Bibliographie

E. Artin, Galoische Theorie (Mathematisch-Naturwissenschaftliche Bibliothek, 28), iv + 86 pages, B. G. Teubner Verlagsgesellschaft, Leipzig, 1959. — DM 5.30

The book is a revised German translation by V. ZIEGLER of the English original (*Galois theory*, 2nd ed., Notre Dame University, Indiana, USA, 1948). Its aim is to make one familiar with the methods and problems of Galois theory. As no preliminary knowledge is presupposed from algebra, it appeals to every moderately advanced student with a sufficiently inquisitive mind.

The work is divided into three parts. The first one studies the basic concepts of linear algebra such as vector spaces over skew fields, systems of homogeneous linear equations, ranks of matrices, and determinants. The second part considers commutative fields and their algebraic extensions, among them separable and normal ones. As applications of the theory given here, the last part studies some special questions. After an account of solvable groups, the familiar criterion for solvability by radicals of an algebraic equation is presented. An equation of degree 5 is given that is not solvable in this way. Finally, construction by ruler and compass is discussed.

Simultaneously with the translation, some major improvements were carried out by the author in this editions such as: the proof of the fundamental theorem of Galois theory is rendered more coherent; a proof, based on E. LANDAU's ideas, of the irreducibility of the cyclotomic polynomials is included in the section on roots of unity; and the last part of the book has been rewritten entirely.

The careful selection and the clear presentation of the material are among the greatest virtues of this book.

A. Máté (Szeged)

G. Asser, Einführung in die mathematische Logik. I. Aussagenkalkül (Mathematisch-Naturwissenschaftliche Bibliothek, 18), VI + 184 pages, B. G. Teubner Verlagsgesellschaft, Leipzig, 1959. — DM 11.25

This is the first book of a planned three-volume introduction into mathematical logics. This volume deals with propositional calculus; and, as is claimed in the preface, the second one is going to discuss first-order predicate calculus; finally, the third one is planned about higher order calculi. The work intends to remain within the bounds of traditional two-valued logics, and no discussion on intuitionism, applications to foundations of mathematics, or philosophical problems, etc., will be included.

Within these limits, the scope of the present volume is fairly comprehensive, though, of course, no book of this size can aim at completeness. It gives a detailed treatment of propositional expressions; a large number of tautologies is listed; a long discussion on the theory of propositional normal forms is presented; deducibility in propositional calculus is dealt with, and its completeness is proved; axiomatizability, consistency, and completeness are studied; independence, propositional matrices, and deductively closed sets are discussed; representability of functions in various extensions (via propositional matrices) of the propositional calculus is dealt with; a syntactic characterization, based on ideas going back to G. GENTZEN, is given of expressions of the classical propositional calculus; the book ends with an account of the generalized notion of calculi.

A. Máté (Szeged)

Leonard M. Blumenthal and Karl Menger, Studies in geometry, XVI+512 pages, San Francisco, W. H. Freeman and Company, 1970.

This book represents on a graduate course level a wide variety of research subjects in modern geometry initiated by works of L. M. BLUMENTHAL and KARL MENGER.

Part 1, written by BLUMENTHAL treats geometrical aspects of lattice theory and of Boolean algebras. After a review of basic concepts and facts normed lattices are introduced and questions concerning their metric geometry discussed. In Boolean algebras a generalized distance of two elements, which is again an element of the algebra, is defined and shown to possess the basic properties of ordinary distance. The Boolean metric spaces thus obtained give rise to a generalized metric geometry on their own.

In Part 2 an exposition of the foundations of projective geometry is given by MENGER, based on the concept of projective structures introduced by him in 1932. This approach having remarkable advantages is worked out for 3 dimensions including results on Arguesian and Pappus planes and Dandelin spaces.

Part 3, written by BLUMENTHAL gives a survey of metric geometry. Basic results concerning metric segments and lines of normed linear spaces are presented first to be applied to various metric characterizations of Banach and euclidean spaces. Some sections treat topics from the metric theory of curves pertaining generally to an at most 3-dimensional situation. This part ends with a concise presentation of the metrization of the Gauss curvature by A. WALD and of related results due to W. A. KIRK.

In Part 4 MENGER gives with characteristic lucidity an expositions of topological curve theory which also embodies results following the edition of his famous *Kurventheorie*.

The whole book is a very suggestive new illustration to the fruitfulness of MENGER's well-known objective to bring traditional topics into the scope of modern mathematics.

J. Szenthe (Szeged)

Functional Analysis and Related Fields, Proceedings of Conference in honor of Professor Marshall Stone, held at the University of Chicago, May 1968. Edited by F. E. BROWDER, 1 portrait VIII+241 pages, Berlin—Heidelberg—New York; Springer-Verlag, 1970. — DM 58.—

The volume contains a number of important contributions. Contents: F. E. BROWDER: Nonlinear eigenvalue problems and group invariance; S. S. CHERN, M. DO CARMO, and S. KOBAYASHI: Minimal submanifolds of a sphere with second fundamental form of constant length; HARISH-CHANDRA: Eisenstein series over finite fields; E. HEWITT: $\mathscr{L}_p$ transforms of compact groups; T. KATO and S. T. KURODA: Theory of simple scattering and eigenfunction expansions; G. W. MACKEY: Induced representations of locally compact groups and applications; L. NACHBIN: Convolution operators in spaces of nuclearly entire functions on a Banach space; E. NELSON: Operants: A functional calculus for non-commuting operators; I. SEGAL: Local non-linear functions of quantum fields; A. WEIL: On the analogue of the modular group in characteristic p; A. ZYGMUND: A theorem on the formal multiplication of trigonometric series; S. MACLANE: The influence of M. H. Stone on the origins of category theory; Remarks of Professor STONE.

