Bibliographie

G. ALEXITS, **Konvergenzprobleme der Orthogonalreihen**, p. 307, Budapest, Verlag der Ungarischen Akademie der Wissenschaften, 1960.

G. ALEXITS, **Convergence Problems of Orthogonal Series**, Translated by I. FÖLDES, p. IX + 350, Budapest, Publishing House of the Hungarian Academy of Sciences, 1961.

Over the course of the recent 50 years there has been accumulated a lot of material concerning convergence and divergence problems of particular and general orthogonal series.

The first more exhaustive account on the subject is in the book „Theorie der Orthogonalreihen" of S. KACZMARZ and H. STEINHAUS which appeared 25 years ago, and therefore does not include a series of important newer contributions, mostly by Hungarian and Russian mathematicians.

It may be found therefore a fortunate event that G. ALEXITS, a distinguished specialist in this field, has published a monographic tract on this subject. ALEXITS' monograph was published in German (1960) and in English version (1961), the latter differring only slightly from the former one. Therefore, when speaking of its contents, we may consider only the English edition.

Chapter I starts from an exposition of the fundamental facts concerning orthonormal systems such as the Riesz—Fischer theorem, completeness etc., but the main stress is laid on presenting a series of important examples of orthonormal systems, more or less to the same extent as given in the book of KACZMARZ and STEINHAUS. Some sections deal with the Haar and Rademacher systems which rightly can be considered as the most interesting among the non-classical orthonormal systems. Relatively not much room has been reserved for the Walsh system, but the reader can find a series of remarks concerning it in the following chapters. It seems, besides, that the Walsh system which shows striking analogies to the trigonometric system, would deserve in the future a separate monographic tract. The author gives a particularly detailed account on the systems of orthogonal polynomials attached to a distribution $d\mu \geqq 0$, in particular to an absolutely continuous one: $d\mu = \varrho(x)dx$. Attention should be called to the local tests for convergence of expansions corresponding to polynomial systems and to the important theorem of G. FREUD giving an estimation of the sum $p_1^2(x)+p_2^2(x)+\dots+p_n^2(x)$, where $\{p_n(x)\}$ is the polynomial system belonging to a density function $\varrho(x)$ satisfying some rather general assumptions.

Chapter II deals with almost everywhere convergence and summability of an orthogonal series

$$(*) \qquad \sum_1^\infty c_n\varphi_n(x),$$

$\{\varphi_n(x)\}$ being an orthonormal system of general type, and

$$(**) \qquad \sum_1^\infty c_n^2 < \infty,$$

by imposing on the coefficients c_n several additional conditions. Typical is the use of conditions with factors $\gamma(n)$, i. e. replacing (**) by the stronger condition

$$\sum_{n=1}^\infty c_n^2\gamma(n) < \infty.$$

It is known e.g. that the above condition with $\gamma(n)=\log^2 n$ guarantees almost everywhere convergence (theorem of RADEMACHER and MENCHOFF) and with $\gamma(n)=(\log\log n)^2$ it guarantees almost everywhere summability $(C, 1)$ of $(*)$ (theorem of MENCHOFF and KACZMARZ). Assumption that the coefficients c_n vanish for certain n's leads to the investigation of the convergence of lacunary series for which the author presents a number of his own results.

It is evident that, when formulating sufficient conditions for convergence, we ought possibly investigate whether they are in fact indispensible or not. Attempts to answer the question to what degree the use of factors as e. g. $\gamma(n)=\log^2 n$ in the Rademacher—Menchoff theorem (or similarly in the Menchoff—Kaczmarz theorem) is essential, lead to the construction of certain orthonormal systems which serve as counterexamples. These constructions are very intricate. The author has successfully got through with the difficulties by limiting himself to the most interesting things, such as the recent counterexamples of K. TANDORI which are much stronger than the older results of MENCHOFF, partly contained already in the Kaczmarz—Steinhaus monography. Let us quote one of TANDORI's results whose full proof has been inserted in Chapter II: If $c_n \downarrow 0$ and $\sum c_n^2 \log n=\infty$, then there exists an orthonormal system in (a, b) such that the series $(*)$ divegres everywhere. Among the other convergence questions dealt with in Chapter II, we mention the problem concerning the almost everywhere convergence of the series $(*)$, in an arbitrary arrangement of its terms („essential" commutative convergence). The author gives, apart from theorems about the „essential" commutative convergence which go back to MENCHOFF and ORLICZ, and can be found also in a number of other monographs, some of his own results generalizing those of MENCHOFF.

