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Bibliographie

Friedrich Bachmann, Aufbau der Geometrie aus dem Spiegelungsbegriff, 2. ergänzte Auflage (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 96), XVI+374 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1973.

This book is the second enlarged edition of the first, which is already a classical textbook on the foundation of geometries based on reflections. This unified, group theoretical treatment shed new light on the systems of axioms in geometry and initiated new progress in this classical subject.

The original material (pp. 1—304) is supplemented with notes and references (pp. 305—310). The new Supplement (pp. 311—357) contains a detailed survey of the recent progress in "reflection geometry". A full bibliography from 1959 to 1972, consisting of 162 items, is also added.

P. T. Nagy (Szeged)

Walter Benz, Vorlesungen über Geometrie der Algebren. Geometrien von Möbius, Laguerre—Lie, Minkowski in einheitlicher und grundlagengeometrischer Behanglung (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 197), XI+368 Seiten, Berlin—Heidelberg—New Yok, Springer-Verlag, 1973.

The geometries of Möbius, Laguerre, Lie and the pseudoeuclidean plane geometry can be treated in an analoguous way based on the geometry of the projective line over an algebra (the algebras of complex, dual complex and anormal complex numbers). The aim of this book is the systematic exposition of the geometry (called "chain geometry") of the projective line over an algebra and of its application to the study of classical geometries over an arbitrary field.

Let L be a commutative algebra over a field K, and consider the analytical projective lines $P(L)$ and $P(K)$ over L and K, respectively. We can regard the projective line $P(K)$ imbedded in a natural way in the line $P(L)$.

The "chain geometry" $\Sigma(K, L)$ is defined as follows: its points are the points of the projective line $P(L)$. The "chains" of $\Sigma(K, L)$ are defined as the set of points of $P(L)$ which are the images of the projective line $P(K)$ under the projectivities on $P(L)$. The incidence relation between points and chains is defined by inclusion. The notions of "tangence" and "angle" can be defined in a natural manner.

The book contains a detailed introduction to the geometries of Möbius, Laguerre, Lie, and to pseudoeuclidean geometry over the reals (Chapter I).

The general chain geometry is explained in Chapter II. There is a discussion of the problems: (i) Is every automorphism of a chain geometry $\Sigma(K, L)$ a projectivity on $P(L)$? (ii) Is each isomorphism of chain geometries induced by an isomorphism of the coordinate algebras? The answer in general is *not*, but in the case of the above-mentioned classical geometries the corresponding theorems are proved.

Chapter III is devoted to the study of questions of axiomatic nature.

Chapter IV deals with models of greater dimension for chain geometry. There is given a glance on the chain geometry over a noncommutative algebra.

The book is written in an always clear and well-readable way and only presupposes familiarity with the basic concepts of algebra and geometry.

P. T. Nagy (Szeged)

D. G. Douglas, Banach Algebra Techniques in Operator Theory (Pure and Applied Mathematics, A Series of Monographs and Textbooks, 49), XVI+216 pages, Academic Press, New York and London, 1972.

Operator theory includes the study of operators and collections of operators arising in mathematics, mechanics and other branches of physics. It is now sufficiently well developed to have a logic of its own.

This book presents a nice introduction to the study of bounded operators on Hilbert space based on powerful and interesting techniques drawn from functional analysis, from the theory of Banach spaces and Banach algebras. The author presumes only that the reader is familiar with general topology, measure theory, and algebra. He does not attempt completeness so that many elementary facts are either omitted or mentioned only in problems, which are of different character: either allow the reader to test his understanding, or indicate certain generalizations, or alert to certain important and related results, or point out open questions.

The book consists of seven chapters, references, and an index.

Chapter 1: *Banach Spaces.* Basic results along with many relevant examples. Discussion of theorems due to Alaoglu, Hahn and Banach, Riesz and Markov, and Banach. Lebesgue spaces L^1 and L^∞, and Hardy spaces H^1 and H^∞.