The topic of most of these papers is intimately related to some aspects of the oeuvre of M. H. STONE (spectral theory, operators, Boolean algebras, general topology, Weierstrass—Stone theorem, harmonic analysis, etc.). Stone's theorem on one parameter groups of unitary operators, and the Stone — von Neumann uniqueness theorem for the Weyl commutation relations in quantum mechanics are, for example, basic for the "imprimitivity theorem" of Mackey and for its consequences, an excellent survey of which is contained in the paper of MACKEY. The topics of the papers of KATO—KURODA, and SEGAL are also closely related to that theorem of Stone.

Less known is the influence of the work of Stone on the origins of Category Theory. This influence is discussed in the very interesting paper by MACLANE. In particular, the influence of Stone's work on adjoint linear operators and on the representation of Boolean rings is considered. In his Remarks, STONE makes a few comments by way of response to some historical questions raised by MACLANE about the concept of adjoint functors or operators, and also discusses some general aspects of mathematical research. He points to "the need for offering our future research mathematicians a broad preparation that will enable them to cope successfully with the increasingly complex interconnections that bind mathematics into a single whole."

Béla Sz.-Nagy (Szeged)

J. C. Burkill and H. Burkill, A Second Course in Mathematical Analysis, VII + 526 pages, Camqridge University Press, 1970.

This book is a continuation of "A First Course in Mathematical Analysis" by J. C. BURKILL, accordingly the abovementioned First Course is the best foundation for this second Course. The book is intended for mathematics students who are familiar with the concept of a limit and its applications to infinite series and to the differential and integral calculus.

The subject is presented in the more abstract setting of metric spaces. The first nine chapters concentrate on general analysis and real functions, and the last five on complex functions. The chapter headings indicate the scope of the book in more detail. They are: 1. Sets and Functions, 2. Metric Spaces, 3. Continuous Functions on Metric Spaces, 4. Limits in the Space R^1 and Z, 5. Uniform Convergence, 6. Integration (The Riemann and Riemann—Stieltjes integrals are treated but the Lebesgue integral is left out), 7. Functions from R^m to R^n, 8. Integrals in R^n, 9. Fourier Series, 10. Complex Function Theory, 11. Complex Integrals. Cauchy's Theorem, 12. Expansions. Singularities. Residues, 13. General Theorems. Analytic Functions, 14. Applications to Special Functions.

The book contains nearly seven hundred exercises and problems with their solutions, and, furthermore the authors give some illustrations of the numerous definitions.

At the end of each chapter there are given notes, some historical ones and others indicating further developments. A list of references and a detailed index are added.

The book is lucidly arranged and the exposition is clear.

L. Leindler (Szeged)

Herbert Busemann, Recent synthetic differential geometry, VI + 110 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1970.

The author's approach to global problems of intrinsic differential geometry presented in his book "*The Geometry of Geodesics*" has proved to be superior to the standard ones in several problems especially in those which concern Finsler geometry. Since the issue of this book in 1955 various results have been achieved on subjects already treated or but proposed in it, and now the author gives a systematic account of these recent results.

In Chapter I questions concerning the foundations of the theory are treated. Since the basic role of the HOPF—RINOW theorem in global differential geometry is in striking contrast with its usually rather loose presentation, a thorough analysis as to the assumptions of this theorem is given first. Next the problem of the topological structure of the so-called G-spaces is considered. These G-spaces are metric spaces satisfying a system of axioms motivated by differential geometry and are the basic concepts in the author's theory. In spite of their geometric versatility the topological structure of these spaces does not admit an easy description. By a modification of the theory of the r-spaces of A. KOSINSKI the author proves that finite dimensional G-spaces possess the property of domain invariance and that small spheres of a G-space are not contractible. The proof of the theorem of B. KRAKUS that every 3-dimensional G-space is a topological manifold is given as well. An earlier result of the author is also presented, stating that a continuously differentiable and regular G-space is a topological manifold, where, of course, continuous differentiability is meant in a metric sense.

Desarguesian G-spaces form the subject of Chapter II. Here the concept of similarity, introduced earlier by the author, is studied first. Then a solution of a problem on the imbedding of desarguesian G-spaces is presented together with results of the author on those r-dimensional areas in n-dimensional affine space for which the r-flats minimize the area. A characterization of HILBERT's and MINKOWSKI's geometries among all straight desargueasian spaces in given by generalizing a theorem of L. BERWALD on desarguesian spaces with constant Finsler curvature. In this context the result of P. KELLY and E. G. STRAUSS on the characterization of hyperbolic geometry among HILBERT's geometries by a metric curvature concept is mentioned.

Results on length preserving maps are presented in Chapter III. Some basic observations on shrinkages, equilong maps and local isometries are made first. Then several conditions are presented under which a G-space cannot have proper local isometries. These conditions have been given by the author, W. A. KIRK, and the reviewer. For elementary spaces all those equilong maps are constructed which are locally finite in the sense that the covering by the closures of their regions of injectivity is locally finite.