I could hardly forbear at this point from quoting an extremely interesting result which has been obtained in 1961 by ULIANOFF and OLEVSKI and which could not have been included in ALEXITS' monograph. Any complete orthonormal system of functions can be arranged in such a way that there exists a square-integrable function whose series expansion with respect to the rearranged system be divergent almost everywhere. This seems to be the most general results presently known among the theorems concerning divergence. In the case of the trigonometric system this theorem was announced without proof by KOLMOGOROFF in 1927, and ZAHORSKI gave in the *Comptes Rendus Paris,* **251** (1960), 501—503, a sketch of its proof; it seems to me that this paper ought to be included in the Bibliography of ALEXITS' monograph.

In Chapter III the investigations on convergence and summability almost everywhere are continued by making use of the order of growth of Lebesgue functions. In case of the ordinary convergence the n-th Lebesgue function is defined as

$$L_n(x)=\int_a^b |K_n(x, t)|\, d\mu(t), \text{ where } K_n(x, t)=\sum_{k=1}^{n} \varphi_k(x)\varphi_k(t).$$

The known integral representation of partial sums of an orthogonal expansion by means of the kernels $K_n(x, t)$ explains the rôle of Lebesgue functions in the investigation of pointwise convergence. This leads in fact to the study of sequences of singular integrals, which is the subject of the last chapter of the book. It has been shown, however, in the paper of KOLMOGOROFF—SELIVERSTOFF—PLESSNER from 1925—26 that the order of increase of the sequence of Lebesgue functions yields an information on the order of increase almost everywhere of the partial sums of expansions of arbitrary square-integrable functions. Ideas contained in the papers by these authors have been developed in those by other mathematicians (as KACZMARZ, ALEXITS, SZ.-NAGY) and applied also to questions of Cesàro and Riesz summability. All fundamental results belonging to these problems are to be found in Chapter III. The possibility of applying general theorems depends upon information about the order of growth of the sequence of Lebesgue functions corresponding to several special orthonormal systems. The author gives attention to this problem and presents in his monograph some of the post-war results of the Hungarian mathematicians (G. ALEXITS, G. FREUD, K. TANDORI). They actually concern orthogonal systems of polynomials with a density function $\varrho(x)$, this being subject to some very general assumptions. ALEXITS' monograph presents still more general classes of systems, the so-called polynomial-like, introduced by him. Interesting applications of the method of estimation of the Lebesgue functions can be found in sections dealing with the so-called multiplicatively orthogonal systems (generalizing the Rademacher system and W-systems (generalizing the Walsh system). These sections give also results of the author published here for the first time.

The subject of Chapter IV are problems of convergence and summability considered from the classical point of view (pointwise resp. uniform convergence). One of the classical methods of investigation depends upon application of general convergence theorems for singular integrals.

The author presents some fundamental facts from the theory of Banach spaces and formulates in terms of singular integrals theorems on convergence and summability of orthogonal polynomial expansions for some typical classes of functions (it seems, by the way, that the Bibliography should quote also, apart from the fundamental paper by LEBESGUE from 1909, the important paper by H. HAHN). Next, in § 3, the author deals with convergence and $(C, 1)$-summability of orthogonal expansions at points of continuity for polynomial and polynomial-like systems, proving among others some generalizations of the theorem of FEJÉR due to ALEXITS and FREUD.

§ 4 is concerned with the phenomena of convergence at Lebesgue-points. Application is made of the theorem on „humpbacked majorant" due to FADDEIEFF. One of the classical methods of dealing with problems of convergence of Fourier series depends upon using approximation theorems in appropriate classes of functions. A good part of these investigations can be performed not only for the trigonometric system. A presentation of the newer results in this respect can be found in §§ 5, 6; applications are made to polynomial and polynomial-like systems. Among theorems which have been included here, and which are not being found in monographs concerning the constructive theory of functions, I would mention e. g. the theorem by B. SZ.-NAGY which supplies a lower estimation of the order of approximation by linear combinations of functions of an orthonormal system in case of typical classes of functions. The last § of this chapter deals with absolute convergence of orthogonal expansions; as well-known, this is also related to approximation problems. We find there also theorems concerning absolute convergence of multiplicatively orthogonal series.

