Bibliographie

C. A. Rogers, Packing and covering, VIII+111 pages, Cambridge University Press, 1964.

The study of packing and covering problems represents the most developed branch of discrete geometry whose results find direct applications in the theory of numbers, in the analysis, in the theory of information, and in other branches of mathematics.

The first monograph dealing with packing and covering is FEJES TÓTH's well-known discrete geometry book (*Lagerungen in der Ebene auf der Kugel und im Raum*, Berlin—Göttingen—Heidelberg, 1953) in which the author confined his attention mainly to packing and covering in two and three-dimensional space of constant curvature. Some results on this subject are treated also in his recent book *Regular Figures* (Oxford—London—New York—Paris, 1964).

The vaste literature of packing and covering, to which many well-known mathematicians (including COXETER, DELAUNAY, DIRICHLET, FEJES TÓTH, GAUSS, HLAWKA, LAGRANGE, MINKOWSKI, ROGERS, SEGRE, THUE, VORONOI) have contributed, has reached the point where a systematization into a theory has become possible, indeed necessary. We can be grateful to Professor ROGERS, who has himself enriched the theory with many beautiful results, that he has undertaken the task to write this monograph.

The book treats mainly problems in n-dimensional Euclidean space, where n is larger than 3, and the packing and covering system is formed by a finite or countably infinite system of translates of a single set which will usually be convex. In the introduction the author gives an excellent historical outline of the subject. In the succeeding chapters he gives a systematic (but by no means exhaustive) account of the general results of the subject and their derivations. For a closer idea about the content of this part let us mention the titles of the chapters: 1. Packing and covering densities; 2. The existence of reasonably dense packings; 3. The existence of reasonably economical coverings; 4. The existence of reasonably dense lattice packings; 5. The existence of reasonably economical lattice coverings; 6. Packings of simplices cannot be very dense; 7. Packings of spheres cannot be very dense; 8. Coverings with spheres cannot be very economical.

The author uses in his work an analytical method and only one diagram illustrates the book, in contrary to FEJES TÓTH who gives preference to the syntetical method, richly illustrating his works.

ROGERS's monograph is thus a useful complement to FEJES TÓTH's discrete geometry monograph. It is written with great care, is easy to read, and its economical and logical structure is quite excellent.

J. Molnár (Bamako—Budapest)

L. Rédei, The theory of finitely generated commutative semigroups, XIII+350 pages, Budapest, Akadémiai Kiadó, 1965.

This book is a translation of the original German text edited in 1963 by Teubner Verlag and Akádémiai Kiadó. The theory contained in this monograph has become a well-known and separate part of the theory of algebraic semigroups. We can omit to give here a detailed report on the work as there are such thorough references at disposal as those of E. A. BEHRENS (MR, **28** (1964), 5130) or Št. SCHWARZ (*Acta Sci. Math.*, **25** (1964), 175—176). One can hope that the English edition rendering this original and important topic available for a wider class of researchmen will start new investigations in this field, concerning especially some possible generalizations, links with other parts of semigroup theory and the application of RÉDEI's results to special classes of semigroups.

G. Pollák (Szeged)

A. F. Timan, Theory of approximation of functions of a real variable, XII+631 pages, Hindustan Publishing Corporation (India), Delhi-7, 1966.

The present book is a translation from the Russian original, published in 1960 by "Fizmatgiz", Moscow. It presents a detailed account of the new results of the theory of approximation. As the author points out in the foreword, this monograph systematically investigates the relationship between the various structural properties of real functions and the character of their possible approximation by polynomials or other simple functions. The investigations carried out in this book are based on the classical approximation theorem of WEIERSTRASS, the concept of TSCHEBYSCHEFF of the best approximation and the converse theorem of BERNSTEIN on the existence of a function with a given sequence of best approximation. The chapter headings give a more detailed outline of the presentation: I. WEIERSTRASS's theorem. — II. The best approximation. — III. Certain compact classes of functions and their structural characteristics. This chapter includes properties of various moduli of continuity, properties of various classes of analytic functions, quasi-analytic classes of functions, and properties of conjugate functions. — IV. Certain properties of algebraic polynomials and transcendental integral functions of exponential type. Here we find interpolation formulae, WIENER—PALEY's theorem, some extremal properties of polynomials and transcendental integral functions. — V. Direct theorems of the constructive theory of functions. — VI. Converse theorems. Constructive characteristics of certain classes of functions. — VII. Additional theorems on the connection between the best approximations of functions and their structural properties. — VIII. Linear processes of approximation of functions by polynomials. Certain estimates connected with them. — IX. Certain deductions from the theory of functions and functional analysis. This includes basic theorems without proof.

