Semigroups in which every proper subideal is a group

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To Professor L. Rédei on the occasion of his sixtieth birthday

Let S be a semigroup. A left ideal of S is a non-vacuous subset $L \subset S$ for which $SL \subset L$ holds. A right ideal is a subset $R \subset S$ with $RS \subset R$. A subset which is both a left and right ideal of S is called a two-sided ideal of S.

If L_1, L_2 are left ideals of S, their union $L_1 \cup L_2$ and their intersection $L_1 \cap L_2$, if it is non-vacuous, is again a left ideal of S. A left ideal L of S is called a minimal left ideal of S if there does not exist a left ideal L' of S such that $L' \subsetneqq L$ holds. The intersection of two minimal left ideals is the empty set. Analogous statements hold for right ideals.

Every semigroup which is not a group contains at least one left or right proper subideal.

Definition. A semigroup S is called to be an *F-semigroup* if it is not a group, but every left and right proper subideal of S is a group.

The purpose of this paper is to describe the structure of all F-semigroups. This is a generalization of a problem treated by POLLÁK and RÉDEI [4] who dealt with semigroups in which every proper subsemigroup is a group.

In section 1 we prove some preliminary lemmas needed in the following. In section 2 we describe the construction of two classes of semigroups that will turn out to be F-semigroups. Section 3 is devoted to the proof of the main Theorem 1. In section 4 we show that the result of [4] is a simple consequence of Theorem 1.

1.

Lemma 1. *Let L be a left ideal of the semigroup S and G a group contained in S. If $L \cap G \neq \emptyset$, then $G \subset L$.*

Proof. Let be $a \in L \cap G$. Then $G = Ga \subset SL \subset L$, q. e. d.

An analogous result holds for right ideals.

Every F-semigroup contains at least one minimal left ideal. For, if S does not contain any proper left subideal of S, then the semigroup S is itself a minimal left ideal of S. If S contains a proper left subideal $L \subsetneqq S$, then, by supposition, L is a group and since a group cannot contain as a proper subset an ideal of S, L is a minimal left ideal of S.

Lemma 2. *Let S be an F-semigroup. Then only one of the following cases can occur:*

A) *either S has a unique minimal proper left subideal;*

B) *or S contains precisely two different minimal left ideals $L_1 \subsetneqq S$, $L_2 \subsetneqq S$ and $S = L_1 \cup L_2$ holds;*

C) *or S does not contain any left ideal $\neq S$ at all.*

Proof. S cannot contain more then two distinct minimal left ideals. For, if there were at least three distinct minimal left ideals, say L_1, L_2, L_3, we would have $S \supset L_1 \cup L_2 \cup L_3 \supsetneqq L_1 \cup L_2 \supsetneqq L_1$. But then $L_1 \cup L_2$ would be a proper subideal of S, which is not a group.

If S contains two different minimal left ideals, say L_1 and L_2, then $S \supset L_1 \cup L_2 \supsetneqq L_1$ implies $S = L_1 \cup L_2$. This proves our Lemma.

Remark 1. We shall see that each of these possibilities really occurs.

Remark 2. Needless to say that an analogous result holds for minimal right ideals.

We recall the following well known fact:

Lemma 3. *If L is a minimal left ideal of a semigroup S and $a \in S$, then La is also a minimal left ideal of S.*

Proof. Suppose that K is a left ideal of S with $K \subset La$. Let be $L_1 = \{x | x \in L, xa \in K\}$. If $z \in S$, we have $zxa \in K$ so that $zx \in L_1$. Hence L_1 is a left ideal. Since L is minimal, we have $L_1 = L$, hence $La = K$, and La has no proper left subideal.

A left ideal L is called to be maximal if there is no left ideal L' such that $L \subsetneqq L' \subsetneqq S$. Maximal right and two-sided ideals are defined analogously.

The following lemma will be useful:

Lemma 4. *Let M be a maximal two-sided ideal of S which is not contained as a proper subset in a left or right ideal of S and different from S. Then*

a) *either $S - M$ is a group;*

b) *or $S - M$ contains a unique element u, with $u^2 \in M$.*

Remark. Analogous theorems have been proved in the paper [6].

