Bibliographie

J. F. Adams, Algebraic Topology—A Student's Guide (London Mathematical Society Lecture Notes Series 4), VI+300 pages, Cambridge, University Press, 1972.

The author's objective is to provide the student with a reliable orientation on the vast and complex material facing him when he tries to master algebraic topology. The unusually long Introduction is the pith of this quite new type of book. The Introduction surveys those topics in algebraic topology which should be studied after a first course in topology, namely categories and functiors; semi-simplicial complexes; ordinary homology and cohomology; spectral sequences; $H^*(BG)$; Eilenberg—MacLane spaces and the Steenrod algebra; Serre's theory of classes of abelian groups (C-theory); obstruction theory; homotopy theory; fibre bundles and topology of groups; generalized cohomology theories. All these topics are dealt with separately, where first the basic ideas underlying the topic are motivated, and then a sketch is given of what the author considers the ideal way of studying the topic. Some very instructive comments are made on the various treatments of the topic available in the literature, where sources recommended in the first place are indicated as well. The second and more voluminous part of the book is a collection of extracts from famous papers. These generally illustrate in a concise way the basic ideas in connection of the above topics.

The book thus cleverly meets the urgent need of those who intend to study the immense and sometimes inconsistent literature of algebraic topology. The fact that it has been written by an author who has authentically indicated the essential points in an overall picture and the technical advantages in different presentations renders this book a very valuable guide for students of algebraic topology.

J. Szenthe (Szeged)

A. Dold, Lectures on algebraic topology (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 200) XI+377 pages, Berlin—Heidelberg—New York, Springer Verlag, 1972.

This book presenting singular homology and cohomology theory, grew out of the author's lectures on algebraic topology and can be highly recommended as a textbook for a one-year course. Chapter I, rather a summary of prerequisites than a detailed presentation, covers categories, Abelian groups and homotopy. Chapter II treats the homology of complexes, and singular homology is introduced in Chapter III. Applications are considered in Chapter IV, where among other things the degree of a map, homology properties of neighborhood retracts in R^n, the Jordan theorem and domain invariance are treated. Cellular decomposition and cellular homology is considered in Chapter V. Functors and complexes are taken up in Chapter VI covering among other things the universal coefficient formula, the Künneth formula and the Eilenberg—Zilber theorem. Various standard products such as the exterior homology and cohomology products, the Pontrjagin product, the cupproduct, the cap-product and some others are introduced in Chapter VII. A very rich material concerning manifolds, including the Poincaré —Lefschetz duality, the Thom isomorphism, the Gysin sequence, intersection of homology classes and other subjects, is presented in Chapter VIII.

As the students of a first course in algebraic topology generally form a rather wide variety the choice and presentation of the matrial for such a course is a perplexing problem. By preferring the singular theory the author seems to have attained an optimum solution. In fact on the one hand an ample algebraic foundation with a great number of motivating examples is provided for those who want to study this subject further; on the other hand for those who are interested only in the applications of algebraic topology the singular theory suits equally the best and the material concerning manifolds covers the most important applications. The presentation also attains an optimum in conciseness and readability.

J. Szenthe (Szeged)

F. F. Bonsall and J. Duncan, Numerical ranges of operators on normed spaces and of elements of normed algebras (London Mathematical Society Lecture Note Series, 2), IV+142 pages, Cambridge, University Press, 1971.

The concept of the numerical range for a linear operator on a normed space was introduced in 1961—62 by Lumer and Bauer in distinct, though related manners both generalizing the classical Hilbert space case. For the elements of a normed algebra the numerical range is defined as that of the corresponding left regular representation operator. The numerical range reflects both the algebraic and the metric structure of the algebra in contrast to the spectrum which depends on the algebra structure only.

The present book is the first self-contained exposition of the subject. The authors use only standard "elementary" material on the theory of linear normed spaces and normed algebras. The power of the numerical range concept is shown e. g. in its application to the problem of metric characterization of C^*-algebras, thus adding a useful tool for the Hilbert space operator theory also.

The book consists of four chapters.