At the end of each chapter there is a section which contains various problems and theorems supplementing the material of the main text.

There is a useful Bibliography (containing some 350 items) and a detailed Index.

The book is well-organized and the presentation is clear.

L. Leindler (Szeged)

Ralph Abraham and **Joel Robbin, Transversal Mappings and Flows,** X+161 pages, W. A. Benjamin, Inc., New York—Amsterdam, 1967.

Since the initiative work of G. REEB, S. SMALE and R. THOM the qualitative theory of ordinary differential equations has merged into a rapidly developing new theory. This process has been marked by the application of differential topology. It has commenced by considering a system of first order ordinary differential equations as a vector field on a differentiable manifold and the solutions of the system as the flows of the latter. The successes of this process, however, were mainly due to the fact that, roughly speaking, the vector fields of a compact differentiable manifold form an (infinite-dimensional) differentiable manifold, which fact proves to be very useful when dealing with questions of the qualitative theory. An exposition of the fundamental ideas and results of the new theory, originating from this process, is the goal of this book.

The first chapter is a review of differential theory. Definitions and basic theorems are given, while in some cases for the proof the reader is referred to DIEUDONNÉ's *Foundations of Modern Analysis* or to LANG's *Introduction to Differentiable Manifolds.* It contains an original result, the converse of TAYLOR's theorem, and the exposition is remarkable in various respects.

The second chapter deals with the topologies of spaces of vector bundle sections and contains a proof of the smoothness of the evaluation map. The latter theorem prepares for the application of transversality technique which has been introduced by R. THOM and is applied here systematically.

The third chapter contains a proof of the SMALE's Density Theorem concerning the regular values of the so-called FREDHOLM mappings. It is obtained as the last one in a sequence of fundamental theorems, each implied by the preceding one. These theorems are: the Rough Composition Theorem of KNESER and GLAESER, and the Density Theorems of MÉTIVIER, SARD and SMALE.

The basic facts of transversality theory are given in the fourth chapter. It contains the Openness and Isotopy Theorems of THOM and the Density Theorem of ABRAHAM.

The fundamental concepts and theorems of the theory of vector fields and their flows are given in the fifth chapter. These are formulated both traditionally and in terms of transversality by which the adequacy of this new technique becomes evident.

Pseudocharts for closed orbits, the FLOQUET normal form and a proof of the Stable Manifold Theorem of SMALE, are the topics discussed in the sixth chapter.

A new proof of the theorem of S. SMALE and I. KUPKA on generic properties of flows is given in the seventh chapter. By the application of transversality technique it is easier than the original one. Important tools of this proof are a theorem of A. TARSKI and A. SEIDENBERG on the structure of semialgebraic sets and the perturbation theory of vector fields.

From the Appendices the first two provide some perequisites, the third, however, is an original research article by AL KELLEY on stable, center-stable, center-unstable, and unstable manifolds.

The above material up to now has been accessible mainly in research papers. The authors' self-contained exposition is worked out carefully, with the intent to attain a maximum of clarity. It serves not only to arouse interest in, but also to yield a very readable introduction into this developing modern subject.

J. Szenthe (Szeged)

H. Halberstam—K. F. Roth, Sequences, Vol. 1, XX+291 pages, Oxford, Clarendon Press, 1966.

The arithmetical properties of special integer sequences (e. g. the distribution of prime numbers in arithmetical progressions, the additive properties of the sequence of squares) are extensively studied in number theory. Many of these properties hold for all or for wide classes of integer sequences. The main theme of this book is the study of those general arithmetical properties which are satisfied for extensive classes of such sequences.