Chapter IV contains results on the behaviour of geodesics. First a theorem of the author is presented which he managed to prove by applying a method due to V. A. EFREMOVIČ and E. S. TIHOMIROVA. By means of this theorem it is then shown that on a closed hyperbolic space form with an intrinsic, but not necessarily symmetric distance there is a class of geodesics and half geodesics which behave uniformly like the hyperbolic geodesics and half geodesics. The facts which make axes of motions interesting have been pointed out by the author and F. P. PEDERSEN. These facts are reproduced here and some remarks are made how results of H. HEDLUND, M. MORSE and G. A. BLISS can be extended and proved in a more elegant manner. Inverse problems of the calculus of variations are considered next and results of the author, H. SALZMANN and L. A. SKORNYAKOV are presented concerning the so-called collineation groups, especially the 1-dimensional and the discrete ones. Due to the fundamental role of the GAUSS-BONNET theorem in the discussion of the behaviour of geodesics in the large on 2-dimensional Reimannian surfaces the question concerning the generalization of this theorem to FINSLER surfaces is essential. Since numerous different generalizations are known it is shown that a generalization comprising the two central features of the theorem does not exist. The result of E. KANN on the generalization of BONNET's theorem concerning the diameter of Riemannian manifolds to G-surfaces is mentioned with some interesting remarks and an observation of E. M. ZAUSTINSKY is presented concerning the divergense property of geodesics on G-surfaces. A brief report is given on results of G. M. LEWIS on conjugacy to points at infinity, a problem taken up initially by J. NASU..

Results on motions are presented in Chapter V. Spaces with a finite or 1-parameter group of motions are considered first and spaces with a group of motions transitive on the set of geodesics second.

Chapter VI contains observations on the contents of the theory and the methods applied in it. A brief outline of the development of the FINSLER geometry is also given with some very interesting criticism on trends of investigations. Several objectives for further research are mentioned.

By commenting on new results and presenting them in the context of the theory as a whole the author has given an authentic review of an advancing trend of investigations following the issue of *"The Geometry of Geodesics"*.

J. Szenthe (Szeged)

O. Bottema, R. Ž. Djordjević, R. P. Janić, D. S. Mitrinović, P. M. Vasić, Geometric inequalities, 151 pages, Groningen, Wolters-Noordhoff Publ. Co., 1969.

Die geometrischen Ungleichungen sind so alt, wie die Geometrie selbst, zahlreiche solche Ungleichungen stammen aber aus den letzten zweihundert Jahren, ja einige sogar aus den kürzlich vergangenen Jahrzehnten. Es handelt sich insbesondere um Ungleichungen für die Seiten eines Dreiecks bzw. von zwei Dreiecken, über spezielle Dreiecke und über Ungleichungen von Vierecken.

Die einfacheren Ungleichugen werden im Buch bewiesen, über die schwereren aber geben uns reichliche Hinweisungen und Anführungen aus der Literatur Auskunft. Die Verfasser haben auch die neueste einschlägige Literatur benutzt.

J. Berkes (Szeged)

K. Chandrasekharan, Arithmetical functions (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 167), XI+231 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1970. — DM 58. —

As is claimed in the introduction, this book can be looked upon as a sequel to the author's *Introduction to analytic number theory,* Berlin—Heidelberg—New York, Springer-Verlag, 1968. (For a review see these *Acta,* **31** (1970), p. 185.) Its title might be slightly misleading: namely, it gives account on an advanced level of problems associated with the distribution of primes, the partition function, and the divisor function. On the part of the reader a moderate acquaintance with the methods of analytical number theory is only assumed. This can be acquired from the work just cited.

The style of the book is clear, the proofs are carefully arranged. Nevertheless, at some places one might wonder whether a little more intuitive description of the ideas behind the proofs would not help the reader to struggle himself through the heaps of symbols that are inevitable in such a context.

To give a more precise description of the contents of the book, here are the chapter headings: I. The prime number theorem and Selberg's method, II. The zeta-function of Riemann, III. Littlewood's theorem and Weyl's method. IV. Vinogradov's method, V. Theorems of Hoheisel and Ingham, VI. Dirichlet's L-functions and Siegel's theorem, VII. Theorems of Hardy—Ramanujan and of Rademacher on the partition function, VIII. Dirichlet's divisor problem. Each chapter ends with notes containing additional information related to the subject; thus the reader is spared the diversion of attention that could have been caused by giving this information in footnotes or interjected remarks. The book ends with a small *List of books* related to the topics discussed and a *Subject index.*

As the above description may show, the book is on the whole really good. The printer's contribution to its value should certainly be mentioned: the format is very pleasing. Considering the large number of formulas in the text, this must have needed a very meticulous effort.

A. Máté (Szeged)

A. H. Clifford and G. B. Preston, The algebraic theory of semigroups, Vol. II, XV+350 pages, Providence, Rhode Island, American Mathematical Society, 1967.

People interested in semigroup theory have been looking forward to the publication of this second part of the monograph of CLIFFORD and PRESTON. The work thus completed is undoubtedly the best one in the field. Though Volume II does not attain the classical compactness of Volume I, this being almost impossible because of the present unaccomplished state of investigations, it gives a clear outline of the trends in the development of the theory. Its best parts, as §§ 6.1—6.4, 9.4—9.5, Chapter 12, and others, will make a pleasure to those liking mathematics not only as science but as an art as well.

The main deficiency of the book is that it makes sometimes the impression of a selection of papers; the authors failed in fusing their material into an organic whole. A simple example: on pp. 108 and 109 there are two constructions. Both amount to an extension of a semigroup by the bicyclic semigroup and yet even the notations used in the two cases are essentially different.