Proof. a) Suppose first that $S-M$ contains at least two elements. Denote $S-M=G$ and choose an element $a\in G$. The left ideal $M\cup\{a\}\cup Sa$ contains M and a, hence $M\cup\{a\}\cup Sa=S$. Since $S-M$ contains more than one element the left ideal $M\cup Sa$ contains M as a proper subset, hence we have also $M\cup Sa=S$. If $x\in M$, we have $M\cup Sx\subset M\cup SM\subset M$. Therefore the set G is characterized by the property that $G=\{x|x\in S, M\cup Sx=S\}$.

We show first that G is a semigroup. To this end it is sufficient to prove: If $M\cup Sa=S, M\cup Sb=S$, then we have also $M\cup Sab=S$. This follows in the following manner: Multiplying the first relation by b we get $Mb\cup Sab=Sb$. Hence $M\cup Mb\cup Sab=M\cup Sb$, i. e. $M\cup Sab=S$.

The relation $M\cup Sa=S$ (which is true for every $a\in G$) can be written in the following manner: $M\cup[M\cup G]a=M\cup G$, i. e. $M\cup Ga=M\cup G$. Since $M\cap G=\emptyset$, we have $Ga\supset G$. On the other side G is a semigroup, hence $Ga\subset G^2\subset G$. Therefore $Ga=G$. Analogously we can prove $aG=G$. The equations $Ga=G, aG=G$ for every $a\in G$ imply that G is a group.

b) Suppose next that $S-M$ contains a unique element, $S-M=\{u\}$. If u is an idempotent, $\{u\}$ forms itself a group. If u is not an idempotent, we have necessarily $u^2\in S-\{u\}=M$. This proves Lemma 4.

2.

In this section we deal with the construction of two types of semigroups that will be needed in section 3. Analogous constructions (in entirely other connections) have been studied previously by CLIFFORD [1], HEWITT [2] and HEWITT—ZUCKERMAN [3].

Lemma 5. *Let G_1 and G_0 be two disjoint groups. Let φ_{10} be a homomorphic mapping of the group G_1 into the group G_0. Let further φ_{00} and φ_{11} denote the identical automorphisms of the groups G_0, G_1, respectively. Consider the set $S=G_0\cup G_1$ in which we introduce a multiplication $\odot$ by the following definition: If $a_i\in G_i, b_j\in G_j$ $(i,j=0,1)$ let be:*

$$a_i\odot b_j=\varphi_{i,ij}(a_i)\varphi_{j,ij}(b_j). \tag{1}$$

Then S is a semigroup with the unit element equal to the unit element of the group G_1.

Remark. The multiplication is defined in such a manner that inside the groups G_0, G_1 it is identical with the original multiplication in these groups. If namely $i=j=1$, i. e. $a_1\in G_1, b_1\in G_1, a_1\odot b_1=\varphi_{11}(a_1)\varphi_{11}(b_1)=a_1b_1$. If $i=k=0$, i. e. $a_0\in G_0, b_0\in G_0$, we have $a_0\odot b_0=\varphi_{00}(a_0)\varphi_{00}(b_0)=a_0b_0$.

Proof. a) For $i \geqq j \geqq k$ $(i, j, k = 0, 1)$ we have clearly

$$\varphi_{ik} = \varphi_{jk}\, \varphi_{ij}. \tag{2}$$

b) Let be $a_i \in G_i, b_j \in G_j, c_k \in G_k$ $(i, j, k = 0, 1)$. Then we have

$$(a_i \odot b_j) \odot c_k = [\varphi_{i,ij}(a_i)\varphi_{j,ij}(b_j)] \odot c_k.$$

Since the expression in the bracket on the right hand side is contained in the group G_{ij} we can further write

$$(a_i \odot b_j) \odot c_k = \varphi_{ij,ijk}[\varphi_{i,ij}(a_i)\varphi_{j,ij}(b_j)] \cdot \varphi_{k,ijk}(c_k).$$

Since $ij \geqq ijk$ $\varphi_{ij,ijk}$ is one of our three mappings $\varphi_{11}, \varphi_{10}, \varphi_{00}$. Further, according to (2) we have

$$\varphi_{ij,ijk}\,\varphi_{i,ij} = \varphi_{i,ijk}, \quad \varphi_{ij,ijk}\,\varphi_{j,ij} = \varphi_{j,ijk}.$$