The monograph has been written in a very clear and suggestive way; one reads it with pleasure. In every chapter numerous passages can be found which supply information on theorems not included. In several places interesting, sofar unsolved questions have been raised. Maybe remarks concerning the possibility of generalizations of some theorems to the case of more variables would be desirable.

ALEXITS' book will prove presumably a fundamental monograph for the specialists in convergence problems of orthogonal series. It will also contribute to a further development of the theory which, judging from the recent publications by the Hungarian, Russian, and Polish mathematicians, does not stop to be attractive. To the ever-widening circle of mathematicians it will provide once again a convincing illustration of the power of the Lebesgue integral. The reader will enjoy the extensive Bibliography at the end of the book. Graphic make-up of both editions is of a very high standard.

W. Orlicz (Poznań)

L. HOLZER, **Zahlentheorie.** Teil I—II (Mathematisch-naturwissenschaftliche Bibliothek, 13/14), 202+126 Seiten, Leipzig, B. G. Teubner Verlagsgesellschaft, 1958—1959.

Band I beginnt mit den Grundbegriffen der Zahlentheorie. Man beweist hier u. a. die Sätze von FERMAT und WILSON, die Lösbarkeit von quadratischen Kongruenzen, die Darstellbarkeit der Zahlen durch gewisse quadratische Formen usw.

Dann wird die Darstellung der ganzen Zahlen als Summen von vier Qadraten betrachtet und ein eleganter Beweis des quadratischen Reziprozitätsgesetzes mit Hilfe der Gaußschen Summen gegeben. (Die genauere Analyse dieser Summen folgt im Band II.)

Es folgt eine Theorie der ganzen algebraischen Zahlen, der kanonischen Körperbasen und der Körperdiskriminanten. Hier finden wir den Minkowskischen Diskriminantensatz, den Begriff der Idealen, die Primidealzerlegung, die Modulbasis der Ideale und die Normen der Ideale. Am Ende des ersten Bandes befindet sich ein Beweis der Endlichkeit der Klassenzahl mit den sogenannten nichttrivialen Gitterpunkten.

Der zweite Band führt die Zahlenkörpertheorie weiter. Der erste Abschnitt bringt den modernen Existenzbeweis der Dirichletschen Grundeinheiten und die Grundbegriffe der Theorie der Relativkörper. Die Hilbertsche Theorie der Galoisschen Körper wird mit ausgiebigen Beispielen (mit dem Aufbau der Kreisteilungskörper und der relativ-zyklischen Körper) erschlossen.

Die folgenden Kapitel bringen die Bestimmung der Klassenzahl durch den Regulator; hier finden wir die Abschließung der im Teil I angekündigten Vorzeichenbestimmung der Gaußschen Summen. Danach werden die Begriffe der unendlichen Primstellen, das Einheitenhauptgeschlecht der relativ-zyklischen Körper mit der Anwendung des Reduktionsprinzips und des Isomorphiesatzes von RELLA beschrieben. (Dieser schöne Satz wird hier zum ersten Male veröffentlicht.) So gelangt man durch den Satz von MORIYA zur Herstellung der Anzahl der ambigen Idealklassen, wodurch der Satz von POLLACZEK (über regulären Primzahlen) auf sehr elegante Weise gewonnen wird.

I. Seres (Budapest)

GUSTAV DOETSCH, Introduction à l'utilisation pratique de la transformation de Laplace. Traduit de l'allemand par M. PARODI. Avec un appendice: Table de correspondences par R. HERSCHEL. VIII+198 pages, Paris, Gauthier-Villars, 1959.

This book is the French translation of the German original, "Anleitung zum praktischen Gebrauch der Laplace-Transformation", and is mainly devoted to physicists, electrical engineers, who are interested in practical applications of this transformation, as an effective, though rather simple method to solve several differential equations. The book is based upon the numerous works of the author concerning the subject, but, according to the above-mentioned purpose, is somewhat different, as, in most cases, omits proofs and though the theory is explained very carefully, the treatment is, however, mainly subjected to the needs of practical applications.

