Local and residual properties in bicategories

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§ 1

In the theory of universal algebras local and residual properties are well known, and they are to some extent dual properties. It is easy to give a categorical definition of these notions, but category theoretically they are not exactly dual. In universal algebra it is proved that any residual property which is preserved under homomorphic images is local but the categorically dual statement is not true even in such a nice category as that of abelian groups (cf. [1], Exercise 3).

The purpose of this paper is twofold. On the one hand, we give a categorical generalization of this connection between local and residual properties. In this way it becomes clear why the dual statement is not true in universal algebra (the reason is GROTHENDIECK's axiom AB 5). On the other hand, as a possible interpretation of the dual statement, we present concrete categories in which it is true. This dual statement, however, yields well known facts of the general topology; we estimate it essential that such a categorical aspect is able to join quite different branches of mathematics.

In our investigations we shall consider a bicategory satisfying some rather natural additional requirements. In § 2 we shall give a categorical definition of local and residual properties with some cardinality-restrictions. Such a subtle definition is suitable with respect to the topological applications. We present also a lemma which establishes an equivalent formulation of a special case of GROTHENDIECK's axiom AB 5. This lemma will be used in the proof of the Theorem of § 3. § 3 is devoted to proving the categorical generalization of the connection between local and residual properties. In § 4 we give concrete categories in which the dual theorem is true (the category of 0-dimensional compact spaces and that of complete metric spaces with closed continuous mappings). By specialization we obtain e.g. that Lindelöf property is preserved by forming inverse limit of countable inverse systems of complete metric spaces.

The author wishes to express his thanks to J. GERLITS for his critical remarks and valuable advices.

§ 2

In general our terminology is based on Mitchell's book [8]. With respect to the interpretations of § 4 it is appropriate to use the notion of bicategory due to Isbell [5] (cf. also Semadeni [9] and Kennison [6]).

Let $\mathscr{C}$ be a category. Let $\mathscr{I}$ and $\mathscr{S}$ be classes of morphisms on $\mathscr{C}$. Then $(\mathscr{I}, \mathscr{S})$ is a *bicategory structure* on $\mathscr{C}$ provided that

(B_1) $\mathscr{I}$ and $\mathscr{S}$ are subcategories of $\mathscr{C}$,

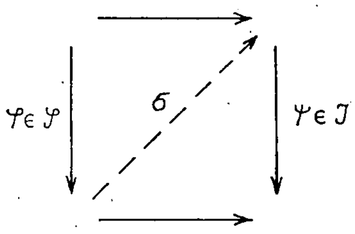
(B_2) $\mathscr{I} \cap \mathscr{S}$ consists exactly of all equivalences;

(B_3) The morphisms of $\mathscr{I}$ are monomorphisms and the morphisms of $\mathscr{S}$ are epimorphisms;

(B_4) Every morphism φ can be factored as $\varphi = \varphi_2 \varphi_1$ with $\varphi_1 \in \mathscr{S}$, $\varphi_2 \in \mathscr{I}$, moreover this factorization is unique to within an equivalence in the sense that if $\varphi = \nu\mu$ and $\mu \in \mathscr{S}$, $\nu \in \mathscr{I}$ then there exists an equivalence γ for which $\nu\gamma = \varphi_2$ and $\gamma\varphi_1 = \mu$.

The morphisms of $\mathscr{I}$ and $\mathscr{S}$ are called *injections* and *surjections*, respectively. A category equipped with a bicategorical structure is called briefly a *bicategory*.

Proposition 1 ([6] Prop. 1. 1). *Let $\mathscr{C}$ be a bicategory. Then*



(1)

(1) *$\varphi\psi \in \mathscr{I}$ implies $\psi \in \mathscr{I}$;*

(2) *$\varphi\psi \in \mathscr{S}$ implies $\varphi \in \mathscr{S}$;*

(3) *Every commutative diagram of the form indicated by figure (1) can be filled in at σ with commutativity preserved.*

If $\alpha: A_1 \to A$ is an injection, then A_1 is a *subobject of* A, if $\beta: B \to B_1$ is a surjection, then B_1 is called a *factorobject* of B.