Therefore

$$(a_i \odot b_j) \odot c_k = [\varphi_{i,ijk}(a_i)\varphi_{j,ijk}(b_j)]\varphi_{k,ijk}(c_k). \tag{3}$$

Analogously we prove

$$a_i \odot (b_j \odot c_k) = \varphi_{i,ijk}(a_i)[\varphi_{j,ijk}(b_j)\varphi_{k,ijk}(c_k)]. \tag{4}$$

Since each of the factors on the right hand side of the equations (3) and (4) is contained in the group G_{ijk}, and the multiplication in this group is associative, we have really $(a_i \odot b_j) \odot c_k = a_i \odot (b_j \odot c_k)$, i. e. S is a semigroup.

c) If e_1 is the unit element of the group G_1, we have for $b_k \in G_k$ (according to (1))

$$e_1 \odot b_k = \varphi_{1,k}(e_1)\varphi_{kk}(b_k) = \varphi_{1,k}(e_1)\, b_k.$$

For $k=1$ we have $e_1 \odot b_1 = \varphi_{11}(e_1) b_1 = e_1 b_1 = b_1$. For $k=0$ (since $\varphi_{10}(e_1) = e_0$) we have $e_1 \odot b_0 = \varphi_{10}(e_1) b_0 = e_0 b_0 = b_0$. Hence $e_1 \odot b_k = b_k$. Analogously we can prove $b_k \odot e_1 = b_k$, i. e. e_1 is the unit element of the semigroup S. This proves our lemma.

Definition. The semigroup S obtained by means of the construction from Lemma 5 will be denoted by $S = S[G_1, G_0; \varphi_{10}]$.

The notation is chosen to emphasize the means needed for the construction of S.

Corollary 5. *In the semigroup* $S[G_1, G_0; \varphi_{10}]$ *the homomorphism* φ_{10} *is uniquely determined by the relation: For* $a \in G_1$ *we have* $\varphi_{10}(a) = a \odot e_0$.

Proof. For $a_1 \in G_1$ we have [by (1)]

$$a_1 \odot e_0 = \varphi_{10}(a_1)\varphi_{00}(e_0) = \varphi_{10}(a_1) e_0.$$

Since $\varphi_{10}(a_1) \in G_0$, we have $a_1 \odot e_0 = \varphi_{10}(a_1)$, q. e. d.

L e m m a 6. *Every semigroup of the type* $S[G_1, G_0; \varphi_{10}]$ *is an F-semigroup in which* G_0 *is the unique proper two-sided subideal.*

P r o o f. First, it is clear that G_0 is a two-sided subideal of S which being a group cannot contain as a proper subset a subideal of S. Next, any ideal which contains an element $\in G_1$ contains the whole group G_1, hence it contains also e_1 and it is equal to S. Therefore our semigroup contains a unique proper subideal which is a group.

L e m m a 7. *Let* G *a group and* u *an element non* $\in G$. *Let* b *be a fixed chosen element,* $b \in G$. *Consider the set* $S = G \cup \{u\}$ *with the multiplication* $\odot$ *defined as follows:*

a) *for* $x, y \in G$ *let be* $x \odot y = xy$;

b) $u \odot u = b^2$;

c) *for* $x \in G$ *let be* $u \odot x = bx$ *and* $x \odot u = xb$. *Then* S *is a semigroup.*

P r o o f. Let be $\xi \in S$. Define

$$\bar{\xi} = \begin{cases} \xi & \text{for } \xi \in G, \\ b & \text{for } \xi = u. \end{cases}$$

In both cases we have $\bar{\xi} \in G$. For every couple $\xi, \eta \in S$ we have clearly $\xi \odot \eta = \bar{\xi}\bar{\eta}$. Therefore for arbitrary three elements $\xi, \eta, \zeta \in S$ we have

$$\xi \odot (\eta \odot \zeta) = \bar{\xi}(\overline{\eta\zeta}) = \bar{\xi}(\bar{\eta}\bar{\zeta}) = (\bar{\xi}\bar{\eta})\bar{\zeta} = (\overline{\xi\eta})\bar{\zeta} = (\xi \odot \eta) \odot \zeta.$$

Hence S is a semigroup.

D e f i n i t i o n. The semigroup constructed in Lemma 7 will be denoted by $S = S[G, u; b]$.