Chapter 2: *Banach Algebras.* Elementary theory of commutative Banach algebras, due essentially to Gelfand and Shilov, the technique of which is very essential in the subsequent chapters. The algebra of all continuous functions on some compact Hausdorff space is discussed here, including the Stone-Weierstrass theorem.

Chapter 3: *Geometry of Hilbert Space.* A short introduction with many examples.

Chapter 4: *Operators on Hilbert Space and C^*-algebras.* After the standard material the notion of a C^*-algebra is introduced and used throughout the rest of the chapter. The commutative Gelfand-Naimark theorem gives here an abstract spectral theorem and functional calculus. Commutative W^*-algebra theory is used to obtain an extended functional calculus. A theorem by Fuglede concludes the chapter.

Chapter 5.: *Compact Operators, Fredholm Operators, and Index Theory.* The approach is somewhat unorthodox: it gives the key results as quickly as possible and adds many examples. Certain ancillary results concerning ideals in C^*-algebras are also proved.

Chapter 6: *The Hardy Spaces.* Various properties of the spaces H^1, H^2 and H^∞ are derived, and results of Hartman, Wintner, Brown and Halmos, Coburn and Widom are treated.

The author adds short notes at the end of each chapter suggesting thus further reading.

The book is a very useful reading for anyone wishing to learn, or make further research in, the theory of operators.

Zoltán Sebestyén (Budapest)

Carl Faith, Algebra: Rings, Modules and Categories. I (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 190), XXIII+565 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1973.

This book is a survey of aspects of ring theory incorporating many of the now classical ring-theoretical ideas with homological ones. In Volume I the emphasis is equally divided between these two influences while Volume II — which has not appeared as yet — will be devoted to ring theory. The book is intended to contain everything that is worth knowing in this subject. There is no possibility here to give the details of the rich material covered in the book and we only give a short sketch of the four parts of Volume I.

After a short foreword on set theory in Part I (Chapters 1—6) the basic concepts and theorems of the theory of rings, modules and categories are presented. Part II (Chapters 7—10) is a discussion of the structure theory of Noetherian semiprime rings. Tensor algebra and the Morita theorems together with their application to the determination of the Picard group are developed in Part III (Chapers 11—13). Volume I is concluded in Part IV (Chapters 14—16) by the theory of Abelian categories including the theory of Grothendieck categories, the Mitchell-Gabriel embedding theorems and the Gabriel-Popesco theorem.

"This book is designed to introduce students to the basic ideas and operations of rings, modules and categories as patiently and as thoroughly as time and space permit, and then bring them to the frontiers of research as rapidly and as comprehendingly as their abilities permit." To this end it contains useful suggestions for reading. The description of the logical dependencies of the chapters makes it easier to peruse the book for those who are interested only in portions of it. A large bibliography of papers closely related to problems occurring in the book is also given. The book can be used as a reference book as well. It will be of great value in promoting and aiding further research on this subject.

Á. Szendrei (Szeged)

Wilhelm Flügge, Viscoelasticity, Second revised edition, VII+149 pages, Springer-Verlag Berlin—Heidelberg—New York, 1975.

The book presents an introductory course in the theory of viscoelasticity. The behaviour of viscoelastic material is described by a mixture of elementary models: the helical spring satisfying Hooke's law and the piston moving in a cylinder with a perforated bottom so that no air is trapped inside. The linear theory of viscoelasticity treated in this book presupposes that the differential equation expressing the connection of stresses, strains and displacements is linear. The reader is supposed to be familiar with some knowledge of Calculus only.

P. T. Nagy (Szeged)

Terence M. Gagen, Topics in Finite Groups (London Mathematical Society Lecture Note Note Series 16), VIII+85, Cambridge University Press, Cambridge—London—New York—Melbourne, 1976.

This book is a well-written explanation of H. Bender's theory of the classification of non-soluble groups with Abelian Sylow 2-subgroups and some related results. The topics covered in the book are of current research interest and were, as yet, accessible only to a very few specialists. The author's aim is to present this rich and original material to a wider community of group theorists.