Here is a short résumé of the contents of the book:

Chapter 6 deals with minimal ideals and minimal conditions. Its main part is constituted by building up ŠT. SCHWARZ's theory of the right (and left) socles of a semigroup, and culminates in the authors' theorem about the structure of the union of the left and right socles. The last section contains some results concerning semigroups with minimal condition on left (or right) ideals.

In Chapter 7 the authors give a survey of results in the theory of inverse semigroups, as those of B. M. ŠAIN about the representations of inverse semigroups by one-to-one partial transformations, the characterization of congruences in inverse semigroups by their classes containing idempotents, and some special questions concerning such congruences (idempotent separating congruences etc.).

Chapter 8 continues the study of simple semigroups, considered already in Chapters 2 and 3, Volume I, and it contains some problems of embedding semigroups in simple semigroups of different types, too. The discussion ends with PRESTON's theorem claiming that any semigroup can be embedded in a bisimple semigroup with identity.

After some preliminary information on free semigroups, Chapter 9 contains RÉDEI's theorem on finitely generated commutative semigroups (all semigroups of this kind are finitely presented), and HOWIE's results concerning the embeddability of a set of semigroups S_i with a common subsemigroup $\cap S_i = U$ into a larger semigroup. As for the first one, P. FREYD has since then found an amazingly simple proof (*Proc. Amer. Math. Soc.*, **19** (1968), 1003). In the last section the authors discuss the method of constructing the minimal cancellative congruence containing a given relation, on a semigroup S.

The first part of Chapter 10 presents the basic results of the theory of DUBREIL's principal one-sided congruences and those of CROISOT in the field of two-sided congruences. Then there follows a specialization for semigroups of GOLDIE's generalized Jordan-Hölder theorem, the analysis of the congruences of completely 0-simple semigroups, and the description of the congruences of the full transformation semigroup of a set X, due to MALCEV.

The next chapter is devoted to the theory of semigroup representations by transformations of a set.

The last chapter seems to be the authors' gift to the reader: it contains, in a highly delectable presentation, the results of LAMBEK and MALCEV concerning embeddability in a group.

Of course, one can always find interesting subjects omitted from a monograph, and there is little use in listing them; nevertheless, it seems to the reviewer that a chapter devoted to different methods of semigroup constructions could contribute not only to the completeness but also to the coherency of the work.

G. Pollák (Szeged)

F. M. Hall, An introduction to abstract algebra, Vol. 2, XII+388 pages, Cambridge University Press, 1969. — £ 3.50

This book, formally the second part of a two-volume textbook for abstract algebra, is a self-contained introduction to the theory of the most important algebraic structures, as groups, rings, fields, vector spaces, and Boolean algebras. It can be used by undergraduate students and their teachers as an auxiliary material, and it is also excellent for self-education.

An informal style without loss of exactness and a didactical construction make this volume an excellent explanation of its subject-matter. The author avoids successfully the customary Scylla and Charybdis in this genre: tedious sophistication and disproportion in the selection of topics. We mention also the well-chosen exercises (there are about seven hundred of them).

B. Csákány (Szeged)

F. Hausdorff, Nachgelassene Schriften. Bände I, II: Studien und Referate. Herausgegeben von G. Bergmann. Bd. I: XXI+538 pages, Bd. II: IX+570 pages, Stuttgart, B. G. Teubner, 1969. — DM 122. —

These two illustrious volumes contain a large number of yet unpublished writings of F. Hausdorff (1868—1942) in form of facsimile reproduction of the original hand-written manuscripts. The writings are selected from studies and reviews originating from the interval March 1934 through March 1938. In about two years, further two volumes are planned, selecting from writings prepared between March 1938 and January 1942. Afterwards, the publication of earlier wirtings may also be contemplated.

It is simply astonishing, and rather unfortunate, that so great a number of Hausdorff's writings remained unpublished. The reason for this is perhaps partly found in Hausdorff's personality itself. But it must not be forgotten that the circumstances in Germany were then unfavourable to the publication of these works. Indeed, in the last years of his life, Hausdorff was actually barred from publishing.

The value of these writings is enormous. A review by Hausdorff has quite exceptional qualities. It is very detailed and thoroughgoing; and if it does not actually pinpoint an unaccuracy, then we may take it as a guarantee of the faultlessness of the reviewed work. As for the studies, they are written with crystal-clear logic and precision. Most of them centre around topics in topology and set theory. It is certainly great loss that these writings have not been available earlier. But even now, they should be of great importance, not only to those turning to Hausdorff's activity with an historical interest, but also to those interested in the subjects discussed; even though the handwritten format seems to favour the former ones.

The handwriting of Hausdorff is quite well legible and the technical quality of the reproduction is excellent.

A. Máté (Szeged)

C. A. Hayes and C. Y. Pauc, Derivation and Martingales (Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 49), VII+203 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1970.

Today we are witnessing a new revival of the theory of derivation and, in a close connection with this, of the modern theory of martingales. But there is no book, to the knowledge of the reviewer, to collect and systematize the essential results of the subject. This is the first attempt, and an excellent one at that, to survey the new methods and results of the modern theory of derivation in a single volume.

The work is relatively self-contained, and all of it can be read by anyone with a preliminary knowledge in the elements of set theory, topology, and measure theory. Most of the notions and results needed from these fields are presented and explained in the text, though in a way presupposing a certain general mathematical maturity of the reader.

