Bibliographie

Eduard Čech, Topological spaces. Revised edition by ZDENĚK FROLIK and MIROSLAV KATĚTOV, 893 pages (errata insert), Publishing House of the Czechoslovak Academy of Sciences, Prague; Interscience Publishers (John Wiley and Sons), London—New York—Sydney, 1966.

E. ČECH directed in Brno from 1936 until 1939 a seminar on Topology. The substance of his book on Topological Spaces arose from this seminar, and the manuscript was completed during the war. However, this book did not appear in Czech language before 1959. Though a number of minor changes was made on the first manuscript, it was impossible to give account in it of the recent and extremely rapid development of General Topology. Therefore, after the death of the author, Z. FROLÍK and M. KATĚTOV decided to prepare a revised edition in English, re-writing the whole book in such an extent that its dimensions grew nearly to the double of the first version. In the same time, the new book got the character of a monograph, while the original was rather a text-book.

The fundamental feature of this book is — quite similarly to the first edition — the effort to investigate all concepts under circumstances as general as possible. Thus, while in most works on General Topology the basic concepts are topology and topological space, here closure operations and closure spaces are principally considered. Under a closure operation on a set E it is understood an operation u for the subsets of E such that $u\emptyset=\emptyset$, $uA\supset A$ and $u(A\cup B)=uA\cup uB$ for A, $B\subset E$; a topology is a closure operation such that $uuA=uA$ for all $A\subset E$. In a similar manner, a proximity on E denotes in the usual sense a relation p for the subsets of E such that $\emptyset$ non pE, ApB implies BpA, $A\cap B\neq\emptyset$ implies ApB, $(A\cup B)$pC holds if and only if ApC or BpC, and (∗) A non pB implies the existence of U and V such that A non p$E-U$, B non p$E-V$. Now, in the terminology of this book, a proximity is a relation p satisfying the above conditions with the exception of (∗). Finally, instead of uniformities, the present book studies semi-uniformities, where a semi-uniformity $\mathcal{U}$ on E is a filter on $E\times E$ such that $U\in\mathcal{U}$ implies $\Delta\subset U$ (where Δ denotes the diagonal of $E\times E$) and $U\in\mathcal{U}$ implies $U^{-1}\in\mathcal{U}$. As well-known, $\mathcal{U}$ is a uniformity if, moreover, $U\in\mathcal{U}$ implies the existence of $U_1\in\mathcal{U}$ with $U_1\circ U_1\subset U$.

It is worth while to note that all these generalizations of the usually investigated concepts of General Topology can be easily described by means of the theory elaborated by the reviewer (*Fondements de la topologie générale*, Budapest et Paris, 1960). As a matter of fact, topologies, proximities (in the usual sense) and uniformities are special cases of the general concept of syntopogenous structures, among which perfect and simple structures correspond to topologies, symmetrical and simple structures to proximities, and perfect and symmetrical structures to uniformities. Now, if in the definition of a syntopogenous structure, axiom (S_2) is omitted, one precisely obtains closure operations, proximites (in the sense of the present book) and semi-uniformities instead of topologies, proximities (in the usual sense) and uniformities. Consequently, it is rather natural that a very extensive part of the theory of the latter "classical" concepts admits a generalization for the former mentioned more general concepts studied in the present book.

The material is divided into seven chapters and an appendix. The first two chapters, written by M. KATĚTOV, have the titles "Classes and relations" and "Algebraic structures and order", and serve as introduction to the remaining part of the book, due to Z. FROLÍK and treating General Topology. These introductory chapters contain an axiomatic but nonformal exposition of set theory, and present much novelty in basic ideas and in methods.

The following chapters (Topological spaces, Uniform and proximity spaces, Separation, Generation of topological spaces, Generation of uniform and proximity spaces) and the appendix (Compactness and completeness) give a detailed exposition of the theory of the above mentioned general structures and their interrelations. Not only the proofs are presented in an easily readable manner but extensive introductions serve to elucidate the concepts and the results of each chapter and each section, and numerous examples and remarks illustrate the meaning of every definition and proposition. A great number of exercises furnish still more illustration and essential additional material.

A short bibliography is added, whereas the text itself does not contain any references to the literature. The terminology differs somewhat from the usually adopted one, but is consequent, and a detailed index facilitates the identification of terms and notations.

Ákos Császár (Budapest)

Hanna Neumann, Varieties of Groups (Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 37), XII+192 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1967.

This book introduces the reader having some familiarity with the basic concepts of the group theory, to the study of the varieties of groups and presents him the results achieved in this new, branch of research. Let us consider the class of all groups satisfying each one of a given set of identities. Such a class is called a *variety*, or a primitive class. For example, the class of all abelian groups as well as the class of all nilpotent groups of class n for any positive integer n are varieties.

The concept of variety may be applied without modification to universal algebras; as a matter of fact, this concept, just for this general case, was introduced by G. BIRKHOFF some three decades ago. The study of varieties of groups obtained an essential impetus only after 1950. Since then, it advanced, however, rapidly and in this progress the author herself, as well as her husband B. H. NEUMANN, her son PETER NEUMANN, and others belonging to this research group have played an important role.

This work, written in a concise and elegant style and having a clear setting up, will surely be a very enjoyable reading for the mathematicians interested in group theory, and wanting to get acquainted with the subject. In spite of its relatively short extent, the book contains a large material including a great number of unpublished results. At the same time, the author illuminates the connections of the subject treated with other chapters of group theory drawing hereby the reader's attention to the fact that varieties prove to be an effective tool in group-theoretical investigations.