In Chapters I, II, and III general laws related to the addition of sequences are established. Chapter I deals with the density relationship. This chapter contains among others the theorems of MANN, DYSON, VAN der CORPUT, BESICOVICH, ERDŐS on the Schnirelmann density and their asymptotic and p-adic analogues, the theorem of LINNIK on the essential components. Chapter II contains the theorem of ERDŐS and FUCHS. Concerning the number of representations of integers as the sum of two summands taken from a given set, this theorem investigates the discrepancy of the asymptotic of the mean value of this number. Chapter III is devoted to the treatment of the probability methods which serve to prove existence theorems for integer sequences with a given growth of the representation function.

Chapter IV gives a very good account of the sieve methods of V. BRUN and A. SELBERG, and of the large sieve.

Chapter V deals with some interesting properties of integer sequences which depend on the multiplicative structure of the integers. Results of ERDŐS, ERDŐS—DAVENPORT, BESICOVITCH, and others, concerning primitiv sequences and sets of multiples are treated.

The style is clear, the authors are masters of their subject.

Imre Kátai (Budapest)

The Theory of Groups, Proceedings of the International Conference held at the Australian National University Canberra, 10—20 August, 1965. Edited by **L. G. Kovács** and **B. H. Neumann,** XVII+397 pages, Gordon and Breach Science Publishers, New York—London—Paris, 1967.

This volume contains more than half a hundred articles which represent all the essential branches of current research in group theory. To make more perceptible the tendencies of development, we list the authors and indicate the results of the more important papers, in the following order: 1) Simple groups, 2) Varieties of groups, 3) Some purely group-theoretical problems, 4) Connections between groups and other algebraic systems.

1. W. FEIT deals with groups having a cyclic Sylow subgroup. Z. JANKO proves that a nontrivial simple group with abelian 2-Sylow subgroups having no doubly transitive permutation representation coincides with the Janko new simple group. R. REE in his article deals with classification of involutions in some Chevalley groups and computes the centralizers of these elements.

2. W. BRISLEY investigates varieties generated by all the proper factors of a critical group. N. D. GUPTA proves some theorems on metabelian groups contained in certain varieties. G. HIGMAN's paper deals with the form of functions, describing orders of relatively free groups. His other article applies the theory of the representation of the general linear groups to varieties of p-groups. L. G. KOVÁCS and M. F. NEWMAN study varieties in which every proper subvariety is a Cross variety. J. D. MACDONALD shows that if critical p-groups generate the same variety then this holds

for the sets of their proper factors too. HANNA NEUMANN summarizes some developments in the field of varieties of groups. In SHEILA OATES' paper we find some investigations on the number of generators of a simple group. SOPHIE PICCARD gives an analysis of the notion of group, free modulo n. P. M. WEICHSEL studies finite critical p-groups which generate join-irreducible varieties.

3. CHRISTINE W. AYOUB studies the minimum number of conjugate classes which a finite p-group can possess. REINHOLD BAER takes part in the volume with three articles. He analyzes the interrelations between the properties characterizing nilpotent groups in the finite case. He characterizes also the polycyclic groups and the noetherian groups possessing a polycyclic subgroup of finite index. In his third paper he discovers some parallelism between theories of artinian and noetherian groups. G. BAUMSLAG gives a review on the present status of the theory of finitely presented groups. H. S. M. COXETER describes some geometric aspects of the isomorphism between the Lorentz group and the group of homographies. J. D. DIXON proves a theorem of Schur—Zassenhaus type. T. HAWKES introduces and investigates the notion of f-Prefrattini subgroup. K. A. HIRSCH presents some results of his student B. WEHRFRITZ, e. g., the proof of the conjugacy of Sylow subgroups in any periodic linear group. N. ITO proves that a nonsolvable transitive permutation group of degree p — where p is a Fermat prime — containing an odd permutation coincides with the symmetric group of degree p. O. H. KEGEL gives a characterization of finite supergroups. R. STEINBERG treats the Galois cohomology of linear algebraic groups. G. SZEKERES determines all finite metabelian groups with two generators. O. TAMASCHKE presents a generalized character theory of finite groups. G. E. WALL constructs a counter-example for a conjecture of D. R. HUGHES. H. WIELANDT gives a survey on subnormal and relatively maximal subgroups and states some problems. In his other article he deals with automorphisms of doubly transitive permutation groups and as application he obtains a special case of SCHREIER's conjecture on the automorphism group of a finite simple group. G. ZAPPA introduces and studies the notion of the Hall S-partition of a group.