An object S of a category $\mathscr{C}$ is called a *cosingleton*, if the following two conditions are satisfied (cf. Semadeni [9]):

(i) For every object A there exists exactly one morphism $\alpha: S \to A$;

(ii) For every object B there exists at least one morphism $\beta: B \to S$.

Throughout § 2 and § 3 we shall assume that the considered category $\mathscr{C}$ is a bicategory, further it satisfies the following axioms:

(A_1) $\mathscr{C}$ has a cosingleton;

(A_2) For every family $\{A_i\}_{i \in I}$, $|I| \leqq \aleph$, of factorobjects of any object A in $\mathscr{C}$ the counion $\bigcup^*_{i \in I} A_i$ exists;

(A_3) $\mathscr{C}$ admits products $\prod_{i \in I} A_i$, if $|I| \leqq \aleph$;

(A_4) $\mathscr{C}$ admits direct limits $\varinjlim \{A_i\}_{i \in I}$ if $|I| \leqq \aleph$.

We shall make use of some statements being easy consequences of (A_1) and (A_3).

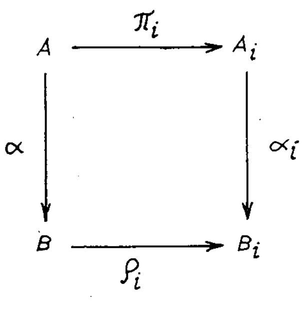
Suppose that S is a cosingleton, and denote by σ_i the only morphism $S \to A_i$. Let us suppose that *the class of all morphisms $A_i \to S$ is a set for every $A_i \in \mathscr{C}$*. Now, by the axiom of choice we can select exactly one morphism $\omega_i: A_i \to S$ for each $A_i \in \mathscr{C}$. Let us define $\omega_{ij} = \omega_i \sigma_j: A_i \to A_j$. By [9], 3,5 for any objects A_i, A_j, A_k we have $\omega_{ij}\omega_{jk} = \omega_{ik}$.

Proposition 2 ([9], 3. 6). *The projection $\pi_i: \prod_{i\in I} A_i \to A_i$, $|I| \leqq \aleph$, is a surjection for each $i \in I$, and there are injections $\sigma_i: A_i \to \prod_{i\in I} A_i$ such that $\pi_i\sigma_i = 1_{A_i}$, $\pi_j\sigma_i = \omega_{ij}$ for $i \neq j$.*

Proposition 3. *Consider $A = \prod_{i\in I} A_i$ and $B = \prod_{i\in I} B_i$, $|I| \leqq \aleph$, with the projections π_i and ϱ_i, $i \in I$, respectively. If $\alpha_i: A_i \to B_i$, $i \in I$ is a family of injections, then there exists a unique injection $\alpha: A \to B$ such that $\varrho_i\alpha = \alpha_i\pi_i$ holds for each $i \in I$.*

The proof will be analogous to that of [7], § 14. 3 in the case when the cosingleton is a zero object.

By the definition of the product there exists a unique morphism (the so-called canonical morphism) α such that diagram *(2)* is commutative for all $i \in I$. We have to show that α is an injection. Consider a factorization $\alpha = \nu\mu$ with $\mu \in \mathscr{S}$, $\nu \in \mathscr{I}$. By Proposition 1 (3) for each $i \in I$ there exists such a morphism σ_i that diagram *(3)* is commutative. Since $A = \prod_{i\in I} A_i$, the canonical morphism $\gamma: C \to A$



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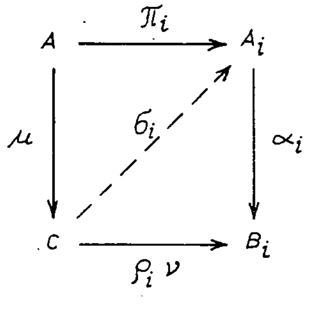


(3)

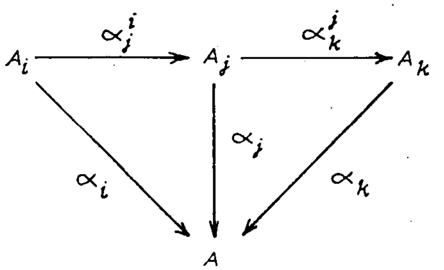
exists. Obviously $\gamma\mu$ has to be 1_A. Hence Proposition 1 (1) implies $\mu \in \mathscr{I}$ and thus α is indeed an injection.