C o r o l l a r y 7. *In the semigroup* $S[G, u; b]$ *the element* b *is uniquely determined by the equation* $b = u \odot e = e \odot u$, *where* e *is the unit element of the group* G.

P r o o f. Puting $x = e$ we get [according to c)] $u \odot e = be, e \odot u = eb$. But since $b \in G$, we have $eb = be = b$; hence $b = u \odot e = e \odot u$.

L e m m a 8. *Every semigroup of the type* $S[G, u; b]$ *is an F-semigroup. Its unique proper (two-sided) ideal is the group* G.

P r o o f. Clearly, G is a two-sided ideal of S. Since G is a group, it is at the same time the minimal two-sided ideal of S. Every ideal I of S different from S must contain the element u. But then we have also $u^2 \in I$, hence $G \cap I \neq \emptyset$. By Lemma 1, we have then necessarily $G \subset I$, hence $S = G \cup \{u\} \subset I$, i. e. $S = I$. Hence G is the unique proper subideal of S.

Remark. If we choose in Lemma 7 for b different elements $\in G$ the semigroups thus obtained need not be isomorphic. This can be shown on simple examples. Let G be the group of second order $G=\{e, a\}$ and choose first $b=e$. Then $S_1=S_1[G, u; e]$ has the following multiplication table:

	e	a	u
e	e	a	e
a	a	e	a
u	e	a	e

Choose next in the same group $b=a$. Then $S_2=S_2[G, u; a]$ has the multiplication table:

	e	a	u
e	e	a	a
a	a	e	e
u	a	e	e

The semigroups S_1 and S_2 are neither isomorphic nor antiisomorphic.

3.

The following theorem gives a solution of the problem mentioned in the introduction.

Theorem 1. *A semigroup is an F-semigroup if and only if it is isomorphic with a semigroup belonging to one of the following classes of semigroups:*

a) *the class of semigroups of the type* $S[G_1, G_0; \varphi_{10}]$ *(see Lemma 5);*

b) *the class of semigroups of the type* $S[G, u; b]$ *(see Lemma 7);*

c) *the class of semigroups of the form* $G\times H$, *where* G *is a group and* $H=\{e_1, e_2\}$ *is a semigroup in which* $e_i e_k=e_i$ $(i, k=1, 2)$;

d) *the class of semigroups of the form* $G\times H'$, *where* G *is a group and* $H'=\{e_1, e_2\}$ *is a semigroup in which* $e_i e_k=e_k$ $(i, k=1, 2)$.

Proof. According to Lemma 2 we have to consider three cases A, B, C.

Case A. Let S be an F-semigroup and suppose that it has a unique minimal proper left subideal L.

Let be $a\in S$. Since La is a minimal left ideal of S (see Lemma 3), we have $La=L$, i. e. $LS=L$; hence L is a two-sided ideal of S. The ideal L cannot be contained in a left (right) ideal L' of S such that $L\subsetneqq L'\subsetneqq S$ holds. For, the ideal L' would be a group and since $\emptyset\neq L\subset L'\cap L$ Lemma 1 would imply $L'\subset L$, i. e. $L'=L$, which is a contradiction. Hence L is a maximal two-sided ideal of S which is not properly contained in a left or right ideal of S different from S.

By Lemma 4, there are two possibilities which are necessary to investigate separately.

a) Let $S-L=G_1$ be a group. Then $S=L\cup G_1$ is a union of two disjoint groups.

Let e_0 and e_1 be the unit elements of the groups L and G_1. If $a\in G_1$, we have $ae_0\in aL=L$. The mapping

$$\psi_{10}:\ a\in G_1\rightarrow ae_0\in L \tag{5}$$

is a homomorphic mapping of the group G_1 into the group L. If namely $a\in G_1$, $b\in G_1$ and $a\rightarrow ae_0$, $b\rightarrow be_0$, we have $ab\rightarrow abe_0=a(be_0)=a[e_0(be_0)]=$ $=(ae_0)(be_0)$.