The book consists of five chapters. The first chapter has an introductory character, the second one deals with the products of varieties. By the product of the varieties $\mathfrak{A}$ and $\mathfrak{B}$, it is meant the class of all groups which are Schreier extensions of a group in $\mathfrak{A}$ by a group in $\mathfrak{B}$. Concerning products let us mention a deep and surprising result: all non-trivial varieties of groups form a free semigroup with respect this product as multiplication. Chapter 3 is devoted to a study of the varieties of nilpotent groups. Chapter 4 deals with some properties of (relatively) free groups in varieties. Among others, all the varieties are described for which the free groups have all their subgroups free also. These are exactly the following: the class of all groups, the class of all abelian groups and the classes of abelian groups with a fixed prime exponent. Finally, Chapter 5 treats the connections between varieties and finite groups belonging to them. The book is completed by a bibliography containing about 150 items.

It may be expected that the appearing of this book gives a further impulse to investigations not only on group varieties, but also on varieties of other types of algebraical systems. To motivate this hope it suffices to refer to a recent article of A. J. MAL'CEV in which some results concerning products of group varieties are generalized for normal varieties of universal algebras.

Béla Csákány (Szeged)

N. Bourbaki, Éléments de mathématique, Fascicule XXXII, **Théories Spectrales,** Chapitres 1 et 2: Algèbres normées, Groupes localement compacts commutatifs (Actualités Scientifiques et Industrielles, 1332), 166 pages, Hermann, Paris, 1967.

Voici la première partie d'un nouveau livre de l'auteur célèbre!

Dans le Chap. 1 on étudie d'abord les algèbres à élément unité, appelées „algèbres unifères", principalement les algèbres normées et les algèbres de Banach, et le spectre d'un élément dans une telle algèbre. L'exposition du calcul fonctionnel holomorphe dans les algèbres de Banach unifères commutatives est faite dans une grande généralité; on y trouve des résultats récents. Puis on passe à l'étude de deux types fondamentaux d'algèbres de Banach: les algèbres de Banach commutatives régulières, qui ont des applications dans l'analyse harmonique, et les C^*-algèbres, appelées „algèbres stellaires" qui sont d'une grande importance dans l'étude des groupes localement compacts: à un tel groupe G on peut associer canoniquement une algèbre stellaire $\mathrm{St}(G)$. On traite aussi les algèbres de fonctions continues sur un espace compact; on a inclus plusieurs résultats de date récente.

Dans le Chap. 2 on étudie les fondements de l'analyse harmonique et la théorie des groupes localement compacts commutatifs. Pour un tel groupe G l'algèbre stellaire St(G), étant commutative, est isomorphe à l'algèbre des fonctions complexes continues, zéro à l'infini, sur un espace localement compact (d'après un résultat fondamental, exposé au Chap. 1). C'est ce théorème qui apparaît ici comme l'outil principal pour démontrer le théorème de Plancherel. Alors la route est libre pour établir la théorie de la dualité pour les groupes localement compacts commutatifs et la formule d'inversion pour la transformation de Fourier des fonctions intégrables (pour la mesure de Haar). Les propriétés fonctorielles de la dualité et la théorie de structure des groupes localement compacts commutatifs sont exposées d'une manière très élégante. La formule de Poisson est établie dans une grande généralité (on peut mentionner un cas qui n'est pas traité ici: la formule s'applique à toute fonction continue, à support compact, telle que la transformée de Fourier soit intégrable pour la mesure de Haar du groupe dual; ce cas est utile dans la Théorie de Nombres et permet aussi de démontrer la Proposition 9, p. 128, d'une manière plus satisfaisante). Enfin l'auteur donne une exposition du théorème taubérien de Wiener et de ses ramifications et généralisations (l'exposition étant d'ailleurs strictement traditionnelle); plusieurs de celles-ci sont données en forme d'exercices (l'exercice 10 c, p. 158, peut s'étendre aux suites telles que la réunion soit fermée).

C'est, en somme, un exposé très riche, qui sera fort utile aux lecteurs sérieux. On attendra avec un grand intérêt les chapitres suivants.

H. Reiter (Utrecht)

William A. Veech, A second course in complex analysis, IX+246 pages, New York, N. Y., W. A. Benjamin, Inc., 1967.

For this "second course" the following topics are chosen: 1. Analytic continuation (Germs and their composition, Covering surfaces, etc.). 2. Geometric considerations (Linear transformations, Noneuclidean geometry, The Schwarz reflection principle, etc.). 3. The mapping theorems of Riemann and Koebe (Lindelöf's lemma, Continuity at the boundary, etc.). 4. The modular function (Schottky's, Picard's, and Bloch's theorems; The Koebe — Faber distortion theorem, etc.). 5. The Hadamard product theorem (Canonical products, The gamma function, etc.). 6. The prime number theorem.

Many problems are added for solution. These are extremely different in level. E. g., one problem (p. 10) is to prove that the value of the integral of $(z-z_0)^{-1}$ on a circle including z_0 does not depend on z_0. (This shows that almost nothing from a standard "first course" is assumed.) As a contrast, a problem (p. 32) asks for a proof of the "Brouwer fixed point theorem" for homeomorphisms of the closed disk.

There is a bibliography on "books which have directly influenced this work, and the foremost among these are the books of Carathéodory". Bieberbach's "Lehrbuch der Funktionentheorie" is not listed, the "Complete Poems of Robert Frost" are.

Béla Sz.-Nagy (Szeged)