In § 2 and § 3 we shall assume that the bicategory $\mathscr{C}$ satisfies condition

(C) *The direct limit $\varinjlim \{A_i\}_{i\in I}$ of every direct family of subobjects $\{A_i\}_{i\in I}$, $|I| \leqq \aleph$ of an object is the union $\bigcup_{i\in I} A_i$.*

Condition (C) without any restriction to the cardinality of I, is fulfilled by every category of any primitive class (i.e. variety) of universal algebras (as it turns out e.g. from [3], § 21) and for a complete abelian category (C) is equivalent to GROTHENDIECK's axiom AB 5 (cf. [4]. Proposition 1,8 or [8]. III Proposition 1,2). GROTHENDIECK [4] has also pointed out that a category satisfying axiom AB 5 as well as its dual one, has to consist of zero objects.

We need also an other form of condition (C). Let $\{A_i\}_{i\in I}$ and $\{B_i\}_{i\in I}$, $|I|\leqq\aleph$ be direct systems of subobjects of the objects A and B with $\bigcup_{i\in I} A_i=A$ and $\bigcup_{i\in I} B_i=B$ and with the commutative diagrams *(4* and *5)* for all $i\leqq j\leqq k\in I$.



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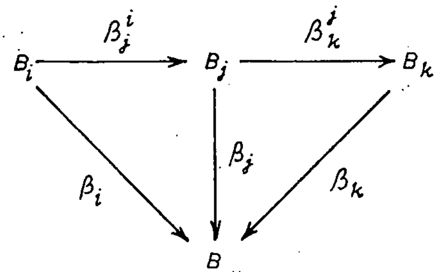
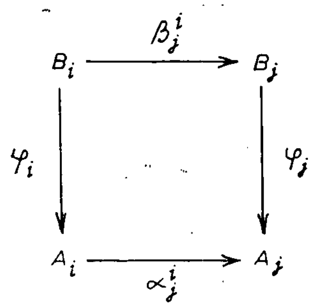


(5)

Lemma. *Assuming that* $\bigcup_{i\in I} A_i$, $|I|\leqq\aleph$, *exists in* $\mathscr{C}$ *for every direct system of subobjects* $\{A_i\}$ *of any object* A, *condition (C) is equivalent to condition*

(D) *If* φ_i: $B_i\to A_i$, $i\in I$, $|I|\leqq\aleph$ *is a system of surjections such that diagram (6) is commutative for all* $i\leqq j\in I$, *then there exists a unique surjection* φ: $B\to A$ $(B=\bigcup_{i\in I} B_i,\ A=\bigcup_{i\in I} A_i)$ *such that diagram (7) is also commutative for all* $i\in I$.



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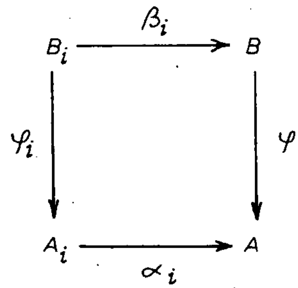
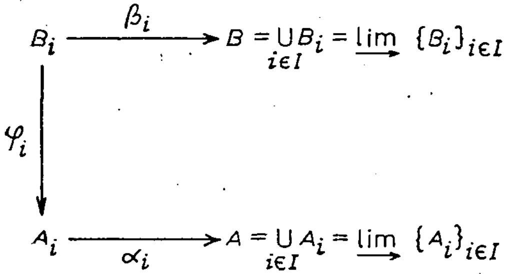