Chapter 1 introduces the numerical range of an element a in a unital normed algebra A:

$V(A, a) = \{f(a): f \in A', f(1) = 1 = \|f\|\}$, A' the dual space of A. It is shown that $V(A, a)$ is a compact convex set in the scalar field (real or complex) depending only on the two-dimensional space spanned by 1 and a. When A is complex and complete, $V(A, a)$ contains the spectrum of a. The numerical radius $v(a) = \max\{|\lambda| : \lambda \in V(A, a)\}$ is immediately seen to be a pseudo-norm on A. For complex A we have $v(a) \leq \|a\| \leq ev(a)$, implying in particular that 1 is a vertex and a point of local uniform convexity of the unit ball of A. The proof of the power inequality $\|a^n\| \leq n!\ (e/n)^n v(a)^n$ and the introduction of the numerical index for A concludes the chapter.

In Chapter 2 those elements h of a complex unital Banach algebra A play central role which have real numerical range. These elements h are called Hermitian and form a real Banach space $H(A)$ such that $i(hk-kh) \in H(A)$ for all $h, k \in H(A)$. It is proved that $h \in H(A)$ if and only if $\|\exp ith\| = 1$ for any real t. Let $J(A)$ denote the set $\{h+ik : h, k \in H(A)\}$. Then $J(A)$ is a complex Banach space which is not necessarily an algebra. It is a subalgebra in A if and only if $h^2 \in J(A)$ for all $h \in H(A)$. The main part of the chapter is the proof of the Vidav-Palmer theorem asserting that A is isometrically isomorphic with a C^*-algebra (i. e. with a norm-closed selfadjoint subalgebra of all bounded linear operators on a Hilbert space) if and only if $A = J(A)$. Among the many applications of this theorem it is shown that a complex Banach $*$-algebra satisfying condition $\|x^*x\| = \|x\|^2$ (B^*-algebra) or $\|x^*x\| = \|x^*\|\ \|x\|$ is isometrically $*$-isomorphic with a C^*-algebra.

Chapter 3 introduces the spatial numerical range of a bounded linear operator T of a normed space X:

$$V(T) = \{f(Tx): x \in X, \|x\| = 1, f \in X', \|f\| = 1\},$$

X' being the dual space of X. The numerical range of T as an element of the Banach algebra $B(X)$ of all bounded linear operators on X is the closed convex hull of $V(T)$. The numerical range $W(T)$ corresponding to a semi-inner-product $[\cdot]$ is defined by

$$W(T) = \{[Tx, x]: [x, x] = 1\}.$$

It is shown that $V(T)$ contains $W(T)$ and that $W(T)$ has the same closed convex hull as $V(T)$. The rest of the chapter deals with spectral, geometric and topological properties of $V(T)$. For a complex Banach space X the authors prove that the closure of $V(T)$ contains the spectrum of T and that the closure of $W(T)$ contains the boundary of the spectrum of T. A theorem of Nirschl—Schneider asserting that eigenvalues of T that lie on the boundary of the closed convex hull of $V(T)$ have index (ascent) one is also proved. The chapter also contains the result that $V(T)$ is connected.

Chapter 4 contains miscellaneous results. The discussion of the numerical range in the second dual of a Banach algebra gives by the Vidav—Palmer theorem that the second dual of a complex B^*-algebra is again a B^*-algebra. The normalized linear functionals $\Omega(A)$ on a complex unital Banach algebra A which are dominated by the spectral radius are called spectral states. These states annihilate elements of the form ab-ba, with $a, b \in A$. A spectral state is multiplicative if and only if it induces a strictly irreducible representation. It is shown in particular that if H is an infinite dimensional Hilbert space then there are no spectral states on $B(H)$. For finite dimensional algebras a complete description of the spectral states in terms of normalized traces (on matrix algebras) is given. The final section of the chapter contains 17 remarks and 16 problems on isolated topics of the numerical range theory.

The book is indispensable for every student or researcher of Banach algebras.