A. P. Huhn (Szeged)

Richard B. Holmes, Geometric Functional Analysis and its Applications, X+246 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1975.

The main purpose of this book is to give a glimpse of applications of functional analysis to optimization theory and in particular to the theory of best approximation. In accordance with this objective it presents only parts of functional analysis having a geometrical nature. Most of the results of this book are based on the concept of convexity; the others generally use outgrowths concerning conjugate spaces or compactness properties, both of which topics are important for the proper setting and resolution of optimization problems. The book is divided into four chapters, all of them containing applications of functional analytic methods to the problems mentioned above.

Chapter I discusses convexity in linear spaces (using only the linear structure). The Hahn-Banach theorem appears in ten different (algebraic and geometric) but equivalent forms, some of which are optimality criteria for convex programs.

In Chapter II the concept of linear topological space is introduced. This chapter contains investigations concerning locally convex spaces, convexity and topology, weak topologies, extreme points, convex functions and optimizations, and some more applications.

Chapter III deals with Banach spaces, examines the questions of completion, congruence, reflexivity and gives some applications of category theorems.

Chapter IV is devoted to studying properties and characterizations of conjugate spaces and isomorphism of certain conjugate spaces. Universal spaces are also investigated.

All of the four chapters end with a rich selection of problems. Some are intended to be of a rather routine nature, many others, however, contain significant further results, converses or counter-examples.

The book is recommended to mathematicians doing research in functional analysis and in its applications, and to students whose mathematical background includes basic courses in linear algebra, measure theory, and general topology.

L. Gehér (Szeged)

C. Hooley, Applications of Sieve Methods to the Theory of Numbers (Cambridge Tracts in Mathematics, 70), XIV+122 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1976.

This book, based on an Adams Prize winning essay of the author, presents some new and less known applications of sieve methods to additive and prime number theory. After a short survey of sieve methods, it proves a series of deep results, most of them due to the author. Emphasis is put on combination of sieve methods with each other and with further techniques. A number theory background is desirable on the reader's part.

L. Lovász (Szeged)

J. E. Humphreys, Introduction to Lie Algebras and Representation Theory (Graduate Texts in Mathematics, Vol. 9), XII+169 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1972.

The book intends "to introduce the reader to the theory of semisimple Lie algebras over an algebraically closed field of characteristic 0, with emphasis on representations".

The first four Chapters (I. Basic concepts, II. Semisimple Lie algebras, III. Root systems, IV. Isomorphism and conjugacy theorems) present the classical parts of the theory, and "might well be read by a bright undergraduate". Chapter V deals with the Poincaré—Birkhoff—Witt theorem and Serre's existence theorem and their consequences. Here a description of the classical simple Lie algebras is given. In Chapter VI representation theory is studied, especially finite dimensional Lie algebra modules. Chapter VII serves as an introduction to the theory of Chevalley algebras and groups, and their applications to Lie algebra representation theory.

Some standard topics are omitted (theorems of Levi and Ado, classification over reals etc.), which are better suited to a second course in the author's opinion.

Each chapter contains references and a lot of exercises. The reader is supposed to be familiar with linear algebra and with the elements of general algebra.

The book is written in a well-readable way. It will be useful to everyone wanting to get acquainted with the representation theory of Lie algebras.

P. T. Nagy (Szeged)

D. L. Johnson, Presentation of Groups, V+204 pages (London Mathematical Society Lecture Note Series 22), Cambridge University Press, Cambridge—London—New York—Melbourne, 1976.

This is a useful and easily readable book for those whishing to learn more group theory than the standard material of an ordinary undergraduate course. The book deals, among others, with free groups, free presentations of groups, Tietze transformations, van Kampen diagrams, coset enumerations, the elements of homological algebra, cohomology of groups, presentations of group extensions and presentations of direct products and wreath products.

A. P. Huhn (Szeged)

O. Neugebauer, A history of ancient mathematical astronomy. In 3 parts (Studies in the History of Mathematics and Physical Sciences, Vol. 1), XXIII, VII, V+1456 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1975.