(7)

Proof. (C)⇒(D). According to (C) we have diagram *(8)*. Since $\alpha_i \varphi_i$ maps B_i into A such that

$$\alpha_j \varphi_j \beta_j^i = \alpha_j \alpha_j^i \varphi_i = \alpha_i \varphi_i \qquad i \leqq j \in I,$$

therefore by the definition of direct limit there exists a unique morphism, the canonical one, $\varphi: B \to A$ such that $\alpha_i \varphi_i = \varphi \beta_i$ holds for all $i \in I$. Moreover, by (B_4) φ can be



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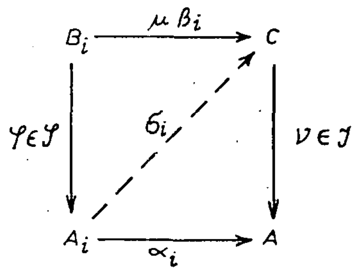
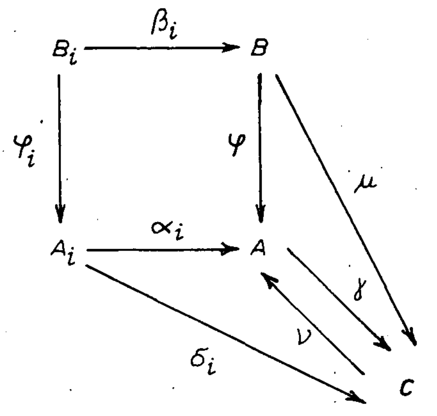


(9)

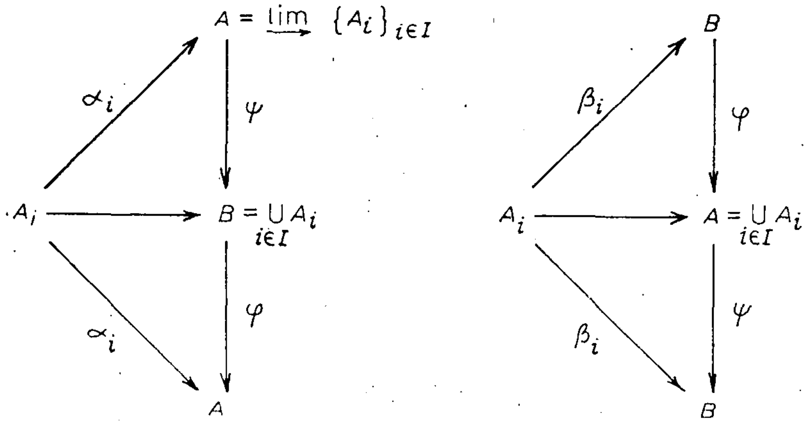
factored as $\varphi = \nu\mu$ with $\mu \in \mathcal{S}$, $\nu \in \mathcal{J}$ and so according to Proposition 1 (3) for each $i \in I$ there exists a morphism σ_i with commutativity preserved in diagram *(9)*. Hence by the definition of the direct limit there exists the canonical morphism γ such that diagram *(10)* is commutative for all $i \in I$.



(10)

Since $A = \varinjlim \{A_i\}_{i \in I}$, therefore $\nu\gamma = 1_A$ follows. Thus by Proposition 1 (2) we have $\nu \in \mathcal{S}$. This implies $\varphi = \nu\mu \in \mathcal{S}$.

(D)⇒(C). Put $A_i = B_i$, $i \in I$. Now by condition (D) there exists a morphism $\psi: \underrightarrow{\lim}\, \{A_i\} = A \to \bigcup_{i\in I} A_i = A$ and $\varphi: \underrightarrow{\lim}\, \{A_i\} = B \to \bigcup A_i = B$ such that diagrams *(11)* and *(12)* are commutative for all $i \in I$. By the uniqueness of φ and ψ it follows $\varphi\psi = 1_A$. Hence $\psi: \underrightarrow{\lim}\, \{A_i\}_{i\in I} \to \bigcup_{i\in I} A_i$ is an equivalence, and so condition (C) is satisfied.



