Cellularity of a suspension arc

By P. H. DOYLE in East Lansing (Michigan, U. S. A.)

Let X be a topological space that is locally euclidean of dimension n except at a single point p. We assume that the suspension $S(X)$ of X is an $(n+1)$-sphere, S^{n+1}. That there are many such spaces X follows from [3]. If v_1 and v_2 are the suspension points of $S(X)$ we study the embedding of $S(p)=v_1p\cup v_2p$, the suspension arc, in S^{n+1}.

Lemma 0. *The arc v_1p in S^{n+1} is cellular.*

Proof. Consider $S(X)-v_1p$. This set is the manifold with boundary v_2X-p with an open collar attached to its boundary. Thus it is topologically E^{n+1} and v_1p is therefore cellular.

Since the above argument applies equally well to v_2p, $S(p)$ is the union of two cellular arcs meeting in a common endpoint p. In general such an arc is not cellular. Example 1.1 of [2] gives such an arc in S^4. If we call this arc A in S^3 and take X to be S^3 modulo A, then $S(X)$ is S^4. However the suspension arc in this case is not cellular since its fundamental group is the non-trivial group $\pi_1(S^3-A)$. We give a sufficient condition that the suspension arc be cellular that is purely geometric.

Theorem. *If there is an n-cell C^n in $S^{n+1}-v_2$ such that v_1 lies in the interior of C^n while the boundary of C^n meets $S(p)$ in just one point, then $S(p)$ is cellular.*

Proof. By the hypothesis it is clear that v_2 does not lie in C^n. Since S^{n+1} is a suspension then given any open set U in S^{n+1} containing v_1 there is a homeomorphism h of S^{n+1} onto itself that carries $S(p)$ onto itself and $h(C^n)\subset U$. But then by Lemma 1 of [1] and Lemma 0 above $S(p)$ is cellular in S^{n+1}.

References

[1] P. H. Doyle, A sufficient condition that an arc in S^n be cellular, *Pacific J. Math.*, **14** (1964), 501—503.

[2] R. H. Fox, and E. Artin, Some wild cells and spheres in three-dimensional space, *Ann. Math.*, **49** (1948), 979—990.

[3] K. W. Kwun, Product of euclidean spaces modulo an arc, *Ann. Math.*, **79** (1964), 104—108.

MICHIGAN STATE UNIVERSITY

(Received April 22, 1966)