The book consists of two parts, in six and four chapters, respectively, a *Complements*, and a *Subject Index*. Each part has its own bibliography, which is short but to the point.

Part I is devoted to the pointwise derivation of scalar set functions employing, in general, abstract derivation bases or blankets, due to R. DE POSSEL and A. P. MORSE, respectively. The principal tool is a Vitali property, of strong or weak type, whose precise form depends on the derivation property studied. The converse problem is also investigated: what covering properties can be deduced from derivation properties of σ-additive set functions.

The "halo" properties furnish the foundation for many of the modern results to establish a Vitali property, or sometimes to produce directly a derivation property. A few sections are concerned with the abstract version of the strong Vitali theorem, modelled after those given by BANACH or CARATHÉODORY. The main results presented are the theorem of JESSEN—MARCINKIEWICZ—ZYGMUND, valid in n-dimensional Euclidean spaces R^n with the interval basis, and the theorem of MORSE on the universal derivability of star blankets. In the context of the former one, the maximal theorem of HARDY—LITTLEWOOD is proved for R and R^2.

Part II begins by studying the notion of increasing stochastic bases with directed index sets on which premartingales, semimartingales and martingales are defined. Convergence theorems, due largely to K. KRICKEBERG, are treated in great detail using various types of convergence: stochastic, in the mean, in L_p-spaces, in Orlicz spaces, and with respect to the order relation. To each theorem for martingales and semimartingales there corresponds a theorem in the atomic case in the theory of cell functions, where cells can be construed as generalized intervals. The derivates concerned are global.

Finally, in a separate chapter, concepts are reintroduced and results on pointwise convergence and on point derivates are deduced from results obtained in the earlier chapters of Part II, under supplementary assumptions. To mention one example, the Radon-Nikodym integrand is defined as a derivative.

The *Complements* consists of such sketches related to topics in Part I and II as derivation of vector-valued integrals, global derivatives in locally compact topological groups, vector-valued martingales and derivation, etc.

The presentation of the book is concise but always clear and well-readable. It should have a great appeal to the students of graduate courses as well as to the mature mathematicians interested in the modern theory of derivation.

F. Móricz (Szeged)

H. P. Künzi and W. Oettli, Nichtlineare Optimierung: Neuere Verfahren. Bibliographie (Lecture Notes in Operations Research and Mathematical Systems, Vol. 16) 180 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1969. — DM 12, —

Das Buch knüpft sich in mehrerer Hinsicht an das im Jahre 1962 erschienene Werk von H. P. KÜNZI und W. KRELLE, *Nichtlineare Programmierung*. Sie ist auch insofern als eine Fortsetzung der erwähnten Arbeit zu betrachten, daß sie nur die Bearbeitung der Forschungsergebnisse enthält, die nach 1960 publiziert wurden.

Die Verfasser betonen, daß sie bei der Ausarbeitung des Themas die Vollständigkeit nicht erzielt haben, es ist aber ohne Zweifel, daß hier die nichtlineare Programmierung und hauptsächlich die wichtigsten neuen Methoden der konvexen Programmierung Platz bekamen. Man betrachtet insbesondere die folgenden Methoden:

Das Schnittebenenverfahren von Kelley; die tangentielle Approximationsmethode von Hartley und Hocking; die modifizierten Schnittebenenverfahren von Kleibohm, Veinott und Zoutendijk; MAP (Method of Approximation Programming) von Griffith und Stewart; die reduzierte Gradientenmethode; die Methode der Penalty-Funktionen; SUMT (Sequential Unconstrained Minimization Technique) von Fiacco und McCormick; die Zentrumsmethode von Huard; und ein Verfahren der zulässigen Richtungen.

Die Beschreibung der Methoden erstreckt sich auf die theoretischen Grundlagen, auf die Gedankenfolge, die zur Methode führt, es geht auch auf ihre Anwendungsweise, auf deren Möglichkeiten und Grenzen ein. Es handelt sich hier auch um Konvergenzprobleme und natürlich um alle weiteren Probleme, die mit der gegebenen Methode in Verbindung stehen. Die Behandlung geht — sehr richtig — auf Einzelheiten nicht ein, zahlreiche Behauptungen sind ohne Beweis angegeben. (Die Beweise sind auf Grund des angegebenen Literaturverzeichnisses vorzufinden.)

Mehr als die Hälfte des Umfangs nimmt eine Bibliographie ein. Hier wurden die Werke aufgeführt, die zu dem Themenkreis der nichtlinearen Programmierung gehören oder mit ihm in enger Verbindung sind, dessen theoretische Grundlage oder seine Anwendung bilden. (Die Arbeiten, die zu dem Kreis der dynamischen, stochastischen und ganzzahligen Programmierung gehören, sind nicht einbezogen.)

Das Buch gibt ein getreues Bild über den heutigen Stand der nichtlinearen Programmierung.

L. Megyesi (Szeged)

Gilbert Helmberg, Introduction to Spectral Theory in Hilbert space, XIII+346 pages, Amsterdam—London, North-Holland Publ. Co., 1969. — Hfl. 60, —

The purpose of the book is to be an introduction to the subject and no part of it is claimed to be original. However, the author expresses "the hope that among the many different people interested in this subject there might be some who find this presentation particularly suited to their personal taste."

Chapters: The concept of Hilbert space. — Specific geometry of Hilbert space. — Bounded linear operators. — General theory of linear operators. — Spectral analysis of compact linear operators. — Spectral analysis of bounded linear operators. — Spectral analysis of unbounded selfadjoint operators.