(11) *(12)*

Consider an abstract property **P** of objects of $\mathscr{C}$, i.e. if A and B are equivalent objects, then either both A and B or none of them has property **P**. Since property **P** divides the objects of $\mathscr{C}$ into two classes, so the fact A has property **P** will be denoted by $A \in \mathbf{P}$.

Let $\aleph$ be a cardinality. By an *$\aleph$-local system* of subobjects of an object A one understands a direct system $\{A_i\}_{i\in I}$ such that $\bigcup_{i\in I} A_i = A$ and $|I| \leqq \aleph$. The object A is said to be *$\aleph$-locally* **P**, if there is an $\aleph$-local system of subobjects of A all A_i belonging to **P**. If every object which is $\aleph$-locally **P** actually belongs to **P** itself, then **P** is said to be an *$\aleph$-local property*. In view of condition (C), an $\aleph$-local property **P** means such an abstract property which is closed under forming direct limits of direct systems having cardinality $\leqq \aleph$.

We define an *$\aleph$-residual system* of an object A to be a system $\{A_i\}_{i\in I}$ consisting of factorobjects of A such that $\bigcup^*_{i\in I} A_i = \overset{}{\underset{\circ}{A}}$ and $|I| \leqq \aleph$. By an *$\aleph$-residual* **P** object we mean an object which has an $\aleph$-residual system consisting of factorobjects belonging to **P**. The property **P** is said to be an *$\aleph$-residual property*, if every object which is $\aleph$-residually **P** actually belongs to **P** itself.

Let us mention that $\aleph$-local and $\aleph$-residual properties are not dual notions.

However, any system $\{A_i\}_{i\in I}$, $|I|\leqq\aleph$, of subobjects of an object A with $\bigcup_{i\in I} A_i=A$ generates a direct system consisting of finite unions $\bigcup_{\text{finite}} A_k$, but $A_i\in\mathbf{P}$, $i\in I$, do not imply $\bigcup_{\text{finite}} A_k\in\mathbf{P}$.

§ 3

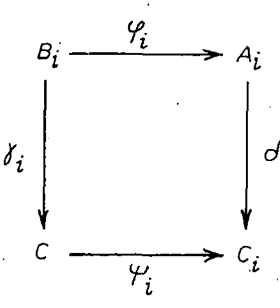
In this section we are going to prove the following

Theorem. *Let the bicategory $\mathscr{C}$ satisfy axioms* (A_1)—(A_4) *and condition (C). If* $\mathbf{P}$ *is an $\aleph$-residual property preserved by surjections then* $\mathbf{P}$ *is an $\aleph$-local property.*

Let us remark that this theorem is valid for any category of Ω-algebras. The corresponding statement without any restrictions to the cardinality, is just Proposition 7,4 of Cohn [1] (there the existence of cosingleton is not supposed).

Proof. The outline of the proof is the following. We shall consider an object A which is $\aleph$-locally $\mathbf{P}$ with an $\aleph$-local system $\{A_i\}_{i\in I}$, $A_i\in\mathbf{P}$, $|I|\leqq\aleph$. From $\{A_i\}_{i\in I}$ we construct an object B which is $\aleph$-residually $\mathbf{P}$, and so by the assumption B will have property $\mathbf{P}$. Further we shall show that there exists a surjection $B\to A$. Hence also the object A will have property $\mathbf{P}$.