Z. Sebestyén (Budapest)

Essays on topology and related topics. Mémoires dédiés à Georges de Rham, publiés sous la direction de André Haefliger et Raghavan Narasimhan, XII+252 pages, Springer Verlag, Berlin—Heidelberg—New York, 1970.

Le premier article de ce volume en hommage est un exposé des travaux de G. de Rham sur les variétés différentiables par Henri Cartan, réunissant un ensemble des résultats aujourd'hui classiques dans leur contexte historique. Tels sont les résultats recouverts par la dénomination collective „le théorème de de Rham" fondamentaux pour l'introduction des groupes de chomologie calculés avec les formes différentielles d'une variété différentiable. Les divers résultats obtenus par la notion féconde de „courant" sont rassemblés en indiquant comment ces distributions permettent d'obtenir une homologie avec les formes différentielles d'une variété différentiable orientée. La théorie des variétés riemanniennes est représenté par les résultats sur les formes harmoniques et sur la réductibilité. Les autres articles du volume sont les suivants: J. Milnor et O. Burlet: Torsion et type simple d'homotopie; M. Atiyah and F. Hirzebruch: Spin-Manifolds and group actions; P. F. Baum and R. Bott: On the zeroes of meromorphic-vector-fields; R. Bott and S. S. Chern: Some formulas related to complex transgression; K. Kodaira: On homotopy $K3$ surfaces; A Borel: Pseudo-concavité et groupes arithmétiques; A. Andreotti and G. Tomassini: Some remarks on pseudoconcave manifolds; J. L. Koszul: Trajectoires convexes de groupes affines unimodulaires; E. Vesentini: Maximum theorem for spectra; N. H. Kuiper and B. Terpstra—Keppler: Differentiable close-embeddings of Banach manifolds; M. W. Hirsch: On invariant subsets of hyperbolic sets; W. Browder and T. Petrie: Semi-free and quasi-free S^1 actions on homotopy spheres; S. P. Novikov: Pontrjagin classes, the fundamental group and some problems of stable algebra; J. Boéchat et A. Haefliger: Plongements différentiables des variétés orientées de dimension 4 dans R^7; C. Weber: Taming complexes in the metastable range; B. Eckmann et S. Maumary: Le groupe des types simples d'homotopie sur an polyèdre; J. Tits: Sur le groupe des automorphismes d'un arbre; M. A. Kervaire: Multiplicateurs de Schur et K-théorie; R. Thom: Topologie et linguistique. Le volume est terminé par une liste des publications scientifiques de G. de Rham.

J. Szenthe (Szeged)

D. T. Finkbeiner II, Elements of Linear Algebra (A Series of Books in Mathematics), XI+268 pages, San Francisco, W. H. Freeman and Company Ltd., 1972.

The book covers the material of a first course in linear albegra for college students. The introductory chapter contains a general survey of linear algebra, the other four chapters provide substantial introductions to linear spaces (mostly real and finite dimensional), linear mappings, and matrix algebra, sysytems of linear equations and determinants, characteristic values and diagonalization problems. All the thirty-seven sections end with numerous exercises and suggestions and answers are provided for almost all exercises at the end of the book. Out of the few unprecise statements in the book we mention that (on p. 132) the author says that the spectral theorem is true for precisely those linear transformations that are linear combinations of projections (he omits the modifier "commuting").

J. Szűcs (Szeged)

F. Gécseg and I. Peák, Algebraic theory of automata (Disquisitiones Mathematicae Hungaricae, vol. 2), XIV+326 pages, Akadémiai Kiadó, Budapest, 1972.

In the last decade, a remarkable number of books, (about fifteen), addressed to give a systematical treatment of the theory of automata, has been published. Even having such a considerable rivalry, it seems to be expectable that the work reviewed now will be ranked among the most valued monographies on this subject.

The authors have delimited with an approvable sense the topics contained in the book. They disregarded a few branches of the theory (experiments with automata, Turing machines) lying far from the basic subject; moreover, the stochastic extensions of the automaton-theoretical investigations (an area which had scarcely been studied before the labour on this book) are likewise missing; these spontaneous restrictions, however, made possible that the volume should contain an approximatively complete, systematical, precise treatment of the researches of algebraic character on the discrete deterministic automata.