4. L. W. ANDERSON and R. P. HUNTER give conditions for the minimal two-sided ideal of a compact connected semigroup to be a group. Their other paper deals also with certain groups connected with the semigroup theory. A. L. S. CORNER gives a characterization of endomorphism rings of countable reduced torsion-free abelian groups as topological rings. L. FUCHS lists the most useful properties of orderable groups and indicates some unsolved problems. M. HALL Jr. presents some applications of block designs to group theory. F. LOONSTRA investigates the ordered set of abelian extensions of an abelian group. P. J. LORIMER generalizes the notion of characteristic for any finite projective plane in a manner eliminating the disadvantages of the earlier generalization. K. W. WESTON presents an interesting connection between group and ring theory. H. SCHWERDTFEGER investigates groups which may be considered as a slightly modified projective plane.

From this list it is clear that this valuable book will be useful for all algebraists interested in group theory.

B. Csákány (Szeged)

Hans Hermes, Einführung in die Verbandstheorie (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 73), XII+209 Seiten, zweite, erweiterte Auflage, Springer-Verlag, Berlin—Heidelberg—New York, 1967.

Die vorliegende zweite Auflage hat den Charakter und den Aufbau der ersten behalten, sie wurde aber mit einigen neuen Paragraphen ergänzt. (Vgl. die Besprechung der ersten Auflage in diesen *Acta*, **16** (1955), 275.) So werden Erzeugungs- und Entscheidungsverfahren für die in verschiedenen, durch Gleichungssysteme definierbaren Verbandsklassen gültigen Termgleichungen angegeben (§ 26, 27); es werden die klassischen und die intuitionistischen Aussagenstrukturen, sowie ihre Beziehungen zu den dualen Primidealen der Booleschen bzw. pseudo-Booleschen (d. h., nach unten beschränkten relativ pseudokomplementären) Verbände behandelt (§ 28, 29; „duales Primideal" wird hier einfach „Primideal" genannt); zur Vorbereitung der letzten dient ein Paragraph über die pseudo-Booleschen Verbände, in dem — unter anderem — gezeigt wird, daß diese Verbandsklasse durch ein Gleichungssystem definierbar ist; ferner werden die Kongruenzrelationen in Verbänden eingehender behandelt (§ 31, 32; hier findet man z. B. den Funayama — Nakayamaschen Satz, hinreichende Bedingungen für die Komplementarität des Kongruenzverbandes und den Satz von HASHIMOTO über die Beziehung zwischen den Kongruenzrelationen und den Idealen). Übrigens wurde der Text der ersten Auflage nur stellenweise abändert; Beispiel 8.4 und Abbildung 19.1 B wurden berichtigt.

G. Szász (Nyiregyháza)

Proceedings of Symposia in Pure Mathematics, vol. X. **Singular integrals**, VI+375 pages, American Mathematical Society, Providence, Rhode Island, 1967.

This volume, edited by A. P. CALDERÓN, contains the material of the Symposium on Singular Integrals and their Applications held at the University of Chicago, in Chicago, April 20—22, 1966. The rich and deep material is dedicated to Professor ANTONI ZYGMUND in celebration of his sixty-fifth anniversary and in recognition of his decisive contribution to the field of singular integrals.

The volume consists of twenty papers written by outstanding authors, furthermore, of an author and subject index. The papers, in general, deal with integral transformations of the form

$$\tilde{f}(x) = \text{P. V.} \int f(x-y) K(y) dy = \lim_{\varepsilon \to 0} \int_{|y| \geq \varepsilon} f(x-y) K(y) dy$$

(P. V. means "principal value") or of similar types and with their various applications, where $x=(x_1, x_2, \ldots, x_n)$, $y=(y_1, y_2, \ldots, y_n)$ are points of the real n-dimensional Euclidean space, $|x| = \left(\sum_{i=1}^{n} x_i^2\right)^{\frac{1}{2}}$, and the kernel $K(y)$ fulfils certain conditions of homogeneity and integrability.