Consider an object A having an $\aleph$-local system $\{A_i\}_{i\in I}$ with injections $\alpha_i: A_i\to A$, $\alpha^i_j: A_i\to A_j$ such that $\alpha_j\alpha^i_j=\alpha_i$, $i\leqq j\in I$, $|I|\leqq\aleph$, and $A_i\in\mathbf{P}$ for all $i\in\mathbf{P}$. Let $\xi_i: A\to D_i$ be an equivalence for all $i\in I$, and form the product $C=\prod_{i\in I} D_i$ with the projections $\pi_i: C\to D_i$. By Proposition 2, every π_i is a surjection. Further, for all $i\in I$, define C_i and C^*_i by $C_i=\prod_{i\leqq j\in I} D_j$ and $C^*_i=\prod_{i>j\in I} D_j$, respectively. (For the empty set $\emptyset$ $\prod_{j\in\emptyset} D_j$ means cosingleton.) According to Proposition 3 both C_i and C^*_i are subobjects of C. The object A_i can be embedded "diagonally" in C_i for any $i\in I$ as follows. The morphism $\delta^i_j=\xi_j\alpha^i_j: A_i\to D_j$ embeds A_i into D_j for every $i\leqq j\in I$. The canonical morphism $\delta_i: A_i\to C_i$ satisfies $\pi'_j\delta_i=\delta^i_j$ where π'_j is the projection $C_i\to D_j$. Hence Proposition 1 (1) implies that δ_i is an injection for each $i\in I$. According to Proposition 3 $B_i=C^*_i\times A_i$ is a subobject of $C=C^*_i\times C_i\times\prod_{i\|k\in I} D_k$ by an injection γ_i such that diagram *(13)* is commutative. Here φ_i and ψ_i denote the projections of B_i and C into A_i and C_i, respectively. Moreover by Proposition 2 they are surjections.



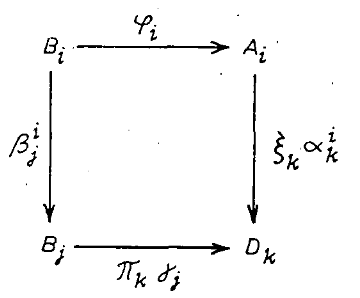
(13)

For any fixed $i<j\in I$, consider the injections $\xi_k\alpha^i_k: A_i\to D_k$ $(i\leqq k<j)$ and $\alpha^i_j: A_i\to A_j$. Now by Proposition 1 (1) the canonical morphism of A_i into $B_j=$

$= C_i^* \times \prod_{i \leqq k < j} D_k \times A_j$ is an injection for $i < j \in I$. So applying Proposition 3 to B_i and B_j we obtain that there exists a unique injection $\beta_j^i : B_i \to B_j$ such that *(14)* and *(15)* are commutative diagrams for $i \leqq k < j \in I$. Hence letting $\beta_i^i = 1_{B_i}$ for any $i \leqq k \leqq j$ we get $\varphi_j \beta_j^i = \alpha_j^i \varphi_i = \alpha_j^k \alpha_k^i \varphi_i = \alpha_j^k \varphi_k \beta_k^i = \varphi_j \beta_j^k \beta_k^i$, and by the uniqueness of β_j^i we have $\beta_j^i = \beta_j^k \beta_k^i$. Thus $\{B_i\}_{i \in I}$ forms a direct system. With respect to condition (C) we have $B = \varinjlim \{B_i\}_{i \in I} = \bigcup_{i \in I} B_i$, and so B is a subobject of C by injection β. Now $\pi_i \beta$ maps B into D_i. Since D_i is a subobject of B_{i+1} and so of B by an injection δ_i, therefore by Proposition 2 for the injection $\delta_i : D_i \to B$ we have

$$\beta \delta_i = \sigma_i \quad \text{and} \quad \pi_i \beta \delta_i = \sigma \delta_i = 1_{D_i}.$$

Thus by Proposition 1 (2) the morphism $\pi_i \beta : B \to D_i$ is a surjection for all $i \in I$.



