On the invariant subspace lattice $1+\omega^*$ (Corrigendum)

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The proofs of Theorems 1 and 2 of the author's paper [2] contain several errors, which come from implicitly assuming that if $\{b_n, \beta_n\}$ is a biorthogonal system with $\{\beta_n\}$ total, then the span of the b_n's is dense. (This is false, even for Hilbert spaces; see [3]). The author was unable to correct this point in Theorem 1; however, the proof given in that paper shows that the following weaker statement is actually true.

Theorem. *Let $\mathfrak{B}$ be a Banach algebra with identity and assume that* $\operatorname{Lat}\mathfrak{B}$ *(the lattice of closed left ideals of $\mathfrak{B}$) is the denumerable chain $\{\mathfrak{M}_n\}_{n=0}^{\infty}\cup\{(0)\}$, where $\mathfrak{M}_0=\mathfrak{B}$ and $\dim\mathfrak{M}_n/\mathfrak{M}_{n+1}=1$ for $n=0, 1, 2, \ldots$. Let t be an arbitrary element of $\mathfrak{M}_1\diagdown\mathfrak{M}_2$; then t is quasinilpotent, $\mathfrak{B}$ is a (necessarily commutative) algebra of formal power series in t, the Gelfand spectrum of $\mathfrak{B}$ consists of a single point and $\mathfrak{M}_n=\operatorname{closure}(t^n\mathfrak{B})$, $n=0, 1, 2, \ldots$. Moreover, if $a=\sum_{n=0}^{\infty} c_n t^n\in\mathfrak{B}$, then there exist constants $\{C_n\}_{n=0}^{\infty}$ independent of a such that $|c_n|\leqq C_n\|a\|$.*

In other words, $\mathfrak{B}$ is the direct sum of a *generalized Banach algebra of power series* in the sense of S. GRABINER [1] and of $\mathbf{C}$ (the constant terms!). The author wishes to thank Professor SANDY GRABINER, who indicated the errors contained in [2] and also provided the correct statement of Theorem 1 given above.

In Corollary 3, it is necessary to make the following change: Instead of "$t\neq 0$", we have to assume that "$t\neq\lambda e$ for all complex λ, where e denotes the identity of $\mathfrak{B}$". (In fact, if $\lambda_0\neq 0$ is a root of the polynomial $p(z)$, then $p(\lambda_0 e)=p(\lambda_0)e=0$, contradicting the thesis of the corollary.)

The result of Theorem 2 is correct, but the preliminary Lemma 5 needs several changes. Recall ([3, Chapter IX]) that $\{b_n, \beta_n\}_{n=0}^{\infty}$ (b_n belongs to the complex Banach space $\mathfrak{X}$ and β_n belongs to the dual space $\mathfrak{X}^*$) is a *biorthogonal system* if $\beta_n(b_m)=\delta_{nm}$ (Kronecker's delta); $\{b_n\}$ is a *(normalized) Markushevich basis* for $\mathfrak{X}$ if the b_n's are the first terms of a biorthogonal system such that $\{\beta_n\}$ is a total set of functionals in $\mathfrak{X}$ and $\{b_n\}$ span a dense linear manifold of $\mathfrak{X}$ (and $\|b_n\|=1$ for all n). For the existence and properties of Markushevich bases, the reader is referred to [3].

Replace Lemma 5 by the following

Lemma. *Let $\{b_n\}_{n=0}^{\infty}$ be a normalized Markushevich basis for the complex Banach space $\mathfrak{X}$ and let $\{\beta_n\}\subset\mathfrak{X}^*$ be chosen so that $\{b_n, \beta_n\}$ is a biorthogonal system. Let $0\leqq\varepsilon_n\leqq(2\|\beta_n\|)^{-1}$ and let $\{b_n', \beta_n'\}$ be a second biorthogonal system, with $\beta_n'=\beta_n+\varepsilon_{n+1}\beta_{n+1}$, $n=0, 1, 2, \ldots$. Then $\{b_n'\}$ is also a Markushevich basis for $\mathfrak{X}$.*

Proof. Clearly, $\|\beta_n\|\geqq\beta_n(b_n)=1$, so that $0\leqq\varepsilon_n\leqq 1/2$. That $\{\beta_n'\}$ is total in $\mathfrak{X}$ follows exactly as in [2, Lemma 5].

It only remains to show that the span of $\{b_n'\}$ is dense in $\mathscr{X}$. By induction over n, it is not difficult to see that

$$b_n'=b_n-\varepsilon_n b_{n-1}+\varepsilon_n\varepsilon_{n-1}b_{n-2}-\ldots+(-1)^n\varepsilon_n\varepsilon_{n-1}\ldots\varepsilon_2\varepsilon_1 b_0 \qquad (n=0, 1, 2, \ldots; b_{-1}=0),$$

so that b_n is a linear combination of $b_0', b_1', \ldots, b_n'$, whence the result follows.

By using this result it is very easy to obtain a correct proof of Lemma 6 and Theorem 2.

After the article [2] was published, a second proof of Theorem 2 was obtained by H. Radjavi and P. Rosenthal in [4], by using a different argument.

References

[1] S. Grabiner, Derivations and automorphisms of Banach algebras of power series, *Memoirs Amer. Math. Soc.*, **146** (Providence, Rhode Island, 1974).

[2] D. A. Herrero, On the invariant subspace lattice $1+\omega^*$, *Acta Sci. Math.*, **35** (1973), 217—223.

[3] J. T. Marti, *Introduction to the theory of bases*, Springer-Verlag (Berlin—Heidelberg—New York, 1969).

[4] H. Radjavi and P. Rosenthal, On transitive and reductive operator algebras, *Math. Ann.*, **209** (1974), 43—56.

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