Chapter 1 treats with the general definitions and some properties of the integral of Laplace:

$$f(s)=\int_0^\infty e^{-st}F(t)dt.$$

Chapter 2 deals with the rules of calculations considering the above integral as a transformation. The applications of the transformation to solve ordinary differential equations, difference equations and partial differential equations, as well as integral equations, are treated in Chapters 3, 4, 5, and 6, respectively, elucidating the general problems by examples. In Chapter 7 the methods, such as the contour integration and expansion in series, are described to find the inversion function, $F(t)$, in the knowledge of $f(s)$. The asymptotic behaviour of both $f(s)$ and $F(t)$ can be found in Chapter 8, together with the application of these properties to the problem of the stability of the physical system considered. The Appendix, compiled by R. HERSCHELL, is a useful collection of the rules of operation and contains numerous pairs $(f(s), F(t))$, where the $F(t)$ are functions of different types. Some of the newly calculated pairs when $F(t)$ is a step function, or piecewise linear function, are of special interest in impulse-technics. The table also contains the solutions of some linear differential equations of order $\leqq 3$ with constant coefficients.

J. Gyulai (Szeged)

J. W. C. CASSELS, An introduction to the geometry of numbers (Die Grundlehren der mathematischen Wissenschaften, Bd. 99), VIII+344 Seiten, Berlin—Göttingen—Heidelberg, Springer-Verlag, 1959.

The aim of this book is twofold. First to give an introduction to the geometry of numbers, and, on the other hand, to show the new methods and problems of the theory.

In the Prologue the author gives a brief summary of the concepts and typical problems which play a leading role in the sequel.

One of the most important concepts of the geometry of numbers, the concept of lattice, is introduced in Chapter I. A lattice is the set of all points of an n-dimensional real euclidean space which may be written in the form $x=u_1a_1+u_2a_2+\ldots+u_na_n$, where $a_1, a_2, \ldots, a$ are linearly independent vectors of the space and $u_1, u_2, \ldots u_n$ are integers. Chapter II deals with the method of reduction of an algebraic form in the sense of MINKOWSKI. We find the important convex body theorem of MINKOWSKI and its generalizations in Chapter III. Some important concepts are also introduced in this chapter (lattice constant, critical lattice, star body). The next chapter contains results on the existence of critical lattice of bounded convex sets and star bodies, besides it contains MAHLER's compactness theorem and its application to diophantine approximation. Chapter VI is devoted to the Minkowski—Hlawka theorem to estimate the lattice constant of a set, and its generalizations. The problem to estimate the number of linearly independent points of a set in common with a lattice is discussed in Chapter VIII. Previously in Chapter VII the concept of the quotient space is introduced, which plays an important role in Chapters VIII and XI, in the latter the inhomogeneous problems are discussed. Chapter IX is related with Chapter III. It deals with several necessary and sufficient conditions that a lattice be admissible for a set with special properties. In Chapter X the author defines the concept of the automorph of a point set (or a lattice) and gives several applications of this concept. An automorph is a homogeneous linear transformation w with the property $wS=S$ (or $w\Lambda=\Lambda$), where S is a point set (Λ is a lattice.)

There is a detailed bibliography at the end of the book.

Z. Papp (Szeged)

I. W. **BUSBRIDGE, The Mathematics of Radiative Transfer** (Cambridge Tracts in Mathematics and Mathematical Physics, No 50), XII+143 pages, Cambridge, University Press, 1960.

Originally, transfer theory was concerned with the transfer of radiation through the atmosphere of a star. Nowadays, its equations occur in many fields of physics, too. The first systematic treatment of it has been given in the Cambridge Tract of E. HOPF: *Mathematical Problems of Radiative Equilibrium* (1934). The subject having grown since then, a new survey became needed: this is the purpose of this book. It gives an account of some mathematical results of transfer theory, incorporating the most lasting parts of HOPF's tract. — Everywhere, language and notation of astrophysics are used, thus the starting point, the equation of transfer has not been derived rigorously; it is accepted as in astrophysics. Probabilistic tools have been avoided.