The purpose of the paper of B. BAJSANSKI and R. COIFMAN is to prove the boundedness of the maximal operator associated with some singular integrals considered by A. P. CALDERÓN. The paper of A. P. CALDERÓN presents the development of the algebraic formalism of singular integral operators as sketched in his earlier papers. In their joint paper A. P. CALDERÓN, MARY WEISS, and A. ZYGMUND investigate the existence of singular integrals and show that, under certain conditions, the singular integrals are operators of weak type (1, 1). M. V. CORDES treats some properties of the mapping $f \to \tilde{f}$ in L^2. E. B. FABES and H. JODEIT, JR. introduce so-called parabolic singular integral operators and apply such operators to boundary value problems for parabolic equations. E. B. FABES and N. M. RIVIERE, continuing their earlier investigations, extend some results of CALDERÓN and ZYGMUND to the case of kernels with mixed homogeneity. K. O. FRIEDRICHS and P. D. LAX deal with symmetrizable differential operators, LARS HÖRMANDER with pseudo-differential operators and hypoelliptic equations, M. JODEIT, JR. with symbols of parabolic singular integrals, B. FRANK JONES, JR. with applications of singular integrals to the solution of boundary value problems for the heat equation. Then the further papers follow: PAUL KRÉE: A Class of Singular Integrals. Pseudo-differential Operators on Non-quasi-analytic Function Spaces, P. D. LAX and L. NIRENBERG: A Sharp Inequality for Pseudo-differential and Difference Operators, J. E. LEWIS: Mixed Estimates for Singular Integrals and an Application to Initial Value Problems in Parabolic Differential Equations, UMBERTO NERI: Singular Integral Operators on Manifolds, JOHN C. POLKING: Boundary Value Problems for Parabolic Systems of Partial Differential Equations, CORA SADOSKY and MISCHA COTLAR: On quasi-homogeneous Bessel Potential Operators, R. T. SEELEY: Complex Powers of an Elliptic Operator, Elliptic Singular Integral Equations, E. M. STEIN: Singular Integrals, Harmonic Functions, and Differentiability Properties of Functions of Several Variables, RICHARD L. WHEEDEN: Hypersingular Integrals and Summability of Fourier Integrals and Series.

The above enumeration shows that this volume, which is very rich in its material, gives a comprehensive view of the modern, developing and important theory of singular integrals.

F. Móricz—K. Tandori (Szeged)

Olivier Dimon Kellogg, Foundations of Potential Theory (Die Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen mit besonderer Berücksichtigung der Anwendungsgebiete, Band XXXI) IX+384 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1967.

This book is the reprint from the first edition of 1929. The first edition was reviewed by F. RIESZ in these *Acta*, **5** (1930—32), 137—138.

D. E. Men'šov, Limits of indeterminacy in measure of trigonometric and orthogonal series (Proceedings of the Steklov Institute of Mathematics, number 99 (1967)), 67 pages, American Mathematical Society, Providence, Rhode Island, 1968.

This book is the translation by R. P. BOAS of the Russian original. It contains the proof of three, very general and complicated theorems concerning the lower and upper limits in measure, or in other words, the limits of indeterminacy in measure, of measurable functions. The first two theorems — somewhat in a less general form — have been published by the author earlier. (On limits of indeterminacy with respect to measure and limit functions of trigonometric and orthogonal series, *Dokl. Akad. Nauk SSSR*, **160** (1965), 1254—1256.) These results are based on the author's earlier investigations related to convergence in measure of trigonometric series (On convergence in measure of trigonometric series, *Trudy Mat. Inst. Steklov*, **32** (1950), 1—99; On the limit functions of trigonometric series, *Trudy Moskov. Mat. Obšč.*, **7** (1958), 291—334) as well as on the results of A. A. TALALJAN (Limit functions of series in bases of the space L_p, *Mat. Sbornik* **56** (98) (1962), 353—374), and at the same time they are improvements of those. The theorems in this book essentially assert that the limits of indeterminacy in measure of the sequences of partial sums of series with respect to a complete orthogonal system, or more generally, to a normalized basis in $L_p(p>1)$, and of series of arbitrary measurable terms, have many properties in common.

