Bibliographie

Zelling S. Harris, Mathematical structures of language (Interscience Tracts in Pure and Applied Mathematics, No. 21), IX+230 pages, New York—London—Sydney—Toronto, Interscience Publishers, John Wiley and Sons, 1968. — 112 s.

The book is an expansion of a lecture given by the author at the Courant Institute of Mathematical Sciences. It does not attempt to present a unified treatment of what is called mathematical linguistics; such a treatment at today's stage of development of the subject could probably not be given. Rather, the book is a report on the author's own work.

The pursued aim is not to build an elegant mathematical theory which has some relevance to linguistics; rather, to define a mathematical structure that comes as close to describing natural languages as seems possible at the present time. The main result of the book is, as the author claims, the definition of such a structure. The structure finally arrived at is rather complicated: it has a family of primitive arguments and five finite families of operators acting on primitive arguments or operators. This is, however, not unexpected if we consider how complicated a natural language really is.

Once such a structure has been defined, one can prove theorems about it. The extent how far these theorems are interpretable as true properties of natural languages may be a good check of how close the given structure comes to describing natural languages. Also, the study of related mathematical structures should be inspiring for linguistics.

The book is written in a lucid style, with many illustrating examples from the English language. Its chapter headings are: 1. Introduction. 2. Properties of language relevant to a mathematical formulation. 3. Sentence forms. 4. Sentence transformations. 5. Structures defined by transformations. 6. Regularization beyond language. 7. The abstract system. 8. The interpretation. The book ends with an Index. At the end of the Introduction a list of works is given that contain more detailed information about parts of the material.

Attila Máté (Szeged)

P. Rosenstiehl and J. Mothes, Mathematics in management: the language of sets, statistics and variables, translated from French, xvi+392 pages, Amsterdam, North—Holland, 1968.

The traditional approach to teaching mathematics in high-school is to provide a basis for those wanting to continue their studies in engineering or science. As a result, up to quite recently, most other people gladly severed all ties with mathematics as something irrelevant to their lives at the age of eighteen. Yet, it was proven quite some time ago, that the applications of mathematics are not restricted to engineering and science. In particular, efficient business management cannot live without them.

Of course, there are specialists in applications of mathematics to business and industry, and business administrators and managers need not have such a specialized knowledge. What they need is an overall picture of where and how to apply mathematics to problems in management. This will enable them to judge when they should invoke the help of specialists; and, in fact, without such a knowledge, they may have extreme difficulty in communicating their problems to the mathematician.

This was kept in mind when, in the beginning of the 1960s, l'Ecole des Hautes Etudes Commerciales started to include a new course in mathematics in its programme. The French original of this book (Mathématiques de l'action, Dunod, Paris, 1968) is based on the experience gathered from this course during several years. This should by itself be a guarantee of the quality of the book.

As it should seem clear from what has been said so far, this book is intended for people whose main interest is not science. This does not mean that low standards of mathematical precision are applied. In fact, the material is presented in a clear and rigorous way. A great number of illustrating examples and excercises are given; many of these help one to grasp the relevancy of the discussed material to problems encountered in management.

The contents of the book can be best illustrated by the chapter and section headings: I. Subsets and partitions of a finite set (1. Elements and sets. 2. The set $\mathscr{P}(E)$ of the subsets of a finite set E. 3. Boolean algebra. 4. Partitions of a finite set). II. Organisation, classification and enumeration (1. General remarks on the statistics of a set. 2. The genealogy of simplexes. 3. Compartments and objects. 4. Morphisms). III. Events and probability (1. The language of events. 2. Probability: a measure of events. 3. Numerical estimation of probabilities). IV. Random variables (1. Discrete random variables. 2. Continuous random variables. 3. Two-dimensional random rariables). V. Common probabilistic models (1. Discrete models. 2. Continuous models. 3. Confrontation of the observations and the model).

Each chapter ends with a summary, practical excercises with solutions, and the description of one of more fields of applications. The book ends with a few tables useful in statistics and a subject index.

As the above description of the contents shows, the special considerations in the preparations of this book do not make its scope so limited as it would seem natural. The book should be useful to everyone who directly or indirectly may be confronted with applications of mathematics, including those interested in various branches of science. A further volume is planned on programming.