Part I (Auxiliary mathematics) begins with the introduction of the transfer equation, however, only some special types of this (of astrophysical background) are considered. They lead to integral equations such as

$$(\mathrm{I}) \qquad I(\tau)-\frac{\omega_0}{2}\int_0^{\tau_1} I(t)E_1(|t-\tau|)\,dt=B(\tau)$$

with given function $B(\tau)$, constants ω_0, $\tau_1 (0<\omega_0<\infty;\ 0<\tau_1\leqq\infty)$, and

$$E_1(t)=\int_1^\infty x^{-1}e^{-tx}dx$$

("Milne-equations"). Then tools like the "H-functions" $H(\mu)$, introduced by V. A. AMBARTSUMIAN and S. CHANDRASEKHAR, are dealt with; they are solutions of $\frac{1}{H(\mu)}=1-\mu\int_0^1\frac{\Psi(x)H(x)}{\mu+x}dx$ with given function $\Psi(x)$, $\Psi(x)\geqq 0$, $\int_0^1\Psi(x)dx\leqq\frac{1}{2}$. Instead of the equations (I) one studies as well the "generalized Milne-equations"

$$(\mathrm{I}^*) \qquad I(\tau)-\int_0^{\tau_1} I(\tau)K_1(|t-\tau|)dt=B(\tau)$$

with

$$K_1(t)=\int_1^\infty \Psi(x^{-1})x^{-1}e^{-xt}dx.$$

Part II (Milne-equations) begins with problems related in astrophysics to semi-infinite atmospheres and isotropic scattering. A chapter on iterative solutions of (I^*, $\tau_1=\infty$) by the Neumann series ("N-series") opens this part. If this series converges to a solution, it will be called the N-solution of (I^*, $\tau_1=\infty$). Most of this chapter — based on the book of E. HOPF — is concerned with N-solutions.

In the sequel, first a modification of the Wiener—Hopf method of solution of homogeneous equations such as the classical Milne-equation

$$F(\tau)=J(\tau)-\int_{-\infty}^{\infty} J(t)K_1(|t-\tau|)\,dt,$$

$J(\tau)=0$ for $\tau<0$, and $F(\tau)=0$ for $\tau>0$) is treated. Then the used technique is applied to the Milne-equations occurring in the book (applying H-functions). Further a technique introduced in 1942 by V. A. AMBARTSUMIAN is discussed, which differs from that of N. WIENER and E. HOPF; this consists of reducing the solution of Milne equations to that of "auxiliary equations of type (I) and (I^*) where $B(\tau)$ has been replaced by $e^{-\sigma\tau}$ ($\sigma\geqq 1$). These auxiliary equations lead to those of the H-functions. This powerful technique combined with the theory of N-solutions can be applied to solve non-homogeneous Milne-equations with general $B(\tau)$. — The last chapters deal with finite atmosphere problems in case of isotropic scattering. Here the H-functions must be replaced by two functions $X(\mu)$ and $Y(\mu)$ (introduced first by S. CHANDRASEKHAR) satisfying some integral equations. Finally, problems concerning the axially symmetric case of a semi-infinite atmosphere (at an anisotropic scattering) are considered. An Appendix sums up some unsolved or incompletely solved problems. — Due to its rigorous treatment, the book will be useful also for mathematicians interested in discussing some special integral (or integro-differential) equations.

P. Medgyessy (Budapest)

HERBERT MESCHKOWSKI, **Ungelöste und unlösbare Probleme der Geometrie**, VIII + 168 Seiten, Braunschweig, F. Vieweg & Sohn, 1960.

Das Buch zerfällt in zwölf Kapitel. In Kapitel I sind jene zehn Probleme aufgezählt, um welche sich die Themen der weiteren Kapitel gruppieren. Es werden (meistens als Einleitung) auch viele gelöste Probleme betrachtet, der Schwerpunkt liegt jedoch an gewissen ungelösten Problemen und an solchen, die lange Zeit hindurch vergeblich bestürmt wurden, über welche es aber heute schon bekannt ist, daß sie im ursprünglichem Sinne „unlösbar" sind.