The book contains the history of ancient mathematical astronomy up to late antiquity. Its objective is exclusively to make the reader familiar with the numerical, geometrical and graphical methods developed in the time period mentioned to describe the mechanism of the planetary system. The plan of the book does not follow strictly the chronological order of discoveries. It begins with a discussion of Almagest since "it is fully preserved and constitutes the keystone to the understanding of all ancient and mediaeval astronomy". Then it goes back in time to Babylonian astronomy for which a fair amount of contemporary original sources is available. Next come a short survey of Egyptian astronomy and then the most fragmentary and most complex section of the book: Greek astronomy and its relation to Babylonian methods. The concluding part of the main body of the work deals with Hellenistic astronomy as known from papyri, Ptolemy's minor works and the "Handy Tables". The material mentioned so far comprises two volumes. There is also a third volume which contains details concerning technical terminology, descriptions of chronological, astronomical and mathematical tools. An abundance of figures and plates, an extensive bibliography and subject index are also given in the third volume.

J. Szűcs (Szeged)

G. Ringel, Map Color Theorem (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, 209), IX+191 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1974.

If you find a problem too difficult to solve, generalize it... In 1890 P. J. Heawood stated a formula which expressed the maximum chromatic number $\chi(S)$ of graphs embeddable in any given surface S, thus generalizing the Four Color Conjecture. It happened that one could obtain a coloration with this many colors by an easy inductive argument — except if S was the sphere. So to completely prove Heawood's formula two tasks remained:

(A) to show that if S is the sphere then $\chi(S)=4$;

(B) to whow that there is a graph on each surface S whose chromatic number attains Heawood's bound.

Problem (A), the Four Color Conjecture, is still unsolved. But (B) did not turn out much simpler either. It suffices to show that the complete graph whose number of points is Heawood's bound can be embedded in S. After the pioneering work of Ringel, Gustin, and others, in 1968 Ringel and Youngs proved this conjecture. The main tool is the construction of certain combinatorial patterns called schemes. This goes quite differently for different residue classes mod 12. The difficulty varies a lot with the residue mod 12 and the different cases give a good picture of the evolution of ideas.

This book contains a very graphic presentation of the solution of this problem. It explains all of the history as well as the graph-theoretical and topological background of the problem. This not only makes it self-contained but also a very enjoyable reading, accessible to students and non-specialists. There are many exercises and open problems. The book is warmly recommended to those who want to get acquainted with topological graph theory.

L. Lovász (Szeged)

J. A. Rosanow, Stochastische Prozesse, eine Einführung (Mathematische Lehrbücher und Monographien, 28), Übersetzung aus dem Russischen, IX+288 Seiten, Akademie Verlag, Berlin, 1975.

Der Theoretiker betrachtet die stochastischen Prozesse als abstrakte Objekte der mathematischen Forschung, während für den Praktiker sind sie Werkzeuge zur Lösung praktischer Probleme. Das vorliegende Buch entstand aus den Vorlesungen des Verf., an dem Moskauer physikalisch-technischen Institut, wendet sich also an die Praktiker. Alle eingeführte Begriffe und erhaltene Ergebnisse werden anschaulich interpretiert, und die Auswahl des Stoffes wird durch die Erfordernisse der Anwendungen bestimmt. Praxisorientiertheit und begrenzter Umfang ziehen notwendigerweise etwas Grosszügigkeit bzgl. mathematische Genauigkeit nach sich, sie wirkt aber nicht störend aus.