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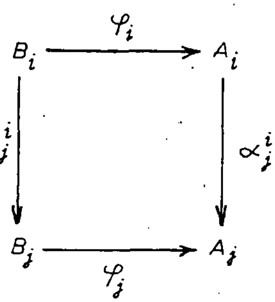


(15)

Now we are able to prove $B \in \mathbf{P}$. To this aim it is sufficient to show $\bigcup_{i \in I}^* D_i = B$ because $\mathbf{P}$ is an $\aleph$-residual property and $\pi_i \beta : B \to D_i \approx A_i \in \mathbf{P}$ is surjection for all $i \in I$. Put $B_0 = \bigcup_{i \in I}^* D_i$. Now there exist surjections $\beta_0 : B \to B_0$ and $\varrho_i : B_0 \to D_i$ such that $\varrho_i \beta_0 = \pi_i \beta$ is valid for all $i \in I$. On the other hand B_0 can be embedded in $C = \prod_{i \in I} D_i$ by the canonical morphism ϱ_0 such that $\pi_i \varrho_0 = \varrho_i$ holds for every $i \in I$. Hence we have $\pi_i \varrho_0 \beta_0 = \pi_i \beta$ and the uniqueness of β yields $\varrho_0 \beta_0 = \beta$. Since β is an injection, so by Proposition 1 (1) also β_0 is an injection. Hence $\beta_0 \in \mathcal{S} \cap \mathcal{I}$, and so B and B_0 are equivalent objects. Thus $B \in \mathbf{P}$ is proved.

By (I) we have $\alpha_j^i \varphi_i = \varphi_j \beta_j^i$ for all $i \leqq j \in I$. Thus with respect to the Lemma there exists a surjection $\varphi : B \to A$ such that $\varphi \beta_i = \alpha_i \varphi_i$ is valid for all $i \in I$. Since property $\mathbf{P}$ is preserved by surjections, so $B \in \mathbf{P}$ implies $A \in \mathbf{P}$. Hence $\mathbf{P}$ is $\aleph$-local, and the theorem is proved.

§ 4

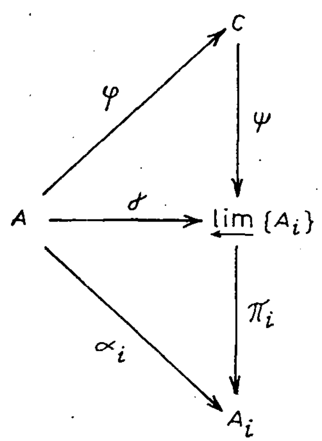
1. Let $\mathscr{C}_B$ be the category of Boolean algebras. In $\mathscr{C}_B$ all the conditions (A_1)—(A_4) as well as (C) are satisfied without any restriction on the cardinality. By the well-known duality between the category $\mathscr{C}_B^*$ of 0-dimensional compact spaces (the so-called Boolean spaces) and that of Boolean algebras, the dual statement of the Theorem holds in $\mathscr{C}_B^*$. (According to the duality we hint to [9].)

2. As an other possibility to interpret the dual statement of the Theorem, let us consider the category $\mathscr{C}_M$ consisting of complete metric spaces (with bounded metric) and closed continuous mappings. For the notions well known in general topology' we refer to ENGELKING's book [2]. $\mathscr{C}_M$ becomes a bicategory by choosing $\mathscr{I}$ and $\mathscr{S}$ to be the class of closed continuous embeddings and that of continuous onto-mappings. The one point space is a singleton in $\mathscr{C}_M$, so the dual condition (A_1^*) of (A_1) is satisfied. If $\{A_i\}_{i\in I}$ is a system of closed subspaces of a space A, then the closure $\overline{\bigcup_{i\in I} A_i}$ of the union of the subspaces will be, clearly, the categorical union of the subspaces A_i, $i\in I$. Hence also (A_2^*) is fulfilled in $\mathscr{C}_M$.

To show the validity of (A_3^*), let us remark that in the category of topological spaces the coproduct is precisely the disjoint union of the spaces. We shall show that the disjoint union of complete metric spaces is again a complete metric space. By [2] Theorem 4. 2. 1 this disjoint union is a metric space. Consider a Cauchy sequence $\{x_n\}$ in the disjoint union $A = \bigoplus_{i\in I} A_i$. Now to any $\varepsilon>0$ there exists a natural number N such that $\varrho(x_n, x_m)<\varepsilon$ holds for every $n, m \geqq N$. This is possible only if x_n and x_m belongs to the same space A_i for a fixed $i\in I$. Since A_i is complete, so the sequence $\{x_n\}$ is convergent in A_i as well as in A.