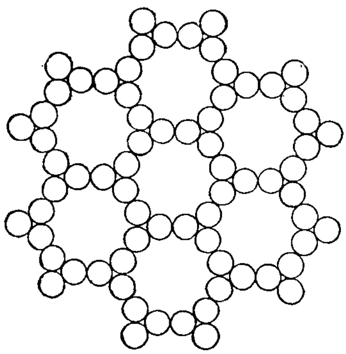

Abb. 1

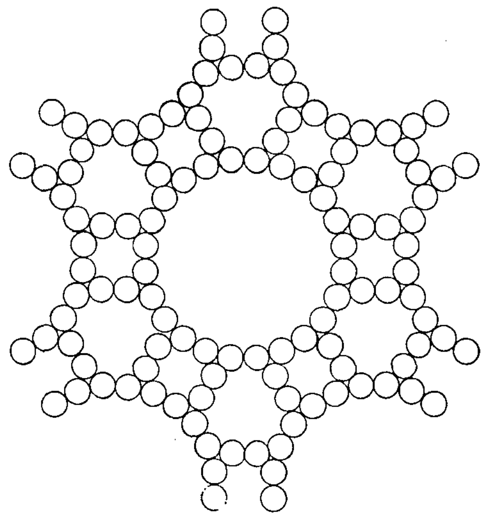

Abb. 2

Kapitel II—IV beschäftigen sich mit sogenannten Lagerungsproblemen. Hier werden Ergebnisse und Vermutungen bezüglich der dichtesten Ausfüllung bzw. dünnsten Überdeckung der Ebene und der Kugel mit kongruenten Kreisen bzw. Kugelkappen, sowie andere verwandte Probleme betrachtet. Als weiteres Problem ist die Bestimmung der dünnsten *festen* Lagerung von kongruenten Kreisen erwähnt, wo eine Lagerung von einander nicht überdeckenden Kreisen in der Ebene fest heißt, wenn jeder Kreis von mindestens drei anderen Kreisen so berührt wird, daß die Berührungspunkte auf keinem der Kreise einen abgeschlossenen Halbkreis frei lassen. Als Vermutung ist hier irrtümlicherweise ausgesprochen, daß die Lösung durch die in Abb. 1 dargestellte Konfiguration geliefert wird. In Abb. 2 geben wir eine „bessere" Lagerung, und wir halten es sogar nicht für unmöglich, daß dieses Problem in dieser Form unter die „unlösbaren" einzuteilen ist, in dem Sinne, daß es keine *dünnste* feste Lagerung gibt. — In demselben Teil befinden sich auch die analogen Probleme für den Raum.

Kapitel V enthält die Bearbeitung des noch ungelösten Lebegueschen Tafelproblems: Es ist jene ebene Figur vom kleinstmöglichen Inhalt gesucht, mit welcher man jede ebene Punktmenge vom Durchmesser 1 bedecken kann. Hier ist auch die verwandte Borsuksche Vermutung erwähnt: Jede beschränkte Punktmenge des n-dimensionalen euklidischen Raumes läßt sich in $n+1$ Teile von kleineren Durchmessern zerlegen.

In Kapitel VI behandelt der Verf. die Zerlegungsgleichheit von Polyedern auf Grund des berühmten Dehnschen Satzes.

Die Grundfrage von Kapitel VII lautet folgendermaßen: Welches ist die kleinste Anzahl inkongruenter Quadrate, in die man ein gegebenes Qadrat zerlegen kann?

Im nächsten Kapitel liest man über das „Nadel-Problem" von KAKEYA: Welcher ist der Bereich vom kleinstmöglichen Inhalt mit der Eigenschaft, daß eine Strecke der Länge 1 innerhalb dieses Bereiches so bewegt werden kann, daß ihr Richtungswinkel sich dabei um 2π ändert? Dieses Problem erwies sich „unlösbar". BESIKOWITSCH hat nämlich gezeigt, daß es Bereiche von beliebig kleinem Flächeninhalt gibt, in denen die verlangte Bewegung der „Nadel" möglich ist. Hier findet man auch interessante Probleme bezüglich Punktmengen mit ganzzahliger Entfernung.

In Kapitel IX und X werden Probleme über Konstruktionen mit Zirkel und Lineal in der Ebene bzw. auf der Kugel behandelt.

Kapitel XI ist mengengeometrischen Problemen und Paradoxen der Zerlegungsgleichheit gewidmet. Von diesen erwähnen wir als typisch den folgenden Satz: Jede Kugel des dreidimensionalen Raumes ist zu zwei punktfremden Kugeln vom gleichen Radius endlich äquivalent.

Das letzte Kapitel behandelt die „methodische Bedeutung" der „unlösbaren" Probleme.