There is an Appendix recalling some pertaining results of the theory of Real Functions.

B. Sz-Nagy (Szeged)

P. Lorenzen, Formale Logik (Sammlung Göschen, Bd. 1176/1176a), second corrected edition, 165 pages, Walter de Guyter & Co., Berlin, 1962. — DM 5,80

A review of the first edition appeared in these *Acta*, **20** (1959), p. 219, where a strong criticism was directed against the book. In what this review was undoubtedly right was the pinpointing of some disturbing misprints. It seems that the editor laid a great care on eliminating them in this second, corrected edition. Also, true enough, as the cited review observed, the slightly complicated notational framework of the book is not to be praised. But the criticism seems certainly unfair in bypassing the virtues of this book.

The main virtue of this small paperback is that it is a pleasure to read it. The style is very lively, full of background explanations. Thereby it is inevitable that the author puts forward his own views on the foundations of logics, and so the treatment is influenced by the author's "operative views" on mathematics (cf. P. LORENZEN, *Einführung in die operative Logik und Mathematik,* 2nd edition (Berlin — Heidelberg — New York, 1969); for a review see these *Acta,* **30** (1969), p. 329). Since this view is not shared by most mathematicians, this feature certainly makes the book less attractive to those starting to learn mathematical logic, but, on the other hand, it gives a special flavour to the book.

The material comprises accounts of syllogisms, of a logic of junctors, of an effective logic of junctors, of a logic of quantifiers, and of logic of equality. Under these titles are also discussed such classical results as the consistence and completeness of the propositional calculus, completeness of the predicate calculus, Church's theorem, etc.

A. Máté (Szeged)

I. J. Maddox, Elements of Functional Analysis, X+208 pages, Cambridge University Press, 1970. — 50s.

As the author states in the Preface, in his view "the field of elementary functional analysis is the ideal place in which to learn some abstract structural mathematics and to develop analytical technique". He presents now a book which provides an introductory, though non-trivial, course on functional analysis which can be followed by every student who has completed basic courses on real and complex variable theory. Most of the examples which are chosen to motivate the basic concepts or to illustrate the strength of the results achieved, involve sequence spaces rather than integration spaces; thus Lebesgue integral theory is not an absolutely necessary prerequisite (however, completeness of the L_p spaces is proved by referring to the relevant theorems on the interchange of limit and integration).

Chapter titles (and some key words): 1. Basic set theory and analysis. (Zorn's lemma.) — 2. Metric and topological spaces. (Category and uniform boundedness.) — 3. Linear and linear metric spaces. (Hamel "base" and Schauder "basis".) — 4. Normed linear spaces. (Open mapping, closed graph, and Hahn-Banach theorems.) — 5. Banach algebras. (Gelfand representation theorem.) — 6. Hilbert space. (Orthonormal sets.) — 7. Matrix transformations in sequence spaces. (Summability. Tauberian theorems.)

There is a great number of exercises, and the last chapter (which concerns an area of special interest to the author) ends with problems for further study, some of them quite difficult.

Operators and spectral theory are barely touched. But within its limits chosen, the book is a useful introduction to the theory, written with much didactical care.

Béla Sz.-Nagy (Szeged)

Jaques L. Mercier, An introduction to tensor calculus, VIII+152 pages, Wolters-Noordhoff Publishing, Groningen, 1971.

The wide variety of topics exposed nowadays generally in tensor form in the technical and scientific literature requires a working knowledge of tensor calculus from the interested reader. This book is intended to help students and engineers to provide themselves with such a knowledge.

Part I is an introduction to the invariant formulation of tensor calculus. The subjects which are exposed here range from the fundamental concepts to such ones as Riemannian tensors. In Part II covariant differentiation is introduced and developed to an extent required by its applications in engineering science.

The topics treated in this book are of course very much the same as in the numerous others of its kind. Yet, by his exceptional care to attain a reasonable maximum of mathematical rigour and by his personal skill in composing an ideal blend of illustrative examples and exercises the author succeeded in writing a book which, owing to these distinguishing qualities, would surely be welcome by a wide class of readers.

J. Szenthe (Szeged)

Jean Pierre Serre, Abelian *l*-adic representations and elliptic curves, New York—Amsterdam, W. A. Benjamin, Inc., 1968.

The book, written in collaboration with JOHN LABUTE and WILLEM KUYK, reproduces with a few complements the lectures of the author at McGill University, Montreal, in 1967. The l-adic representations treated here have been introduced by Y. TANIYAMA in 1957 and are the algebraic analogue of the locally constant sheaves of topology.

In Chapter I the definition and some examples of l-adic representations are given first. Then assuming that the ground field is a number field rational l-adic representations are considered. The attaching of L-functions to rational l-adic representations is discussed too.

Chapter II contains the construction of some abelian l-adic representations of a number field, which is originally due to G. SHIMURA, Y. TANIYAMA and A. WEIL, and is given here with some modifications.

In Chapter III the question is considered whether an abelian l-adic representation of a number field can be obtained by the method of the preceding chapter, and in this respect a necessary and sufficient condition is given. The problem whether any abelian rational semi-simple l-adic representation of a number field is ipso facto locally algebraic is also taken up and proved for the case when the field is a composite of quadratic fields.

Chapter IV is concerned with the l-adic representation defined by an elliptic curve. Its aim is to determine, as precisely as possible, the image of the Galois group, or at least the corresponding Lie algebra.