(A_4^*) $\mathscr{C}_M$ admits inverse limit of countable inverse systems. Since the Cartesian product of a countable number of complete metric spaces is again such a space ([2] Theorem 4. 3. 7), so taking into account that the inverse limit is a closed subspace of the Cartesian product, the validity of (A_v^*) is obvious.

(C*) Let us consider an inverse system $\{A_i, i=1, 2, \ldots\}$ of quotient spaces $A_i\in\mathscr{C}_M$ of a space $A\in\mathscr{C}_M$. First of all we shall show that the canonical map $\gamma: A\to \varprojlim \{A_i\}$ is an onto-mapping. By (B_4) γ can be factored as $\gamma=\psi\varphi$ with $\psi\in\mathscr{I}$ and $\varphi\in\mathscr{S}$ such that diagram *(16)* is commutative for $i=1, 2, \ldots$. Consider an arbitrary element $a\in\varprojlim \{A_i\}$. Obviously a has the form $(\ldots, a_j, \ldots, a_i, \ldots)$ with $\pi_j^i a_i=a_j$. The inverse image $F_i=(\pi_i\psi)^{-1}(a_i)$ is a closed subset of C,



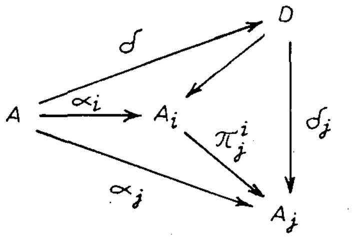
(16)

moreover $F_1 \supset F_2 \supset \dots$ holds. If $\delta(F_i)$ denotes the diameter of F_i, then by [2] Theorem 4. 2. 2 we have

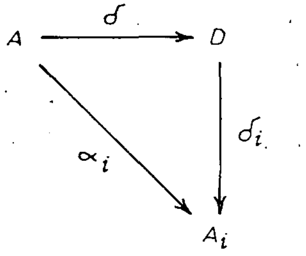
$$\delta(F_i) \leqq \delta(\pi_i^{-1}(a_i)) \leqq \frac{1}{2^i}.$$

Thus $\lim_{i\to\infty} \delta(F_i)=0$, and so the completeness of C implies that the intersection $\bigcap_{i=1}^{\infty} F_i$ is not empty. For $b \in \bigcup_{i=1}^{\infty} F_i$ we have, clearly, $\psi(b)=a$, and so ψ is a surjection too. Hence γ is indeed an onto-mapping.

If $\delta: A \to D$ is a surjection such that diagram *(17)* is commutative for $i=1, 2, \dots$, then diagram *(18)* is also commutative.



(17)



(18)

Hence by the definition of the inverse limit, for the canonical map $\delta': D \to \underline{\lim}\ \{A_i\}$ we get $\delta'\delta=\gamma$. Now $\gamma \in \mathscr{S}$ implies $\delta' \in \mathscr{S}$, and so $\underline{\lim}\ \{A_i\} = \bigcup_{i=1}^{\infty}{}^* A_i$ is proved. Thus $\mathscr{C}_M$ fulfils condition (C^*).

A reformulation of the dual statement of the Theorem is

Theorem.* *Let* **P** *be a topological property of complete metric spaces such that it is inherited for closed subspaces, and it is preserved to the closure of the union of countable many subspaces belonging to* **P**. *Then property* **P** *is preserved by forming inverse limit of countable inverse systems of complete metric spaces.*

To motivate Theorem*, let us choose property **P** as follows:

a) **P** means the *Lindelöf property*;

b) **P** means that *the space A has weight* $w(A) \leqq \mathfrak{m} (\geqq \aleph_0)$.

Let us recall that in $\mathscr{C}_M$ the Lindelöf property is equivalent to the separability, and $w(A) \leqq \mathfrak{m}$ means exactly that A contains a dense subset of cardinality $\leqq \mathfrak{m}$ (cf. [2], Chapter 4).

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