F. Móricz—K. Tandori (Szeged)

Pál Révész, The Laws of Large Numbers, 176 pages, Akadémiai Kiadó, Budapest, 1967.

The laws of large numbers have always occupied an important position in the history of the calculus of probability, referring to both theoretical and practical applications. Although there exists already a huge literature on this subject, there appeared no monograph which would systematically elaborate all known laws. The present work fills this gap, giving a general survey of the results and the most important methods of proof in this field. Occasionally, when the proof of a theorem requires very special methods, it is omitted. Several open questions are also mentioned.

To define exactly the field of the laws of large numbers seems to be very difficult. We can say, in an attempt to obtain a definition, that a law of large numbers asserts the convergence, in a certain sense, of the average

$$\eta_n = \frac{\xi_1+\xi_2+\dots+\xi_n}{n}$$

of the random variables $\xi_1, \xi_2, \dots$ to a random variable η (p. 8). By making use of different modes of convergence, different types of the laws of large numbers can be obtained. Three kinds of convergence are considered in the book: stochastic convergence (or convergence in measure), convergence with probability 1 (or convergence almost everywhere), and mean convergence. According to this, mainly, the weak laws, the strong laws, and the mean laws of large numbers are studied.

In connection with the laws of large numbers the author investigates the rate of convergence. Hence, the laws of the iterated logarithm are also treated in the book.

Theorems on the convergence of a series of the form $\Sigma c_k\xi_k$, where $\{\xi_k\}$ is a sequence of random variables and $\{c_k\}$ is a sequence of real numbers, cannot be considered as a law of large numbers. However, this class of theorems is occasionally also studied because the convergence of a series of the above form immediately implies a law of large numbers by using the Kronecker lemma.

The book contains eleven chapters with a complete bibliography and author index.

In Chapter 0 the author has collected the most important definitions and theorems without proof which are applied in the book. The reader should be familiar with the most fundamental results and concepts of probability, stochastic processes, measure theory, ergodic theory, functional analysis, etc.

Chapter 1 deals with the special concepts and general theorems of the laws of large numbers.

Chapter 2 is devoted to the laws of large numbers of independent random variables. The author presents some fundamental results of KOLMOGOROV such as the Kolmogorov inequality, the so-called three series theorem, another theorem that gives a necessary and sufficient condition for the weak law of large numbers, etc. One virtue of the book is that there are occasionally given more essentially different proofs of the same theorem, e. g. the gap method and the method of high moments for the proof of the strong laws of large numbers. The end of this chapter studies the asymptotical properties of weighted averages, and the case of convergence to $+\infty$.

Chapter 3 contains the author's own results that are analogues of some theorems of the previous chapter for the case in which the random variables are not independent but only strongly multiplicative. These investigations are based on an inequality, also due to him, which can be considered as a generalization of the Rademacher—Menšov inequality, well-known in the theory of orthogonal series.

Chapter 4 discusses the laws of large numbers for stationary sequences by making use of the results of ergodic theory. The difficulties from the special point of view of probability lie in the investigation of the condition of ergodicity. This investigation seems to be very difficult and not quite solved.

The main problem of Chapter 5 is the following one: under what conditions can we find a subsequence of an arbitrary sequence of random variables obeying a law of large numbers? Among others, the author investigates this question in the case of Walsh functions $\{w_n(x)\}$ (and, presents the analogous results related to the sequence $\{\sin nx\}$), and he proves that if we consider a subsequence $w_{n_k}(x)$ (resp. $\sin n_k x$) for which $n_{k+1}/n_k \geqq q > 1$ then we can obtain practically the same results as for independent sequences. The main result of this chapter, due also to the author, asserts that from any sequence of uniformly bounded random variables we can choose a subsequence which has properties similar to those of an independent system.

