Bibliographie.

Joseph Fels Ritt, Differential algebra (American Mathematical Society Colloquium Publications, Volume XXXIII), VIII + 184 pages, New York, American Mathematical Society, 1950.

The subject of this interesting book has its origin in an algebraic foundation of differential equations given by the author in his earlier book *Differential equations from the algebraic standpoint,* published in 1932 in the same Publications. Since that time a number of new contributions were made by several authors under the leading of Prof. Ritt, giving thereby „fresh substance and new color to the subject." Therefore, one may be very greatful to the author for having presented an up-to-date discussion of the subject in this accurately and clearly written excellent book.

In the theory, there is a base field $\mathcal{F}$ of characteristic 0 in which an operation ′ called differentiation is defined such that $(a+b)' = a' + b'$ and $(ab)' = a'b + ab'$. This differential field $\mathcal{F}$ serves for the coefficient domain of so-called differential polynomials (d. p.), involving a finite number of indeterminates $y_1, \ldots, y_n$ and their derivatives $y_i^{(j)} (j = 1, 2, \ldots)$. An algebraic ideal in the ring of these d. p. is called a differential ideal if it is closed under differentiation. The notion of perfect ideals is fundamental in the investigations a perfect deal is one which coincides with its own radical, i. e., it contains all d p. which have a power in the ideal. For perfect ideals the analogue of Hilbert's basis theorem holds. This is the Ritt—Raudenbush basis theorem which states that every perfect ideal has a finite base, a fact which occupies a central position in the discussions. (This implies at once that any infinite system of simultaneous differential equations in finitely many unknowns is equivalent to a finite system.)

Chapter II is devoted to algebraic differential manifolds. A theory analogous to the known case of polynomial ideals is developed; for example it is shown that each differential manifold is the union of a finite number of irreducible ones. The next chapter deals with some structural problems concerning a single d. p. and then applications are made to systems of pure algebraic equations. In three chapters the author discusses constructive methods, the special case of analytical functions and the intersections of differential manifolds. The following chapter contains an interesting existence theorem due to Riquier and the final Chapter 9 is concerned with an algebraic discussion of partial differential equations. It is shown that many of the previous results retain their validity even in this case.

We are convinced that this valuable work will exert an influence in a considerable extent to the future development of the theory of differential algebra.

L. Fuchs.

O. F. G. Schilling, The theory of valuations (Mathematical Surveys, Number IV), VIII + 254 pages, New York, American Mathematical Society, 1950.

The recognition of the analogy between the theory of algebraic number fields and the theory of algebraic functions of a single variable may be considered as a source of the present subject. It was K. Hensel who discovered the p-adic numbers corresponding to the power series expansions in a point of a Riemann surface. The first systematic valuation-theoretic discussion is due to J. Kürschák. Of the numerous authors whose ideas are of fundamental importance in the valuation theory we mention only the names of A. Ostrowski, W. Krull, C. Chevalley and the author himself

The present book contains a clear and systematic development of the theory of valuations and its applications to several questions of algebra. The advantage of a unified valuation-theoretic treatment is well exhausted, making clear the power of the methods in the simplifications of certain discussions. The book presents a very rich material in a clear style.

The beginning chapter deals with the definitions and basic properties, defined for the non-commutative case. One of the most important concepts is the rank 1 valuation, with the characterizing property of having a value group satisfying the archimedean axiom. Then a general method is given how to extend a field of rank 1 valuation so as to obtain a larger field which is complete with respect to this valuation. Of fundamental importance is Hensel's reducibility lemma, on which a large part of valuation theory is based. The ramification theory of valuations, in Chapter 3, generalizes Hilbert's ramification theory of algebraic number fields. Among several interesting results some existence theorems for certain fields with a prescribed Galois group are proved. Next the ideal theory of algebraic number fields and fields of algebraic functions of one variable is treated from the viewpoint of valuations; the ideal theory for the infinite case is also included. Then the classical ideal-theoretic results are extended to simple algebras of finite rank over a complete field. Chapter 6 is devoted to an extensive discussion of the local class field theory treated by means of the theory of algebras. In the final chapter the author investigates the structure of complete fields by topological methods. Two appendices are added, one on the infinite Galois theory and one collecting the needed facts on algebras over a field. Each chapter closes with a bibliography which enhances the value of this excellent work.

L. Fuchs.