The book presupposes only minimal previous knowledge; it reckons, however, on readers having mature mathematical abilities. The presentation is exact and concise. The latest mentioned property seems to follow necessarily from how a rich material is condensed in a volume of modest size.

In Chapter 1 (Concept of automaton) the fundamental concepts being in connection with automata are dealt with, the minimization procedure is also included. Chapter 2 (Analysis and synthesis Algebra of events) contains, among others, Kleene's theorems on the regular expressions the results of Red'ko and Salomaa on the axiomatizability of regular expressions, investigations of Brzozowski and Yoeli on the derivation of events and on some special event types. In Chapter 3 (Some special classes of automata), the notions of commutative, nilpotent, definite, linear and push-down automata are introduced and studied. Chapter 4 (Composition of automata. Automation mappings) is mainly devoted to Gécseg's results on the properties of families of automaton mappings and on the metrical completeness of automaton systems with respect to various composition concepts. In Chapter 5 (Automata and semigroups), the characteristic semigroups, endomorphism semigroups and automorphism groups of automata are studied, including a number of theorems due to Peák. In the Appendix of the book (Structural systems), the basic concepts of the structural automaton theory are introduced, together with the theorem of Post and Jablonskij on the functional completeness of truth functions and Quine's method for determining the minimal representations of these functions by disjunctive normal forms.

The volume terminates with a bibliography consisting of about 250 items.

A. Ádám (Budapest)

George Grätzer, Lattice theory (First concepts and distributive lattices), XV+212 pages, W. H. Freeman and Co., San Franciso, 1971.

The aim of this book, divided into three chapters, is to give a detailed presentation of distributive lattices.

In the first chapter the basic concepts of lattice theory are introduced and free lattices, which take an essential part in the book, are very carefully discussed.

The second chapter starts with a skillful development of various characterizations and representations of distributive and Boolean lattices. It follows a section on congruence relations in distributive lattices. The next one, including some new results of the author, deals with some generation problems of distributive lattices, Boolean lattices and Boolean algebras. (Boolean lattices and Boolean algebras are strictly and consistently distinguished here.) Then it comes Stone's representation theory of distributive lattices by the topological spaces defined on the partially ordered sets of their prime ideals. Next, one can find the results of the author and H. Lakser, on free distributive products. The last section of the chapter deals with the characterizations of mono- and epimorphisms as well of certain projective subclasses and all the injective ones in the categories of the distributive lattices, bounded distributive lattices and Boolean algebras.

The third chapter discusses distributive lattices with pseudocomplementation including Stone lattices. All equational classes of them and the subdirectly irreducible ones are completely described. Moreover, Stone lattices are characterized among the distributive ones and conditions are given for a Stone lattice to be injective. Most of the results discussed in this chapter were obtained by the author, partly with R. Balbes and H. Lakser.

Each section is followed by many exercices, and at the end of each chapter several unsolved problems are mentioned.

The author says in the preface that he was to break with the traditional approach to lattice theory, which proceeds from partially ordered sets to general lattices, semimodular lattices, modular lattices and finally distributive lattices. But he does not realize his decision perfectly because he introduces the lattices as partly ordered sets of special kind. In the rest of the book the central part is taken by the distributive lattices indeed and the more general facts needed are introduced just when and where they are applied the first time. This construction, logical and attractive, evidently has disadvantages, too, appearing for instance in the fact that some of the general theorems are discussed in such a section where the reader would not think to find them.

The author strives successfully after estimating the results and showing the inner connections of the matter. But, in the opinion of the reviewer, the remark on priority after Theorem 6.4 is superfluous. The Further Topics and References at the ends of the chapters cannot provide, of course, a complete survey of lattice theory, but some applications to algebra and geometry would have been worthy enough to be mentioned.

The composition of the book is careful. There is only one fault found by the reviewer: one reads the sentence "if L has 0 and 1" on page 23, while these elements will be defined on page 56. only.

