Accretive operators: Corrections

By BÉLA SZ.-NAGY and CIPRIAN FOIAŞ

1. In Chapter IV of the monograph [1] we made a statement (Lemma 5. 2) which in general is false; the error stemmed from an incorrect use of Schwarz's inequality for non necessarily symmetric real bilinear forms[1]). However, a somewhat weaker statement (Lemma 5. 2 below) is sufficient for the concluding part of the proof of Proposition 5. 5 (i.e., Langer's uniqueness theorem for the accretive nth roth of a maximal accretive operator). Lemma 5. 3 (which is also needed in the proof of Proposition 5. 5) can be given an independent proof.

The two lemmas and their proofs should read as follows.

Lemma 5. 2. *Let A be a linear operator in the Hilbert space $\mathfrak{H}$, densely defined and such that*

$$|\arg (Ah, h)| \leqq \alpha\pi/2 \quad \textit{for some} \quad \alpha (0 \leqq \alpha \leqq 1) \quad \textit{and all} \quad h \in \mathfrak{D}(A).$$

If $\alpha < 1$ then $(Ah, h) = 0$ implies $h = 0$.

Proof. The binary forms $(g|h)_\pm = \mathrm{Re}[e^{\pm i(1-\alpha)\pi/2}(Ag, h)]$ on $\mathfrak{D}(A)$ are bilinear with respect to real coefficients and satisfy $(h|h)_\pm \geqq 0$. Therefore the Schwarz type inequality

$$\left|\tfrac{1}{2}(g|h)_\pm + \tfrac{1}{2}(h|g)_\pm\right|^2 \leqq (g|g)_\pm \cdot (h|h)_\pm \tag{5. 12}$$

holds and as a consequence $(Ah, h) = 0$ implies

$$\mathrm{Re}\,\{e^{\pm i(1-\alpha)\pi/2}[(Ag, h) + (Ah, g)]\} = 0 \quad \text{for all} \quad g \in \mathfrak{D}(A).$$

Suppose $\alpha < 1$. Then $\pm(1-\alpha)\pi/2$ are not congruent modulo π, and hence we infer that

$$(Ag, h) + (Ah, g) = 0 \quad \text{for all} \quad g \in \mathfrak{D}(A).$$

---

[1]) We are indebted for this remark to Professors RICK CAREY, J. E. KERLIN and A. L. LAMBERT at the University of Kentucky in Lexington, U.S.A., and UWE BÖCKER at the University of Frankfurt/Main, Germany.

This holds for ig as well as for g so we also have

$$(Ag, h)-(Ah, g) = 0,$$

and therefore $(Ah, g)=0$ for all $g \in \mathfrak{D}(A)$. As $\mathfrak{D}(A)$ is dense in $\mathfrak{H}$, we conclude that $Ah=0$.

Lemma 5. 3. *For any closed accretive operator A in $\mathfrak{H}$, the set*

$$\mathfrak{N} = \{g\colon\ g \in \mathfrak{D}(A),\ Ag = 0\}$$

is a subspace of $\mathfrak{H}$ reducing A.

Proof. As A is linear and closed, the set $\mathfrak{N}$ is also linear and closed, i.e. a subspace of $\mathfrak{H}$. For $h \in \mathfrak{D}(A)$ and $g \in \mathfrak{N}$ we have $(Ah, g)=0$ as a consequence of inequality (5. 12) for $\alpha=1$. Thus if $h \in \mathfrak{D}(A)$ then $P_{\mathfrak{N}}Ah=0$, where $P_{\mathfrak{N}}$ denotes orthogonal projection onto $\mathfrak{N}$. On the other hand, $AP_{\mathfrak{N}}h=0$ obviously holds for every $h \in \mathfrak{H}$. Thus we have $P_{\mathfrak{N}}A \subset AP_{\mathfrak{N}}$, and hence $\mathfrak{N}$ reduces A.

2. We use this opportunity to correct the Notes to Chapter IV of [1]. There it is asserted that Proposition 4. 2 (on the simultaneous extension of some dually coupled accretive operators) is new. Although it was independently found by the authors, the result is essentially contained in Ref. [2].

References

[1] Béla Sz.-Nagy—Ciprian Foiaş, *Analyse harmonique des opérateurs de l'espace de Hilbert* (Budapest, 1967), and also the translations in English (Budapest, 1970) and Russian (Moscow, 1970).

[2] R. S. Phillips, The extension of dual subspaces invariant under an algebra, *Proc. Internat. Symposium on Linear Spaces, Jerusalem* 1960 (Jerusalem, 1961), 366—398.

(Received December 5, 1971)