If further $a\in G_1$, $b\in L$, we have

$$ab=a(e_0b)=(ae_0)b=\psi_{10}(a)b,$$
$$ba=(be_0)a=b(e_0a)=[b(e_0a)]e_0=(be_0)(ae_0)=b(ae_0)=b\psi_{10}(a).$$

Put — for a while — $L=G_0$ and denote by ψ_{00} and ψ_{11} the identical automorphisms of the groups $L=G_0$ and G_1. We then have for $a_i\in G_i$ and $b_j\in G_j$

$$a_i b_j=\psi_{i,ij}(a_i)\psi_{j,ij}(b_j).$$

Hence, in the notations of Lemma 5, we have necessarily $S=S[G_1,L;\psi_{10}]$. Hereby, in accordance with Corollary 5, ψ_{10} is defined by the relation (5).

Conversely, we know from Lemma 6 that $S[G_1,L;\psi_{10}]$ is an F-semigroup.

b) Let $S-L=\{u\}$, where $u^2\in L$. Denote by e the unit element of the group L. Denote further $b=ue$.

Since $b\in uL\subset L$, we have $b=eb=eue$. Since $eu\in Lu=L$ we have $(eu)e=eu$. Hence we have also $b=eu$.

Further $u^2\in L$ implies $u^2=eu^2e=(eu)(ue)=b^2$.

Finally for $x\in L$ we have $ux=u(ex)=(ue)x=bx$ and $xu=(xe)u=$ $=x(eu)=xb$. Our semigroup is necessarily of the type $S[G,u;b]$, where, in accordance with Corollary 7, we have $b=ue$.

Conversely, we know (see Lemma 8) that every semigroup of the type $S[G,u;b]$ is an F-semigroup.

Case B. Let S be an F-semigroup. Suppose that it contains precisely two minimal left ideals L_1, L_2. Then, by Lemma 2, we have necessarily $S=L_1\cup L_2$. Denote by e_1, e_2 the unit elements of the groups L_1 and L_2.

We show first that S cannot contain a proper right subideal $R \neq S$. If R is a right ideal of S, we have $\emptyset \neq RL_1 \subset R \cap L_1$. Since the group L_1 has a non-empty intersection with the right ideal R, we have (by Lemma 1) $L_1 \subset R$. Analogously $L_2 \subset R$. Hence $S = L_1 \cup L_2 \subset R$, i. e. $S = R$.

S is therefore a so called right simple semigroup containing idempotents.

It is known (see f. i. [5]) that in every right simple semigroup T containing idempotents every idempotent is a left unit and the semigroup itself is a union of disjoint isomorphic groups. The set of left units $H \subset T$ forms clearly a subsemigroup of T. Further it is known that the semigroup T is isomorphic to the direct product $G \times H$, where G is a group (namely the abstract group isomorphic to the groups whose union is T itself).

In our case the right simple semigroup S contains two idempotents e_1, e_2, hence we have necessarily $S \cong G \times H$, where G is a group and $H = \{e_1, e_2\}$ has the multiplication table

$$\begin{array}{c|cc} \cdot & e_1 & e_2 \\ \hline e_1 & e_1 & e_2 \\ e_2 & e_1 & e_2 \end{array} . \tag{6}$$

The left ideals L_1, L_2 of S are then isomorphic to the group G.

Conversely, if G is an arbitrary group and H a semigroup with the multiplication table (6), then $G \times H$ is a semigroup without a proper right subideal. It contains precisely two proper left subideals, namely $G \times \{e_1\}$ and $G \times \{e_2\}$, both being groups (and both isomorphic to G). Hence $G \times H$ is an F-semigroup.

Case C. Suppose that S is an F-semigroup which does not contain a proper left subideal. Hence $Sa = S$ for every $a \in S$.

Let R be a minimal right ideal of S. The set $SR = \bigcup_{a_\nu \in S} a_\nu R$ is a two-sided ideal of S, hence $SR = S$. Since, by Lemma 3, every summand $a_\nu R$ is a minimal right ideal of S, we conclude that S is the union of its minimal right ideals. By Lemma 2 (formulated for right ideals) we conclude further that a) either S does not contain a proper right subideal at all, b) or S is the sum of two minimal right ideals of S (each of which is a group).

a) The case that S does not contain a proper right subideal is impossible. For then we would have also $aS = S$ for every $a \in S$. The relations $Sa = S$, $aS = S$ for every $a \in S$ imply that S is a group, contrary to the supposition that S is an F-semigroup.