After presenting the fundamental theorem of symmetrically dependent random variables, the results of Chapter 4 concerning the strong laws of large numbers are sharpened in Chapter 6. At the end of this chapter the author discusses the connection between the equinormed strongly multiplicative systems and the quasi-multiplicative systems.

Chapter 7 deals with the laws of large numbers of Markov chains. The laws of large numbers as well as the limit theorems for non-homogeneous Markov chains are based on the different kinds of measures of ergodicity. The author introduces some of them, and several theorems are mentioned.

Chapter 8 contains some general laws of large numbers which are not related to any concrete class of stochastic processes. In this chapter there are no restrictions on the kind of dependence, only on the strength of it. After introducing the notion of mixing for a sequence of random variables, general theorems are treated.

Up to the Chapter 9 random variables taking values on the real line occur. In general, similar results can be obtained for random variables taking values in a finite dimensional Banach space. The situation is not much more complicated if the values of the random variables are in a Hilbert space. The real difficulty is in the treatment of the random variables taking values in a Banach space. The author follows the treatment of BECK.

In general, a law of large numbers states that the average of the first n terms of a sequence of random variables is practically constant if n is large enough. In many practical applications the number of the experiments (i. e. the integer n) depends on chance. Chapter 10 deals with the questions of the sum of a random number of independent random variables.

The results of the previous chapters are applied in Chapter 11 to number theory, to statistics, and to information theory. To begin with, two classes of expansions of the numbers $x \in [0, 1]$ are studied: the first is the so-called Cantor series, the second is a very general expansion introduced by RÉNYI. In connection with this second expansion only the case of the continued fractions is treated in detail. As regards applications in statistics, the book investigates the estimation of the distribution and of the density functions.

The book is well-readable and, in spite of its relatively short extent, the most important results of the laws of large numbers are presented in it with complete proofs.

F. Móricz (Szeged)

Pál Révész, Die Gesetze der grossen Zahlen (Lehrbücher und Monographien aus dem Gebiete der exakten Wissenschaften, Mathematische Reihe, Band 35), 176 Seiten, Birkhäuser-Verlag, Basel und Akadémiai Kiadó, Budapest, 1968.

The German edition is the translation of the English original, reviewed above.

András Ádám, Truth functions and the problem of their realization by two-terminal graphs, 206 pages, 34 figures and 11 tables, Akadémiai Kiadó, Budapest, 1968.

The concept of truth functions appeared firstly in mathematical logic, mainly by studying the simplest functions such as negation, disjunction, conjunction, and equivalence. The functions in question are called frequently Boolean functions too, for the logician BOOLE investigated them firstly systematically. The discovery of the applicability of these functions to engineering and cybernetics has given rise to the development of a self-reliant theory.

The author gives a systematic survey on the main directions of the theory of truth functions from a mathematician's point of view. According to this, the author mentions only in short remarks the technical origine or significance of the matter presented.

The book is divided into two parts; it contains ten chapters, an appendix, bibliography, and indices of names and subjects. For the convenience of the reader, the symbols which are used in the book are also summarized. The first part treats the theory of truth functions considered from the point of view of "discrete analysis", the second one deals with the problem of their realization by two-terminal graphs, mainly by using graph-theoretical methods.

The content of the book is almost self-contained; the presupposed knowledge of the reader does not exceed some fundamental notions and results of mathematics (lying mainly in the theories of sets, numbers and groups).

The majority of the results contained in the book has appeared solely in the original articles of several authors; only a small part of them was already elaborated in books. Such results also occur which have never been published before. A virtue of the book is the fact that the proofs presented are always given in a complete form, and this makes the subject more readable than in the original publications.

In Chapter 1 the fundamental concepts of this topics are considered. § 5 contains an unpublished theorem of T. BAKOS that is useful for the design of logical machines, more precisely, for the planning of technically advantageous enumeration of the places of the definition of a truth function.

Chapter 2 is devoted to the presentation of the fundamental researches of QUINE on prime implicants and disjunctive normal forms of minimal length. These results have an important applicability to the problems of simplification of electrical networks. The author presents two algoritms of QUINE to determine all the prime implicants of a truth function, and a further method to find all the representation of a truth function by irredundant disjunctive normal forms. The end of this chapter contains some results of the author on relations between repetition-free superpositions of truth functions and prime implicants.