Attila Máté (Szeged)

S. A. Naimpally—B. D. Warrack, Proximity spaces (Cambridge Tracts in Mathematics and Mathematical Physics, No. 59), X+128 pages, Cambridge University Press, 1970.

The idea of using the relation of "nearness" of two subsets of a space as a basic tool of introducing a structure goes back to a congress talk of F. Riesz in 1908. However, this idea was not systematically developed earlier than the work of V. A. Efremovič (1952), who defined proximity spaces axiomatically. Since that time, the theory of these spaces produced interesting and deep results and found important applications, so that a monograph on this subject fills a serious gap in the literature of general topology.

The authors divided the book into four chapters preceded by a short account on historical background. The first of them presents basic definitions and facts, the second gives the theory of Smirnov compactification (with the help of the method of clusters). In the third chapter we find the most important interrelationships between proximity and uniformity, including some generalized concepts of uniformity (Alfsen—Njåstad uniformities, contiguities). The main subject of the final chapter is a survey of various kinds of generalized proximities and further generalizations of uniformities (syntopogenous spaces of the referee, generalized topological spaces of D. Doičinov, sequential proximites of S. G. Mrówka, generalized proximities of S. Leader, M. W. Lodato, W. J. Pervin, generalized uniformities of C. J. Mozzochi, etc.); a similar subject (local proximity) was previously presented in Chapter 2. Each chapter is followed by a series of references to the fairly complete bibliography standing at the end of the volume.

This monograph is very useful for a reader interested in modern developments of general topology. Although nothing else is postulated than basic knowledge from the theory of topological spaces, it is advantageous to be familiar with the theory of uniform structures because some concepts (e. g. that of a uniformly continuous mapping) are used without a definition. The referee succeeded in finding only a very small number of misprints and errors.

Á. Császár (Budapest)

Josef Stoer—Christoph Witzgall, Convexity and optimization in finite dimensions. I (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 163), IX+293 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1970. — DM 54, —

This book provides an excellent summary of mathematical results which are basic for the linear and nonlinear continuous variable programming in finite dimensions. The results of the various authors are discussed in the frame of a unified theory in a clear, elegant manner. The book consists of six chapters. Chapter 1 is devoted to the algorithmic solution of linear inequalities originated by Fourier. Farkas' theorem, the main transposition theorems, the duality theorem of linear programming and the complementary slackness theorems are deduced from the theory obtainable from the elimination procedure. Chapter 2 contains the basic theory of convex polyhedra. Beyond the classical results of Minkowski, Farkas, Carathéodory, Motzkin, Weyl, attention is paid to the important later results, among which we mention the combinatorial type Gale diagram characterizing the face structure of convex polyhedra: Chapter 3 deals with convex sets, their topological, combinatorial, extremal properties, supporting sets, separation and fixed point theorems. Chapter 4 deals with the properties of convex functions, the conjugate function theory of Fenchel and various generalizations of convexity. Chapters 5 and 6 are devoted respectively to the strongly related duality theory and saddle point theorems. Fenchel's duality theorem was generalized by Rockafellar and this is again generalized in Chapter 5 and then the previous theorems (proved by Gale, Kuhn, Tucker for linear programs and Dennis, Dorn, Eisenberg and Cottle for nonlinear programs) are shown to be special cases of this one. A similar line is followed in Chapter 6. The classical theorems of von Neumann and Kakutani were generalized by Sion while this generalization is extended in this book to the noncompact case. This contains as a special case the Kuhn—Tucker saddle point theorem. A direct approach to the Kuhn—Tucker theory and explanation of its connection with classical calculus is also given. In the foreword the authors promise to treat the algorithms of convex optimization in a subsequent volume.

A. Prékopa (Budapest)

H. Störmer, Semi-Markoff-Prozesse mit endlich vielen Zuständen (Lecture Notes in Operations Research and Mathematical Systems, Vol. 34), VII+126 Seiten, Berlin—Heidelberg—New York, Springer Verlag, 1970. — DM 12, —

Der Begriff der Semi-Markoff-Prozesse wurde von R. Pyke eingeführt. Diese Prozesse sind durch endlich viele oder abzählbar unendlich viele Zustände, durch die Übergangswahrscheinlichkeiten und durch die Verteilungsfunktionen für die Zustandsdauern angegeben und enthalten als Spezialfälle die Klasse der Erneuerungsprozesse, die Klasse der Markoff-Ketten und der Markoff-

Prozesse mit stetigem Zeitparameter. Die Semi-Markoff-Prozesse sind besonders geeignet zur Beschreibung einer großen Anzahl von Zufallsvorgängen in Natur, Wirtschaft und Technik; man kann z. B. sie für die Betrachtung der Wachstumspozesse, der Lagerhaltungs- und Warteschlangenprobleme oder der Probleme der Zuverlässigkeit von Systemen anwenden.