b) In the second case, if $S = R_1 \cup R_2$, and R_1, R_2 are two different minimal right ideals of S, we can use the result proved sub B by interchanging the role of left and right ideals. If e_1, e_2 are the unit elements of the groups R_1, R_2, we conclude that the semigroup is necessarily isomorphic to the direct product $G \times H'$, where G is a group and H' is a semigroup with the multiplication table

$$\begin{array}{c|cc} \cdot & e_1 & e_2 \\ \hline e_1 & e_1 & e_1 \\ e_2 & e_2 & e_2 \end{array} \tag{7}$$

Conversely, every semigroup of the type $G \times H'$, where G is a group and H' is a semigroup with the multiplication table (7), is an F-semigroup without proper left subideals, containing precisely two proper right subideals each of which is a group.

This completes the proof of Theorem 1.

4.

In this section we show that the result of paper [4] is an immediate consequence of Theorem 1.

We shall use the following notations.

Let S be a semigroup and $a \in S$. The cyclic subsemigroup of S generated by a will be denoted by $[a]$. An element $a \in S$ is called to be of finite order if $[a]$ is contains only a finite number of different elements. If every element of S is of finite order, S is called a torsion semigroup. If a is of finite order, $[a]$ is called to be of the type (m, n), if n is the least integer such that there is an integer $m < n$ with $a^m = a^{n+1}$. If $[a]$ is of the type (m, n), $[a]$ contains exactly n different elements and it is well known that $\{a^m, a^{m+1}, \ldots, a^n\}$ is the greatest group contained in $[a]$.

D e f i n i t i o n. A semigroup S is called to be an E-semigroup if every proper subsemigroup of S is a group.

T h e o r e m 2 (POLLÁK—RÉDEI [4]). *A semigroup is an E-semigroup if and only if S belongs to one of the following types of semigroups:*

a) *S is a torsion group;*

b) *S is a cyclic semigroup $[a]$ of the type $(2, n)$, where $n > 2$ is an integer;*

c) *$S = \{e_1, e_0\}$, where $e_0^2 = e_0 e_1 = e_1 e_0 = e_0$, $e_1^2 = e_1$;*

d) *$S = \{e_1, e_2\}$, where $e_i e_k = e_i$ for $i, k = 1, 2$;*

e) *$S = \{e_1, e_2\}$, where $e_i e_k = e_k$ for $i, k = 1, 2$.*

P r o o f. An E-semigroup is clearly a torsion semigroup. For if there were an $a \in S$ which is not of finite order, then $[a] = \{a, a^2, \ldots\}$ would contain the subsemigroup $\{a^2, a^3, \ldots\}$ which is not a group.

An E-semigroup is necessarily either a group or an F-semigroup. Since a torsion group is clearly an E-semigroup, we have only to discuss the four cases of Theorem 1.

a) Let $S=S[G_1, G_0; \varphi_{10}]$ and suppose that S is an E-semigroup. Let e_1, e_0 be the unit elements of the groups G_1 and G_0. The two-element set $T_1=\{e_1, e_0\}\subset S$ with the multiplication table

	e_1	e_0
e_1	e_1	e_0
e_0	e_0	e_0

forms a semigroup which is not a group. Hence $S=T_1$. Conversely, T_1 is clearly an E-semigroup.

b) Let $S=S[G, u; b]$ and suppose that S is an E-semigroup. Consider the cyclic semigroup $[u]\subset S$. Since $[u]$ is not a group (i. e. it is not of the type $(1, n)$), we have necessarily $[u]=S$. If $[u]$ were of the type (m, n) with $m\geqq 3$ the semigroup $\{u^2, u^3, \dots u^n\}\subsetneqq[u]$ would be a proper subsemigroup of S which is not a group. It remains the case that $[u]$ is of the type $(2, n)$. Conversely, in this case S is obviously an E-semigroup.

c) Let be $S\cong G\times H$. Denote by e the unit element of the group G. The semigroup $\{e\}\times H$ is a subsemigroup of $G\times H$ which is not a group. Hence $S\cong\{e\}\times H$. But $\{e\}\times H\cong H$, thus $S\cong H$. Conversely, H is obviously an E-semigroup.

d) The case $S\cong G\times H'$ can be settled analogously. This completes the proof of Theorem 2.

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