Summarizing, Grätzer's book is a good introduction to some problems concerning the class of distributive lattices and some of its important subclasses.

G. Szász (Budapest)

W. Hahn, Stability of Motion (Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, 138) XI+446 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1967. — DM 72, —

In the last two decades the number of publications concerning the stability problems of functional equations (differential-, differential-difference-, and difference-equations) has grown enormously. In general, these papers are connected with practical problems and investigate the qualitative behaviour of the given special systems.

The aim of this monograph is twofold: to summarize from a uniform point of view the results of the stability theory of functional equations as a rigorous mathematical discipline, and to present important and comprehensive applications to the theory.

The first part of the book deals with the stability of solutions of special functional equations drawing the attention to analogies between the various results. In Chapter 5 the general concept of motion is defined; then, by the aid of comparison functions, a classification of stability types is given. There follows a number of stability criteria in a general form based on the direct method of Liapunov. The results are illustrated by examples and applications to ordinary differential-, difference-, and partial differential equations. In the further chapters special questions on motions generated by ordinary differential equations, are discussed.

Altogether it is a characteristic feature of monograph that in the setting of problems, interpretation of results and choosing of terminology, it combines in a lucky manner the classical treatment with a treatment aimed at modern physical and technical applications. It deals with problems and methods concerning the stability of automatic control systems and servomechanisms (e. g. speed control by means of a centrifugal pendulum, regulation of the water level in a container, Popov's criterion and so on).

The monograph is beautifully organized and highly readable. It is not only for specialists but a very valuable text book for beginners too.

L. Hatvani (Szeged)

K. Hinderer, Grundbegriffe der Wahrscheinlichkeitstheorie, VIII+247 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1972.

Das Buch gibt eine kurze, moderne und präzise Einleitung in die Theorie der Wahrscheinlichkeitsrechnung, und bietet eine sichere Basis für weitergehende Studien. Da fast alle Hilfsmittel vorbereitet sind, kann man den Text ohne höhere mathematische Vorkenntnisse studieren. Nach den einzelnen Punkten gibt es einige Aufgaben und Ergänzungen, in denen sich historische Anmerkungen und Hinweise auf weitere Literatur und auf verschiedene Anwendungsmöglichkeiten befinden. Bei der sehr präzisen und lückenlosen Betrachtungsweise sorgt Verf. darauf, daß die Schwierigkeiten der mathematischen Modellbildung gut erleutert seien. In Kapitel I werden die diskreten Wahrscheinlichkeitsräume betrachtet. (Auch die Grundformeln der Kombinatorik werden bewiesen.) Kapitel II bietet die wichtigsten Hilfsmittel der abstrakten Maß- und Integrationstheorie. Kapitel III beschäftigt sich mit den allgemeinen Wahrscheinlichkeitsräumen. Gewisse einfache stochastische Prozeße (Markoffsche Ketten, Poisson-Prozeße), weiterhin die einfachsten Gesetze der großen Zahlen, und Verteilungssätze werden auch betrachtet.

Das Buch benützt die moderne Terminologie. Die Betrachtungsweise ist gedrängt, so ist das Buch nur für solche Leser empfehlenswert, die schon eine gewiße Lesefertigkeit von mathematischen Texten besitzen.

K. Tandori (Szeged)

F. John, Partial Differential Equations (Applied Mathematical Sciences, 1) VIII+221 pages, New York—Heidelberg—Berlin, Springer Verlag, 1971.

Perhaps the following few words, taken from the preface, are most characteristic of the book which grew out of a course held by the author in 1952—53: „Though the field of Partial Differential Equations has changed considerably since those days, particularly under the impact of methods taken from Functional Analysis, the author feels that the introductory material offered here still is basic for an understanding of the subject. It supplies the necessary intuitive foundation which motivates and anticipates abstract formulations of the questions and relates them to the description of natural phenomena."

The chapter headings are: I. The single first order equation, II. The Cauchy problem for higher order equations, III. Second order equations with constant coefficients, IV. The Cauchy problem for linear hyperbolic equations in general. — There is also a list of books for further study.