Chapter 3 treats interrelations between conjunctive and disjunctive normal forms in order to reach all prime implicants of a truth function in a more economical way than in the preceding chapter. From the theoretical point of view the theorem of NELSON, from the view-point of practical applications the method of VOISHVILLO solve completely the problem of finding all the prime implicants starting with an arbitrary conjunctive normal form of a truth function.

Chapter 4 deals with the characterization of the systems of truth functions which are functionally complete concerning superposition. In §§ 14—15 the functionally complete systems of truth functions, in §§ 16—17 the complete systems of certain special automata consisting of a function and a non-negative integer number expressing time delay are considered.

Chapter 5 is devoted to the questions of uniqueness of the "deepest" decomposition by repetition-free superpositions. The main result is due to KUZNETSOV whose theorem asserts that if a truth function is given, then its deepest repetition-free superpositional decompositions are necessarily "almost conciding" with each other.

Chapter 6 deals with numerical questions, particularly, with the number of certain sets consisting of truth functions. The author presents three permutation groups in connection with the set of the truth functions of n variables, and discusses the following problem: What is the number of classes of the essentially different truth functions? The "essentially different" is meant in three distinct senses, namely, two truth functions belong to the same class if one of them is originated from the other either by applying a permutation of its variables or by substituting some variables by their negatives or by applying simultaneously both previous processes. The treatment of this subject is based chiefly upon the results of G. PÓLYA.

Chapter 7 presents notions and results concerning linearly separable functions, i. e. treats the possibility of assigning real numbers to the variables of a truth function in such a manner that the function value is true exactly if the sum of the numbers, assigned to the true variables, exceeds a given threshold.

The first part of Chapter 8 presents general preliminaries of graph theory. The author deals in detail with the questions of series-parallel decomposition of 2-graphs (2-graphs always means a strongly connected two-terminal graph), of canonical decomposition of indecomposable 2-graphs, etc. The results of §§ 38—39 are due to the author. Introducing the notion of completable and separating pair of edges and that of quasi-series decomposition, he gives a new method of building up 2-graphs from simpler ones.

In Chapter 9 the author introduces several concepts of realization of truth functions: (i) by functional elements, (ii) by 2-graphs that is attributed to SHESTAKOV and SHANNON, (iii) by threeterminal graphs that was proposed by L. KALMÁR. In the rest the repetition-free realization of truth functions by 2-graphs is studied. § 45 contains some own results of the author concerning the problem of existence, after introducing the notion of the quasi-series decomposition of truth functions. The chapter ends with the result of TRAHTENBROT on the solution of the problem of unicity.

Chapter 10 mentions some aspects of the problem of optimal realization containing two theorems of LUNTS.

In the Appendix the author mentions some possibilities of the future development by presenting the notion of stochastic truth functions introduced by JOHN VON NEUMANN.

The book will be useful both for the theoretical-minded mathematicians who either want to make research in the theory of truth functions or to be thoroughly acquainted with the more essential results of this topic, and expectably also for scientists, well-educated in mathematics, who may apply the theory of truth functions in their work.

F. Móricz (Szeged)

K. Reidemeister, Vorlesungen über Grundlagen der Geometrie (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 32.), berichtigter Nachdruck, X+147 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1968.

Seit Erscheinung der Originalausgabe (für ihre Besprechung siehe: diese *Acta*, **5** (1930—32), S. 250) hat sich die Grundidee des Buches, bei der Begründung der ebenen affinen Geometrie dem Begriff der Gewebe eine Hauptrolle zu erteilen, bekanntlich als fruchtbar erwiesen. Die für die Grundlagen der Geometrie Interessierten werden also sicher diesen Nachdruck begrüssen. Durch Hinweise auf die nach 1930 erschienene Literatur werden die inzwischen erreichten Ergebnisse und die Anhaltspunkte zu aktuellen Forschungsfragen angegeben.

J. Szenthe (Szeged)

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