-1}(q_i) = \{q_{i_1}, \ldots, q_{i_l}\}$ for some $l \geqq 1$, then
a) $\xi_f(\mathbf{i}(m, i)) = \mathbf{i}(i_1, p+1)$,
b) $\xi_f(\mathbf{s}(i_1, p+1)) = \xi_g(\mathbf{i}(m, i))$,
c) $\xi_f(\mathbf{s}(i_k, p+1)) = \mathbf{i}(p+1, i_{k-1})$ if $1 < k \leqq l$,
d) $\xi_f(\mathbf{s}(p+1, i_k)) = \mathbf{i}(i_{k+1}, p+1)$ if $1 \leqq k < l$,
e) $\xi_f(\mathbf{s}(p+1, i_l)) = \mathbf{s}(\min(m, i), \max(m, i))$,
f) $\xi_f(\xi_g^{-1}(\mathbf{s}(\min(m, i), \max(m, i)))) = \mathbf{i}(p+1, i_l)$;

(iv) for any other $a \in \{\mathbf{i}(i, j) \mid 1 \leqq i \neq j \leqq p\} \cup \{\mathbf{s}(i, j) \mid 1 \leqq i \neq j \leqq p$ and $f(q_i) \neq f(q_j)\}$, $\xi_f(a) = \xi_g(a)$.

Again, let $i, j \in [p+1]$, $f(q_i) = q_j$. We prove that
a) for every $1 \leqq r \neq s \leqq p+1$ (3)

$$\eta_i \circ \xi_f(\{\mathbf{i}(r, s), \mathbf{i}(s, r)\}) = \{\mathbf{s}(r, s), \mathbf{s}(s, r)\}, \text{ and}$$

b) for every $s \neq j$

$$\eta_i \circ \xi_f(\mathbf{i}(j, s)) = \mathbf{s}(\min(j, s), \max(j, s))$$

(3)/a follows from the fact that all the attributes but the last one of path $(\eta_i \circ \xi_f, \mathbf{i}(r, s))$ are in the set $\{\mathbf{s}(i, j), \mathbf{i}(i, j) \mid \{f(q_i), f(q_j)\} = \{q_r, q_s\}\}$ and there are only two ways out of this circle which lead to $\mathbf{s}(r, s)$ and $\mathbf{s}(s, r)$. To prove (3)/b we distinguish three cases.

1) $i=p+1$.
   a) $f^{-1}(q_s)=\emptyset$: consider (ii),
   b) $f^{-1}(q_s)=\{q_{s_1}, \ldots, q_{s_l}\}$ for some $l \geq 1$:

$$\mathrm{path}\,(\eta_l \circ \xi_f, \mathbf{i}(j, s)) = (\mathbf{i}(s_1, p+1), \mathbf{s}(p+1, s_1), \ldots, \mathbf{i}(s_l, p+1), \mathbf{s}(p+1, s_l), \mathbf{s}(\min(m, s), \max(m, s)));$$

2) $i \neq p+1$ and $s \neq m$.
   a) $s=p+1$: consider (i),
   b) $s \neq p+1$: $\mathrm{path}\,(\eta_l \circ \xi_f, \mathbf{i}(j, s)) = \mathrm{path}\,(\eta_l \circ \xi_g, \mathbf{i}(j, s))$;
3) $i \neq p+1$ and $s=m$.

Let $f^{-1}(q_j)=\{q_{i_1}, \ldots, q_{i_l}\}$ $(l \geq 1)$, $i=i_k$ for some $1 \leq k \leq l$ and

$$\mathrm{path}\,(\eta_l \circ \xi_g, \mathbf{i}(m, j)) = \alpha.$$

Then

$$\mathrm{path}\,(\eta_l \circ \xi_f, \mathbf{i}(m, j)) = \beta \times \alpha,$$

where $\beta=(\mathbf{i}(i, p+1), \ldots, \mathbf{i}(i_r, p+1), \mathbf{s}(i_r, p+1), \ldots, \mathbf{s}(i_1, p+1))$ for some $r \leq k$. Since the last attribute of α is $\mathbf{s}(\max(m, j), \min(m, j))$, (2)/a implies that $\xi_f(\mathbf{i}(j, m))=\mathbf{s}(\min(m, j), \max(m, j))$. Finally, (2) and (3) imply (1).

It must be noticed, however, that (1) holds only under one particular ordering of Q. Let us fix an arbitrary order, i.e. suppose that $Q=[n]$. Then by steps 1 and 2 we in fact construct $\xi_{f'}$, where $f'=\varrho^{-1} \circ f \circ \varrho$ for some bijection ϱ. Since $f=\varrho \circ f' \circ \varrho^{-1}$, we can take $\xi_f=\xi_\varrho \circ \xi_{f'} \circ \xi_{\varrho^{-1}}$. (Recall that η_i is an arbitrary representant, and the construction of ξ_ϱ and $\xi_{\varrho^{-1}}$ can be carried out directly.)

Now define $h: X(L, R) \to M(A_n, Y)$ as follows. For $x \in X$ consider the mapping $f: [n] \to [n]$ for which $f(i)=j$ if $\delta(i, x)=(j, w)$. Let
   (i) $\mathrm{attr}\,(h(x))=\xi_f$;
   (ii) for each $i \in [n]$
   $\mathrm{out}\,(h(x), \mathbf{s}(i))=w$ if $\delta(i, x)=(j, w)$;
   (iii) for each $1 \leq i \neq j \leq n$
   $\mathrm{out}\,(h(x), \mathbf{s}(i, j))=\mathrm{out}\,(h(x), \mathbf{i}(i, j))=\lambda$.

Extend h to a homomorphism of X^* into $\mathbf{M}(A_n, Y)$. An easy induction shows that for any $u \in X^*$ $\delta(i, u)=(j, w)$ implies that
   a) $\mathrm{attr}\,(\eta_i \circ h(u)) \equiv \eta_j$;
   b) $\mathrm{out}\,(\eta_i \circ h(u), \mathbf{s}(1))=w$.

Thus, to make $\mathbf{T}$ and the injective SDAST $(A_n, X, L, R, Y, h, a_0)$ equivalent we only have to set:
   (i) $a_0=\mathbf{s}(1)$;
   (ii) if $\bar{q}=i$ and $\delta(i, L)=(j, w)$, then $h(L)=\eta_j$ with the modification $\mathrm{out}\,(h(L), \mathbf{s}(1))=w$;
   (iii) $h(R)(a)$ is defined iff $a=\mathbf{s}(i)$ $(i \in [n])$ and $\delta(i, R)(=(j, w))$ is defined. In this case $h(R)(\mathbf{s}(i))=(\mathbf{s}(i), w)$.

In [3] we proved that injective SDAST mappings are closed under composition. Thus, using Lemmas 1 and 2 we get the following result.

Theorem. SDAST, injective SDAST and 2DFT define the same class of mappings.

Corollary. ([1], [2]). 2DFT mappings are closed under composition. Other results of [1] and [2] concerning 2DFT with regular lookahead (which are called quasideterministic in [2]) and the reverse run of 2DFT can also be derived from this theorem.

Abstract

The result indicated in the title is achieved as a corollary of the following four statements.

1. 2-way deterministic finite state transducers (2DFT) and simple deterministic attributed string transducers (SDAST) are equivalent.
2. Every SDAST mapping is the composition of two 1DFT mappings and an injective SDAST mapping.
3. 1DFT mappings can be defined by injective SDAST.
4. Injective SDAST mappings are closed under composition.

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