The book is an excellent introduction to the classical theory of partial differential theory.

L. Pintér (Szeged)

E. A. Maxwell, Fallacies in Mathematics, 95 pages, Cambridge, University Press, 1963.

The book of Dr. Maxwell is more than an entertaining piece of reading. In the first chapter the author distinguishes among three kinds of mathematical errors: the simple mistake, causing only a momentary aberration, the howler, which leads innocently to a correct result, and the fallacy, leading by some guile to a wrong but plausible conclusion. The author investigates different types of fallacies, in particular fallacies in geometry, algebra, trigonometry, differentiation and integration. Especially interesting is the "Isosceles Triangle Fallacy" that plays an important role in subsequent chapters, one of them containing a digression on elementary geometry and the analysis of fallacies. The last three chapters are on: "Fallacy be the Circular Points at Infinity", "Some Limit Fallacies" and "Some Miscellaneous Howlers". The finding of suitable tricks together with the author's comments are very interesting and amusing reading for students of any age. Teachers of mathematics in high schools, colleges and universities will also find some useful and intriguing problems to be used in their work.

I. Szalay (Szeged)

J. L. Lions, Optimal Control of Systems Governed by Partial Differential Equations (Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, 170), XI+396 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1971.

The content of the book developed from a course given at the Faculty of Sciences, University of Paris.

Practical necessities as well as investigations of optimal control theory in infinite dimensional spaces suggest in a natural way to examine models described by a family of partial differential operators. So an entirely new type of controls appear, the control by boundary values. This new character of controls and the complexity of partial differential equations in comparison with ordinary differential equations give the high difficulties of the problem.

Relying on his former book on partial differential equations the author uses a quite general technique and treats the material in a clear formulation, grouping it into five chapters according to five different areas. First, minimization of functions in a Hilbert space is presented as an introduction to infinite dimensional extremum problems. Then controls of systems governed by elliptic, parabolic and hyperbolic partial equations follow. Each of these chapters deals, besides the general problems of observability, controllability and existence of optimal controls, also with some special problems according to the character of the control system. Of course the results concerning the various types of elliptic systems are used in the sequel in the mixed problems with control on the boundary. Parabolic systems have the most interesting and ramifying relations, for instance the asymptotic behaviour of control and the connections with the Hamilton—Jacobi theory of variations. The book ends with regularization, approximation and penalization problems.

Each chapter begins with a detailed plan sketching out the scope of the chapter and closes with bibliographic notes and indications on unsolved problems.

A considerable number of comparatively simple examples complete the book and make it useful for those interested in this developing branch of mathematics and in applications of the modern theory of partial differential equations.

T. Matolcsi (Szeged)

H. H. Schaefer, Topological Vector Spaces (Graduate Texts in Mathematics), Third printing corrected, XI+294 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1972.

The book is intended to be a systematic text on topological vector spaces. At its first appearance it had a pioneering character, and it still preserved its popularity and usefulness, which justified the new edition.

After shortly recalling some prerequisits on general topology and linear algebra, topological vector spaces are treated in general. The next chapter deals with locally convex spaces including important special types of such spaces. Linear and bilinear mappings form the subject of another chapter, which has most important sections on special problems such as topological tensor products, nuclear mappings and spaces. These subjects are indispensable tools for modern functional analysis and its applications. The chapter on duality discusses $\mathfrak{S}$-topologies and reflexivity, and some other special parts of the theory, again with great emphasis on tensor products and nuclear spaces. The content of the last chapter is usually not involved in the investigations concerning topological vector spaces: this chapter has the order structures as its subject.

At the end of each chapter there is a vast amount of exercises devoted to further supplements, in particular to examples and counter examples.

The book is well organized, the presentation of material is concise but completely understandable. It may be very useful both for introduction and for reference.

T. Matolcsi (Szeged)

H. Schubert, Categories, XI+385 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1972. — DM 93,—

Categorial methods of speaking and thinking are becoming more and more widespread in mathematics. In this textbook, which only assumes that the reader has elementary prior knowledge of set theory and algebra, the central ideas of category theory are developed. The presentation of material is very concise, however definitions and notions are illuminated by a vast amount of examples and occasionally even special cases are treated to give a base for later abstractions.