Im ersten Teil des Buches wird ein Abriß der Erneuerungstheorie angegeben; die Ergebnisse der Erneuerungstheorie liefern nämlich die wesentlichen mathematischen Hilfsmittel für die Behandlung der Semi-Markoff-Prozesse. Im zweiten Teil werden die für die verschiedenen Anwendungen wichtigsten Resultate der Theorie der Semi-Markoff-Prozesse hergeleitet. Nur die Semi-Markoff-Prozesse mit endlich vielen Zuständen werden diskutiert; diese haben nämlich für die Anwendungen besondere Bedeutung, und diese kann man mit den einfachen Mitteln des Matrizenkalkuls behandeln. Die Betrachtungsweise ist sehr klar und das Buch eignet sich vorzüglich dafür, daß man daraus eine Übersicht über diese für die Anwendungen wichtige Theorie gewinnt.

K. Tandori (Szeged)

F. Ferschl, Markovketten (Lecture Notes in Operations Research and Mathematical Systems, Vol. 35), VI+168 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1970. — DM 14,—

Dieses Buch ist die Ausarbeitung einer Vorlesung, die in den Jahren 1969—70 für Studenten der Volkswirtschaft gehalten wurde. Die wichtigsten Grundbegriffe und Ergebnisse der Theorie von Markovketten mit abzählbar unendlich vielen Zuständen und ihre wichtigsten Anwendungen in der Volkswirtschaft (Theorie der Warteschlangen, Erneuerungstheorie, Ruinprobleme) werden kurz, aber klar zusammengefaßt. Die Titel der einzelnen Kapitel sind die folgenden: Die Definition stochastischer Prozesse; Die Definition von Markovketten; Übergangswahrscheinlichkeiten; Die graphentheoretische Analyse von Markovketten; Das Rückkerverhalten von Markovketten; Stationäre- und Gleichgewichtsverteilungen; Transienz- und Rekurrenzkriterien; Algebraische Methoden zur Berechnung der Übergangswahrscheinlichkeiten. Am Anfang der einzelnen Kapitel — wo es notwendig ist — werden die entsprechenden Hilfsmittel (z. B. Hilfsmittel aus der Wahrscheinlichkeitstheorie, aus der Graphentheorie, aus der Reihentheorie) betrachtet. Es ist erwähnenswert, daß der Verfasser zur Einführung der verschiedenen Zustände der Markovketten die Begriffe von gerichteten Graphen und von der Theorie der Relationen anwendet. Mit dieser Betrachtungsmethode wird es möglich, die verschiedenen Begriffe klar einzuführen; diese abstrakte Betrachtungsmethode ist aber nur für Mathematiker interessant. Das Buch betrachtet ausführlicher auch die Methoden der Matrizenrechnung, und so gibt es praktisch handhabbare Rechenmethode für die Bestimmung der Potenzen von stochastischen Matrizen. Am Ende des Buches gibt es ein Literaturverzeichnis, in welchem die wichtigsten Werke über Markovketten kurz rezensiert werden.

K. Tandori (Szeged)

F. Bartholomes and G. Hotz, Homomorphismen und Reduktionen linearer Sprachen (Lecture Notes in Operations Research and Mathematical Systems, Vol. 32), XII+143 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1970.