A considerable number of instructive exercises and abundant motivation by remarks and references serve to make this book a very readable exposition.

J. Szenthe (Szeged)

C. A. Rogers, Hausdorff measures, viii+ +179 pages, London, Cambridge University Press, 1971. — £ 3.8.

The theory of Hausdorff measures was initiated by C. CARATHÉODORY in 1914 when he studied linear and p-dimensional measures in n-dimensional Euclidean space, thus giving a general solution to the problem of measuring surfaces. This theory was further developed by F. HAUSDORFF; as an illustration of his results he showed how to assign a positive finite measure to Cantor's ternary set in a natural way. Since then the progress in the field has been enormous, largely due to the work of A. S. BESICOVITCH and his students. Despite this progress, until now there has been no book discussing the fundaments of the subject.

A book of this size cannot serve as general reference on a subject with such diversified applications as the theory of Hausdorff measures, but this one does achieve a lot. It outlines the main core of the theory; it sets down a standard terminology and restates many results scattered in the mathematical literature or known as "folklore", in a sufficiently general from. These features make the book indispensable for research mathematicians in measure theory; but the simple style makes it also very attractive to students.

Measures have two, more or less distinguishable, major roles in mathematics. First, they can serve for sizing sets, and, second, they can be used to define integrals. In the present work, naturally, the first aspect prevails. We are going to give a closer description of the contents:

The book is divided into three parts. The first one studies the general aspects of measure theory. The author departs from the standard terminology by calling measure what is usually referred to as outer measure (we shall do the same below). The construction from pre-measures and the properties of measures are discussed, with a special stress on measures in topological and in metric spaces, and on non-σ-finite measures in general. The chapter ends with an account of the Souslin operation. The second chapter deals with the more general aspects of Hausdorff measures. After their definiton, their behaviour with respect to mappings and their use for measuring surface areas are considered. Existence and comparison theorems are studied extensively. Accounts of Souslin sets, of consequences of the increasing sets lemma, and of comparable net measures follow. Finally, a special section is devoted to the investigation of non-σ-finite sets. The topic of the last chapter is the applications. The first section here is a general survey of them, mentioning only the most important ones. The first of the applications given in detail is an account of JARNÍK's pioneering investigations concerning sets of real numbers defined in terms of their expansions into continued fractions. The second one describes a part of the study of S. J. TAYLOR and the author on additive set functions in Euclidean space (this is the only part of the book where integration is also considered). The book ends with an extensive bibliography and an index.

A. Máté (Szeged)

I. Singer, Bases in Banach Spaces. I. (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen mit besonderer Berücksichtigung der Anwendungsgebiete, Band 154), VIII+ +668 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1970.

The concept of basis is a fundamental tool to investigate the structure of various abstract spaces in functional analysis such as Banach spaces, F-spaces, topological linear spaces, etc. Although there are at present possibly more than a thousand publications in existence on the theory of bases, there has to date been a serious gap in the textbook literature. The recent appearance of two books helps to fill this gap, and it will most certainly be a major stimulant to the development of the subject.

The books are, in order of appearance, *Introduction to the theory of bases,* by J. T. MARTI (for a review, see e. g. these *Acta,* **31** (1970), 379—380) and the book under review. The two books complement each other, the former being an elegant introduction which beautifully conveys the spirit of the subject; and the latter a monograph which contains an encyclopaedic discussion of the results known today on bases in Banach spaces as well as in other spaces and of some unsolved problems on them. Some so far unpublished results and observations of the author have also been included in this latter work.

To make the book under review accessible to a larger circle of readers the author gives exact references to textbooks when basic results from functional analysis are used without proof. Results which have appeared only in periodicals are usually proved as lemmas.

Chapter I begins with the basis problem, i. e. whether or not every separable Banach space possesses a basis, and enumerates some of its reformulations. The basis problem was first raised explicitly in the famous book of BANACH, and despite many efforts in solving it, it has remained one of the most significant open problems of functional analysis.

Next, the reader will find a thorough discussion of some of the deeper properties of the most important types of bases for Banach spaces in a great detail. The reviewer should especially like to emphasize the exhaustive presentation of the following two topics. The first concerns properties of strong duality and weak duality, which can be formulated as follows: given a basis $\{x_n\}$ for a Banach space E, what can we say about the sequence $\{f_n\}$ of coefficient functionals associated with $\{x_n\}$ when both E and the conjugate space E^* are endowed with their norm-topologies or weak topologies, respectively. The converse problem is also dealt with. The second one concerns stability theorems of PALEY-WIENER type, in particular the famous KREIN—MILMAN—RUTMAN theorem. These theorems assert that various properties of a sequence $\{x_n\}$ in a Banach space are "stable" in the sense that they are preserved by every sequence $\{x_n\}$ "sufficiently near" the sequence $\{x_n\}$.

Chapter II contains a relatively full account of special classes of bases in Banach spaces. It is divided into two parts: I. Classes of Bases not Involving Unconditional Convergence, and II. Unconditional Bases and Some Classes of Unconditional Bases.

Separate sections are devoted to a systematic study of each of the following particular classes of bases: monotone and strictly monotone bases, normal bases, positive bases, rétro-bases in conjugate spaces, shrinking bases and boundedly complete bases, both considered and studied first in detail by R. C. JAMES, Besselian and Hilbertian bases, bases of types P and P^*, bases of types l_+ and $(l_+)^*$, etc; such special classes of unconditional bases as orthogonal and hyperorthogonal bases, which are the "unconditional analogues" of monotone bases, symmetric and subsymmetric bases, perfectly homogeneous bases, etc; and, furthermore, absolutely convergent bases and uniform bases.