After the introductory chapters including also the treatment of additive categories and categories of functors, attention is concentrated upon the concept of a representable functor and its variations: limits, colimits and adjoint functor pairs. Most of the chapters are devoted to or connected with this area of the theory. In the sequel, an elementary and then a functional exposition of algebraic structures and of Abelian and Grothendieck categories are the main subjects, followed by such other topics as fractional calculus, Grothendieck topologies and triples.

The clearly written book can serve as a good guide to those interested in this novel branch of mathematics.

T. Matolcsi (Szeged)

W. Freiberger—U. Grenander, A Short Course in Computational Probability and Statistics (Applied Mathematical Sciences 6), XII+155 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1971.

One of the most critical points in any presentation of applied mathematics is to preserve the unity of theory and applications. The solutions of textbook problems are often easy consequences of the theorems presented so the reader could have the impression that every problem arising in practice can be solved in five minutes, provided that one knows the adequate theorems. However, the solution of a real problem is much more difficult. From model building until numerical computation each step of the approach must be chosen carefully, otherwise the success is threatened.

The purpose of the present book is to help the reader with his work in real problems and as the authors state, the book "has been designed with the aim of making students and perhaps also faculty aware of some of consequences of modern computer technology for probability theory and mathematical statistics". It is pointed out that computational probability does not equal writing of statistical programs, and what is analytically possible need not be computationally feasible. Special attention is paid to model building and to the most effective combination of analytic and computational methods.

The material is a discussion of about 20 problem groups, the most interesting ones are as follows: random number generation, different Monte Carlo methods,i nsurance problems, growth and renewal models, Bayesian decision problems, stochastic approximation, design of experiments, analysis of variance, time series analysis, signal detection, etc. Different approaches to the problems are compared, and to each one complete APL programs are given. (In the reviewer's opinion the use of a more wide-spread programming language would have made the understanding of these programs easier.) The material presented in the book does not cover the whole area of applied probability and statistics, but the understanding of the given methods enables the reader to solve problems from other spheres of applied mathematics, too.

By its nature the book is perhaps better suited for a subject of discussions in a seminar than for individual study. To use all of its advantages, the reader must have solid knowledge in calculus, linear algebra, probability theory and statistics as well as some experience with computers.

Summing up, the present book is an excellent introduction to the ambitious applications of probability theory and statistics, and presents a new approach in the education of applied mathematics.

D. Vermes (Szeged)

H. Bühlmann, Mathematical Methods in Risk Theory (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 172), XII+210 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1970.

Insurance mathematics is one of the oldest applied mathematical disciplines, its origins go back into the 17th century. Resulting from its vigorous development in the last decades the present actuarial mathematics "undertakes to solve the technical problems of all branches of insurance, and it concerns itself particularly with the operational problems of the insurance enterprise". The mathematical basis is probability theory, but as a result of its long history, insurance mathematics has its own language, which appears somewhat strange for the non-specialist.

The present book "attempts to create a synthesis out of a selection made by the author of modern scientific publications in the field of actuarial mathematics, with the goal of presenting a unified system of thought".

The reader, who is only supposed to be familiar with the elements of probability theory, will find this book a useful introduction into the modern spheres of insurance mathematics. The chapter headings are: 1. Probability aspects of risk, 2. Risk processes, 3. The risk in the collective, 4. Premium calculation, 5. Retentions and reserves, 6. The insurance carrier's stability criteria.

D. Vermes (Szeged)

Martin Gardner's Sixth Book of Mathematical Games from Scientific American, 262 pages, San Francisco, W. H. Freeman and Co., 1972.

Martin Gardner's book is a brilliant set of mathematical puzzles, games and plays completed by historical curiosities concerning mathematics. Most of the themes are novel and quite intriguing. The book will give great delight to a wide circle of readers, in particular to students in high schools and universities, but it is a useful help for the teachers of mathematics at all levels as well. The text makes for an easy reading, the figures are clear and well arranged, and there are a few but well chosen references at the end of every section. From the topics: topology, combinatorial theory, board games, three-dimensional maze, prime numbers, graph theory, ternary system, cycloid, mathematical magic tricks, Pythagorean theorem, infinite series, lattice of integers, op art.