A. Heppes (Budapest)

Proceedings of symposia in applied mathematics, Volume XII. **Structure of language and its mathematical aspects.** Edited by ROMAN JAKOBSON, VI+279 pages, Providence, R. I., American Mathematical Society, 1961.

Linguistic expression is a linar sequence of linguistic elements and, insofar, subject to the laws which govern linear arrangements in general. The linguist who wishes for certain reasons to make use of that property of language must again acquaint himself with the laws governing such arrangements. Hence the present work is of primary interest for linguists. But it is addressed to mathematicians with at most an amateur interest in linguistics too. The papers are written by philosophers, mathematical logicians and linguists well versed in mathematics. They show that linguists are attracted by the most various mathematical disciplines as mathematical logic, the theory of recursive functions and automata, the theory of communication and probabilistic models, the topological, algebraic and quantitative facets of mathematics. R. JAKOBSON writes about the coincidences and convergences between the latest stages of linguistic analysis and the approach to language in the mathematical theory of communication. J. LAMBEK is searching for an effective rule for distinguishing sentences from nonsentences, in which not only linguists but also mathematical logicians may be interested. N. CHOMSKY gives in his paper "Rule of Grammar" a precise formulation of the notion "structural description of a sentence", and a precise account of the manner in which structural descriptions are assigned to sentences by "grammatical rules", which has a theoretical value for modern linguistics. The paper of HASKELL B. CURRY is dealing with the formal properties of grammars revealing the close connection between mathematical logic and language referring to grammatical structure. The other papers, written by W. V. QUINE, H. PUTMAN, H. HIŻ, N. GOODMAN, Y. R. CHAO, M. EDEN, M. HALLE, R. ABERNATHY, H. G. HERZBERGER, A. G. OETTINGER, V. H. YNGVE, G. E. PETERSON, F. HARARY, H. A. GLEASON, B. MANDELBROT, CH. F. HOCKETT, and R. WELLS may all be read with great interest both by mathematicians and linguists. Their results are not only indispensable for the further development of applied linguistics such as machine translation of languages, but contribute to stating more precise notions in linguistics independent from the semantical meaning and valid for all languages.

F. Kiefer (Budapest)

KIYOSHI OKA, **Sur les fonctions analytiques de plusieurs variables,** VI+234 pages, Tokyo, Iwanami Shoten, 1961.

This book performs the welcome service of making accessible the fundamental researches of the author on the subject of several complex variables. It consists of nine papers which were published during the years from 1936 to 1953. Although much of the material has been incorporated into the theory of analytic spaces and is now available in several standard treatments of the subject, much remains that cannot be found elsewhere. Even those parts of the book which have been thoroughly reworked by other investigators are worth reading for the insights they afford. The book will of course be a valuable reference; it can also serve as a good place to begin the study of the theory of several complex variables. There is naturally not space in the following short review of the contents of these nine articles to do more than indicate the variety of material which they contain.

The first paper, "Domaines convexes par rapport aux fonctions rationnelles", gives the solution to the first Cousin problem — the problem of finding a meromorphic function with given principal parts — for rationally convex domains in the space C^n of several complex variables. The solution employs the basic technique of realizing the given domain as an analytic surface in a higher-dimensional space. In the second paper, "Domaines d'holomorphie", the same technique is used to extend the solution to domains which are holomorphically convex. An example is given in a later paper, "Domaines d'holomorphie et domaines rationnellement convexes", of a domain in C^2 which is holomorphically convex but not rationally convex.

The third paper, "Deuxième problème de Cousin", contains in essence the demonstration that COUSIN's second problem — the problem of finding a holomorphic function with given zeros — can be solved for a holomorphically convex domain whose second cohomology group with integral coefficients vanishes. Necessary and sufficient conditions are given for a solution to exist when the cohomology group in question does not vanish. An example is given for which COUSIN's second problem has no solution.

After a preparatory paper which extends the Weil integral to holomorphically convex domains, the author undertakes the investigation of those domains spread over C^n which satisfy a pseudo-convexity condition at each boundary point. It is shown that such domains are holomorphically convex, thereby giving a characterization of holomorphic convexity in terms of local properties of the boundary. This is accomplished in the sixth paper, „Domaines pseudoconvexes", for univalent domains in C^2. It is not until the last paper, „Domaines finis sans point critique intérieur", that the general case is treated. The proof makes strong use of properties of the class of pseudoconvex functions (called plurisubharmonic functions in the recent literature) introduced by the author for the study of pseudoconvex domains.