One of the main problems that are considered for each special class of bases is whether or not there exists in every separable Banach space a basis belonging to the respective class. In finite dimensional Banach spaces the solutions, in general, are known, and with a few exceptions they are obvious. In infinite dimensional Banach spaces the answer to the corresponding existence problems is either negative or unknown (an affirmative answer would also imply an affirmative answer to the basis problem). In the first case counter examples are given, while in the second case one considers the more restricted problem of the existence of bases of that class in infinite dimensional Banach spaces with bases.

In the course of these investigations, a number of interesting special properties of particular classes of bases are considered, and certain interrelations between these classes are established.

Both chapters end with "Notes and remarks", which have a double purpose. They contain references to original papers in which the principal results in question have been discussed, and in addition, they contain references to some results related to but not included in those given in the text.

The author tried on the whole to adhere to the traditional terminology and notation; there are only a few exceptions. At any rate, a *Notation*, an *Author*, and a *Subject Index* have been provided to give guidance to the reader. Moreover, known interconnections between related concepts and results are sometimes summarized in the form of a table.

The bibliography concerns only the material of Volume I and does not aim at being complete, but wants merely to give a useful orientation to the reader. The bibliography for Volume II will be given separately in that volume.

The book has been carefully and accurately written. The style of its presentation is tight, with hardly a word wasted. There is a disturbing lack of motivation in some places: the author usually states the theorems without background explanations.

To sum up, the present volume contains a great wealth of information in a concise and polished form, and, what is especially welcome, a great number of problems in an explicit form. It will certainly indicate the location of the weak and of the strong spots in the edifice of the theory built so far, and thereby facilitate both the study of the subject, as it exists today, and future research on it.

The second volume, in preparation, will treat upon the following topics: Generalizations of the notion of a basis; Applications to the study of the structure of Banach spaces; Some properties of bases in concrete Banach spaces; Bases in general (not necessarily separable) Banach spaces; Bases in topological linear spaces.

F. Móricz (Szeged)

Lajos Takács, Combinatorial Methods in the Theory of Stochastic Processes (Wiley Series in Probability and Mathematical Statistics), XI+262 pages, New York—London—Sidney, John Wiley & Sons, Inc., 1967.

As the author writes in the Introduction, the aim of this valuable book is to show that for wide classes of random variables and stochastic processes the problem of finding the distribution of the supremum for both sums of random variables and sample functions of stochastic processes can be solved in an elementary way, and this problem, in turn, frequently arises in various fields in the theory of probability. Most of the results presented were achieved by the author during the period 1961—1966 and some have already been published in a series of papers. The book is diveded into eight chapters.

Chapter I (Ballot theorems) contains a generalization of the following classical ballot theorem of J. Bertrand. If in a ballot candidate A scores a votes and candidate B scores b votes, $a \geqq b$, then the probability that A is leading throughout the counting of the votes is $(a-b)$ $(a+b)$, provided all the possible voting records are equally probable. TAKÁCS's generalization, which serves as a base for all the subsequent considerations of the book, is the following. *Let $\varphi(u)$ be a nondecreasing function on $[0, t]$ for which $\varphi'(u)=0$ almost everywhere and $\varphi(0)=0$. Let $\varphi(t+u) = \varphi(t)+\varphi(u)$ for $u \in [0, t]$. Define $\delta(u)=1$ if $\varphi(v)-v \leqq \varphi(u)-u$ for every $v \in [u, u+t]$, and $\delta(u)=0$ otherwise. Then $\int_0^t \delta(u)du = t-\varphi(t)$ whenever $\varphi(t) \leqq t$.**)

Chapter 2 (Fluctuations of sums of random variables) deals with the determination of the distribution of the maximum of sums of cyclically interchangeable, interchangeable and independent, identically distributed random variables and gives also a discrete generalization of the classical ruin problem.

Chapter 3 (Fluctuations of sample functions of stochastic processes) determines the distribution of the supremum of stochastic processes whose increments are cyclically interchangeable, interchangeable, or stationary independent, and proves a continuous generalization of the classical ruin problem.

Chapter 4 (Random walk processes) treats special processes of the above kind and a random walk process and the Brownian motion process.

Chapter 5 (Queuing processes), Chapter 6 (Dam and storage processes) and Chapter 7 (Risk processes) demonstrate further applications of the preceding general theorems in the theories determined by the chapter headings in the brackets.

Chapter 8 (Order statistics) starts with another extension of the ballot theorem, then by this extension the author gives new proofs and generalizations of the results of Gnedenko, Koroljuk Mihalevič, Smirnov, Birnbaum, Pyke, Tingey and others concerning the exact distribution of Kolmogorov — Smirnov — Rényi type statistics and also computes the exact distribution of the one sided random sample-size M. Kac and similar statistics.

There is an Appendix containing various topics referred to in the text. After every chapter there are some problems (most of them are not merely exercises, but are intended to be extensions of the material covered) whose complete solutions may be found at the end of the book. Each chapter ends with a carefully compiled bibliography and the author should be praised also for commenting historically most of the problems. The good didactical structure and the clear style make the book very well readable.

S. Csörgő (Szeged)

*) We remark that L. GEHÉR has given a further generalization of Takács's theorem the proof of which is very simple. His paper appeared in these *Acta*, **29** (1968), 163— 165.