J. Major (Budapest)

K. Jordan, Chapters on the classical calculus of probability (Disquisitiones Mathematicae Hungaricae 4) XXV+619 pages, Budapest, Akadémiai Kiadó, 1972.

This is a translation of the original Hungarian edition that appeared in 1956. It contains a preface written by Béla Gyires and a list of Károly Jordan's works.

Károly Jordan is a classical figure in Hungarian mathematics. His results in probability theory and in the theory of finite differences are well-known. It was peculiar to his work that he was an eminent expert of applications. Although the major part of his research falls in the time before Kolmogoroff's foundation of probability theory became widely known and accepted, the book is a highly valuable and useful contribution to the literature and very enjoyable reading. The reader will find many interesting facts about the development of probability theory and philosophical disputes on the notion of probability. The author's mastery of the combinatorial methods and the presentation of various applications also increase the book's value.

Károly Tandori (Szeged)

R. Nevanlinna, Analytic Functions, 373 pages, Springer-Verlag, Berlin—Heidelberg—New York 1970.

This monograph on analytic functions is the revised translation of "Eindeutige analytische Funktionen", 2nd edition 1953 (Grundlehren der mathematischen Wissenschaften, Vol. 46).

This work coincides to a large extent with the presentation of the modern theory of single-valued analytic functions given in the author's earlier works "Le théorème de Picard—Borel et la théorie des fonctions méromorphes", Gauthier—Villars (Paris, 1929), and "Eindeutige analytische Funktionen" (Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Vol. 46), Springer-Verlag (1st ed. Berlin, 1936, 2nd ed. Berlin—Göttingen—Heidelberg, 1953).

As the author says in the preface, this new edition contains some changes and additions, particularly concerning the Second Main Theorem in the Theory of Meromorphic Functions (IX. Chapter). In the derivation of this theorem the author uses a version of F. Nevanlinna's differential geometrical method, which makes the main theorem easier for access.

The chapter headings are: I. Conformal Mapping of Simply and Multiply Connected Regions, II. Solution of the Dirichlet Problem for a Schlicht Region, III. Function Theoretic Majorant Principles, IV. Relations Between Noneuclidean and Euclidean Metrics, V. Point Sets of Harmonic Measure Zero, VI. The First Main Theorem in the Theory of Meromorphic Functions, VII. Functions of Bounded Type, VIII. Meromorphic Functions of Finite Order, IX. The Second Main Theorem in the Theory of Meromorphic Functions, X. Application of the Second Main Theorem, XI. The Riemann Surface of a Univalent Function, XII. The Type of a Riemann Surface, XIII. The Ahlfors Theory of Covering Surfaces.

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F. W. Stevenson, Projective planes, X+416 pages, San Francisco, W. H. Freeman and Company, 1972.

The greatest difficulty for the beginner in the study of axiomatic geometry is presented by the large number and the many kinds of axioms involved. There are essentially two kinds of axioms: some of them have an algebraic character, the others are metrical or topological. The same holds for the axiomatic theory of projective planes, cf. the standard book on the subject: G. Pickert, *Projektive Ebene* (Berlin, 1955).

The present book, as claimed in the preface, is intended to serve as a textbook for the theory of projective planes from the point of view of ring theory.

It is divided into three parts. Part 1 introduces the reader to the basic concepts and methods necessary for Parts 2 and 3. Part 2 is devoted to the classical theorems of Desargues and Pappus. (Chapters: Desarguesian planes, Pappian planes, Planes over division rings and fields, Coordinatizing planes.) In Part 3 it follows the study of non-Desarguesian planes, coordinatized by various generalized rings: planar rings with associativity, quasifields, planar nearfields, semifields, and alternative rings.

The topological aspects of the theory of projective planes are not studied.

There are many interesting examples and exercises.

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