The seventh paper, "Sur quelques notions aritmétiques", contains the proof of the fundamental result that (in current terminology) the sheaf of relations of finitely many holomorphic functions is a coherent analytic sheaf. The eighth paper, "Lemme fondamental", proves the equally basic result that the sheaf of normal-holomorphic functions on an analytic variety is a coherent analytic sheaf. This is equivalent to the existence of the normalization of the variety.

Errett Bishop (Princeton, N. J.)

ROBERT SCHATTEN, **Norm ideals of completely continuous operators** (Ergebnisse der Mathematik und ihrer Grenzgebiete, neue Folge, Heft 27), VI+81 Seiten, Berlin—Göttingen—Heidelberg, Springer-Verlag, 1960.

Completely continuous operators on a Hilbert space or Banach space have received considerable attention since HILBERT's papers on integral equations and completely continuous quadratic forms of infinitely many variables. E. SCHMIDT's and F. RIESZ' classical work in this field is well-known. However, interest in *spaces* of completely continuous operators is comparatively new. Some results of this type may be found implicit in the early work of E. SCHMIDT, other ones are "generally known" and cannot be found explicitly in print. Therefore, it was a quite actual task to present a unifying theory of the spaces of completely continuous operators on a Hilbert space, and that is what the author has accomplished in the present survey.

This book contains the major part of the author's previous book, *A theory of cross spaces* (Princeton Univ. Press, Princeton, 1950) and some closely related theorems due to other authors wich were not included in the first book.

The setting of the discussion is a complex Hilbert space $\mathfrak{H}$. $\mathbf{A}$ denotes the full algebra of bounded linear operators on $\mathfrak{H}$, and $\mathbf{C}$ and $\mathbf{R}$ denote the subalgebras of $\mathbf{A}$ consisting of all completely continuous or finite-rank operators on $\mathfrak{H}$, respectively. The bound of an operator A ($\in \mathbf{A}$) will be denoted by $||A||$.

Chapter I contains fundamental theorems on the spectrum of completely continuous operators, further, theorems on the ideals of such operators, such as the theorem of CALKIN on the characteristic sets of ideals of $\mathbf{C}$ and a theorem of KAPLANSKY on uniformly closed left ideals of $\mathbf{C}$. Chapter II and III are devoted to the operators of the Schmidt-class and the Schmidt-norm, and to the operators of the trace-class and the trace-norm, respectively. In chapter IV, the author describes the successive conjugate spaces of $\mathbf{C}$, the latter being considered as a Banach space with $||A||$ as its norm. It is also shown that, in the case of an infinite-dimensional Hilbert space, **C** is not the conjugate space of any Banach space. Chapter V constitutes the main part of the work. It is concerned with the crossnorms and norm ideals. A norm α on an ideal **T** of **A** is a *crossnorm* if it possesses the "cross-property", i. e. $\alpha(A) - ||A||$ for all operators A of rank 1. α is termed *unitarily invariant* if $\alpha(UAV^* - \alpha(A)$ for $A \in \mathbf{T}$ and any pair of unitary operators U, $V \in \mathbf{A}$; α is termed *uniform* if $\alpha(XAY) \leqq ||X||\ ||Y||\alpha(A)$ for $A \in \mathbf{T}$ and any pair X, $Y \in \mathbf{A}$. An ideal $\mathbf{T} \subset \mathbf{A}$ is called a *norm ideal* if on it there is defined a uniform crossnorm with respect to which **T** is also complete. A norm ideal is *minimal* if none of its proper subspace is also a norm ideal. In the first part of chapter V, the author develops the direct connection between the unitarily invariant crossnorms on **R** and the symmetric gauge functions on the linear space of all infinite sequences of real numbers having only finite number of non-zero terms, and shows that the class of unitarily invariant crossnorms on **R** coincides with that of uniform crossnorms. In the second part of this chapter, the author determines all minimal norm ideals and characterizes their conjugate spaces which again may be interpreted as norm ideals of operators.

The proofs are concise, but very clearly written, and the book is very readable. It will without any doubt induce many new workers in this field.

I. Kovács (Szeged)