Correction to my paper "Systems of equations over modules".*)

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L. KOVÁCS has kindly called my attention to the fact that in the solving formula occurring in Theorem 8 of my paper (*) the parameters cannot be chosen quite arbitrarily. As a consequence Corollaries 1 and 2 of Theorem 8 also loose their validity. Here we are giving the appropriate correction.

Let R be a semi-simple ring and $[M,\varphi]$ a compatible system of equations

(1) $$f_\beta(\ldots, x_\alpha, \ldots) = g_\beta \qquad (\in G;\ \alpha \in A;\ \beta \in B)$$

over the unitary R-module G_1.[1]) If F is the free unitary R-module generated by the free generators $x_\alpha\,(\alpha \in A)$ then by Lemma 5 of (*) a direct representation

(2) $$F = M + N$$

holds, where

(3) $$N = \sum_{\delta \in D} \{s_\delta x_\delta\} \qquad (s_\delta \in R)$$

D being a subset of the index set A. By the proof of Theorem 7 in (*) there exists a solution

(4) $$x_\alpha = c_\alpha \qquad (\alpha \in A)$$

of the system of equations (1), c_α being a linear combination over R of a finite number of the g_β's.

Consider now the homogeneous system $[M,\psi]$ of equations corresponding to the system $[M,\varphi]$, i. e. let $M\psi = 0$. In order to get all solutions of this system we construct all possible R-homomorphisms $\overline{\psi}$ of the free unitary R-module F into G_1 for which

(5) $$M\overline{\psi} = 0$$

holds. In view of (2) and (3) $\overline{\psi}$ is determined by the images

(6) $$(s_\delta x_\delta)\overline{\psi} = h_\delta\,(\in G_1;\ \delta \in D)$$

*) *Acta Sci. Math.*, **18** (1957), 207—234. In the sequel this paper will be denoted by (*).

[1]) We are using the notations and terminology of the paper (*).

where $O(h_\delta) \supseteq O(s_\delta x_\delta)$.[2]) On the other hand any system of elements $h_\delta (\in G_1, \delta \in D)$ for which $O(h_\delta) \supseteq O(s_\delta x_\delta)$, induces with ψ by (6), (3) and (2) an R-homomorphism $\overline{\psi}$ of F into G_1 with (5). Since, by (2) and (3) there holds a representation

(7) $$x_\alpha = m_\alpha + \sum_{\delta \in D} d_{\alpha\delta}(s_\delta x_\delta) \qquad (m_\alpha \in M;\ d_{\alpha\delta} \in R)$$

for the elements $x_\alpha (\in F;\ \alpha \in A)$,[3]) it follows moreover from (6) and (5) that all solutions in G_1 of the homogeneous system $[M, \psi]$ of equations are obtained from the formulae

$$b_\alpha = x_\alpha \overline{\psi} = \sum_{\delta \in D} d_{\alpha\delta} h_\delta,$$

where the parameters h_δ running over G_1 are subjected only to the conditions $O(h_\delta) \supseteq O(s_\delta x_\delta)$. Taking into account the fact that (4) is one of the solutions of the system of equations (1) we obtain

Theorem 8. *If R is a semi-simple ring, then any compatible system $[M, \varphi]$ of equations* (1) *over the arbitrary unitary R-module G_1 possesses a solution in G_1 and all solutions in G_1 can be obtained by the system of formulae*

(8) $$x_\alpha = c_\alpha + \sum_{\delta \in D} d_{\alpha\delta} h_\delta,$$

where the parameters h_δ run over G_1, being bound only to fulfil the conditions $O(h_\delta) \supseteq O(s_\delta x_\delta)$ $(\delta \in D)$; the constants s_δ, $d_{\alpha\delta} (\in R)$ are defined by (3) *and* (7) *resp. (depending on the direct representation* (2)*), and the constants $c_\alpha (\in G_1)$ are (finite) linear combinations over R of the elements g_β standing on the right-hand side of the equations* (1).

In the special case $G_1 = R_{(R)}$ we have

Corollary 1. *An arbitrary compatible system of linear equations over a semi-simple ring is solvable in the ring and all solutions are yielded by the system of formulae* (8).

Another immediate consequence of Theorem 8 is

Corollary 2. *If R is a semi-simple ring, then a compatible system $[M, \varphi]$ of equations over an arbitrary unitary R-module G_1 admits exactly one solution in G_1 if and only if for a direct representation* (2) *with* (3) *and for all elements $h (\in G)$ and all indices $\delta (\in D)$ the relation $O(h) \supseteq O(s_\delta x_\delta)$ implies $h = 0$.*

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[2]) If y is an element of an R-module, then $O(y)$ denotes the set of all those elements r of R for which $ry = 0$. $O(y)$ is always a left ideal of R.

[3]) Naturally for a (fixed) α there are but a finite number of δ's for which $d_{\alpha\delta} \neq 0$.