Es ist bekannt, daß man die linearen Chomsky-Sprachen als direkte Verallgemeinerungen der endlichen Automaten betrachten kann, im Sinne, daß die durch endliche Automaten darstellbaren Mengen genau mit den Satzmengen der einseitig linearen Sprachen zusammenfallen. Auf Grund dieser Tatsache darf man erwarten, daß sich eine ganze Reihe von Ergebnissen der Theorie von endlichen Automaten auf lineare Sprachen übertragen läßt. Das Hauptziel dieser Monographie ist die Realisation dieses Programms dadurch, daß man zu jeder linearen Sprache eine endlich erzeugte

Kategorie zuordnet, deren Objekte und Morphismen Wortmengen mit einer ganz speziellen Struktur bzw. die Klassen gewisser Ableitungen dieser Sprache sind. Die systematische Anwendung der Methoden der Theorie von Kategorien gestattet einen Überblick darüber, welche Sätze über lineare Sprachen *rein algebraischer und welche spezifisch sprachentheoretischer Natur sind.*

In § 1 wird es gezeigt, daß jede durch endliche Automaten darstellbare Menge als Satzmenge einer linkslinearen Sprache auftritt. § 2 enthält gewisse spezielle kategorien-theoretische Vorbereitungen. Hier wird es sich zeigen, daß eine umkehrbar eindeutige Beziehung zwischen den aus der Automatentheorie bekannten normalen Standardereignissen und den endlich erzeugten freien Kategorien existiert. In §§ 3 und 4 werden der Homomorphiesatz und der Begriff des Reduktionsverbandes der endlichen Automaten auf die linearen Sprachen übertragen und nachher für endlich erzeugte freie Kategorien formuliert. § 5 enthält Untersuchungen über die Homomorphismen und Reduktionen der linearen Sprachen. Die Reduktionen sind im wesentlichen surjektive Funktoren zwischen den den linearen Sprachen zugeordneten freien Kategorien. In § 6 findet man einige Bemerkungen über die lokal eindeutigen und eindeutigen linearen Sprachen.

I. Peák (Szeged)

Paul F. Byrd—Morris D. Friedman, Handbook of Elliptic Integrals for Engineers and Scientists (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 67), XVI+358 Seiten, zweite, verbesserte Auflage, Berlin—Heidelberg—New York, Springer-Verlag, 1971.

Die erste Auflage dieses Buches erschien 1971. Der vorliegenden zweiten, verbesserten Auflage ist eine ergänzende Bibliographie hinzugefügt, die mehrere Hinweise auf die numerischen Näherungsmethoden und auf die entsprechenden Algorithmen für Rechenapparaten enthält. Das Buch umfaßt umgefähr 3000 verschiedene Formeln und im Appendix mehrere Werttabellen, die die Auswertung von elliptischen Integralen erleichtern. Die entsprechenden Beweise sind nicht diskutiert, nur die notwendigen Begriffe und die Formeln sind mitgeteilt. So its dieses Buch in erster Reihe für diejenigen Fachleute brauchbar, die in ihrer Tätigkeit nicht-elementare Integrale auswerten sollen.

Károly Tandori (Szeged)

D. S. Mitrinović, in cooperation with **P. M. Vasić, Analytic inequalities** (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 165), XII+400 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1970.

From the author's *introduction*: "If it is true that 'all analysts spend half their time hunting through the literature for inequalities which they want to use and cannot prove', we may expect that 'Analytic inequalities' will be of some help to them."

The aim of the present monograph is mainly to collect inequalities not dealt with in the classical work "Inequalities" by Hardy, Littlewood, and Pólya, and the book "Inequalities" by Beckenbach and Bellman. Some overlap was of course inevitable. However, as is claimed in the preface, even in the presentation of classical inequalities new facts have been added.

The collection is very rich, although it was impossible to strive for completeness. Where proofs or details could not be included for lack of space, references are given to original works. The first part, entitled "Introduction", concentrates on convex functions. The author considers the second part, entitled "General inequalities", the main part of the book. It is subdivided into twenty seven sections, some of which are further subdivided.

Studied here are, among many other topics: Young's and Hölder's inequality, the inverse of Hölder's inequality due to Dias, Goldman, and Metcalf, inequalities involving means, the λ-method of Mitrinović and Vasić, which may be used to connect various, seemingly unrelated inequalities, Steffenson's and Turán's inequalities, integral inequalities involving derivatives, and inequalities for vector norms. The third part, entitled "Particular inequalities", collects over 450 special results.

This collection should be very useful as a reference book for any research mathematician in analysis, but it may be useful to other people, like engineers, physicists, statisticians, etc., who might encounter inequalities in their works, and students may also benefit from parts of the book.

Attila Máté (Szeged)