Die erste Hälfte des Buches enthält eine übliche Einführung in die Wahrscheinlichkeitsrechnung von Grundbegriffen bis zum zentralen Grenzwertsatz (in der Ljapunowschen Form). Vorausgesetzt sind nur Grundkenntnisse aus der Differential- und Integralrechnung. Der Umfang des behandelten Stoffes von dem Gebiet der stoch. Prozesse illustrieren die Kapiteltitel der zweiten Hälfte des Buches: 1. Definitionen und Beispiele; 2. Markowsche Ketten, Klassifikation der Zustände, stationäre Verteilungen; 3. Markowsche Ketten mit stetiger Zeit; 4. Verzweigungsprozesse; 5. Einige stoch. Prozesse in der Bedienungstheorie und Irrfahrten; 6. Stoch. Prozesse in linearen Systemen; 7. Stationäre Prozesse; 8. Diffusionsprozesse; 9. Prognose und Filtration stoch. Prozesse.

Grosser Vorteil des Buches ist seine Kürze, Anschaulichkeit der Darlegungen und die enge Verbindung mit den Anwendungen. Schade, dass keine Literaturangaben die weitere Orientation helfen.

D. Vermes (Szeged)

Murray Rosenblatt, Random Processes (Graduate Texts in Mathematics 17), second edition, IX+228 pages, New York—Heidelberg—Berlin, Springer-Verlag, 1974.

Since the middle of the fifties the theory of random processes has been renewed by a revolutionary development, initiated by the works of Prohorov, Skorohod, Ito, Hunt, Dynkin, and others. The new point is the inclusion of sample space properties into the investigation, while in the classical theory only distributions were considered.

The present book, an enlarged edition of the original published in 1962 by Oxford University Press, aims to give a first introduction to the classical parts of the theory. To make the book understandable for students in lower semesters, a short introduction in probability theory is included while functional analytical methods and more complicated proofs are avoided. After the introduction and the basic definitions, Markov chains, ergodic theory of stationary sequences, Markov processes in continuous time (approach via Kolmogorov equations), the spectral decomposition of weakly stationary processes and convergence theorems for martingales are presented. Problems are included at the end of each chapter.

The book is very useful in giving a first insight into the classical theory of random processes and also as a textbook for a one or two semester course requiring only the elements of calculus and matrix algebra as background.

D. Vermes (Szeged)

C. L. Siegel—J. K. Moser, Lectures on Celestial Mechanics (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 187), X+290 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1971.

The book is a revised and enlarged translation of the very successful "Vorlesungen über Himmelsmechanik" by the first author (Grundlehren der mathematischen Wissenschaften, Bd. 85, 1956). Although new sections have been added to reflect recent work in the field, the basic organization of Siegel's original book has not been altered. As in 1957 there appeared a review of the German original in the 18th volume of these *Acta* (pp. 145—146) this time we shall discuss only the new parts added in the English text. Nevertheless, let us quote only one sentence from the English translation of the preface to the first edition in which Siegel says that his aim was "to develop some of the ideas and results that have evolved over the period of the past 70 years in the study of solutions to differential equations in the large, in which of course applications to Hamiltonian systems and in particular the equations of motion for the three body problem occupy an important place". Just as in the German original the authors again did not attempt to give a complete presentation of the subject, for example they do not discuss the now revitalized measure-theoretical methods of mechanics. The new parts added to the first edition are the following: Two sections in the first chapter on triple collision in the three-body problem by Siegel. The only relevant difference in Chapter II is the addition of the convergence proof for the transformation into Birkhoff's normal form of an area-preserving map near a hyperbolic fixed point. The main additions can be found in Chapter III. One can find a new and simpler proof for Siegel's theorem on conformal mapping near a fixed point as well as 5 sections on stability theorems for systems of two degrees of freedom and the existence theorem for quasi-periodic solutions, these 5 sections being based on the work of Arnold, Moser, and Kolmogorov.

J. Szűcs (Szeged)

Allan M. Sinclair, Automatic continuity of linear operators (London Mathematical Society Lecture Note Series, 21), 92 pages, Cambridge—London—New York—Melbourne, Cambridge University Press, 1976.

These notes are based on postgraduate lectures given at the University of Edinburgh during the spring of 1974. They contain a good amount of the results on automatic continuity of intertwining operators of Banach space operators and on homomorphisms of Banach algebras that were obtained between 1960 and 1973. They do not deal with axiomatic results such as Wright's asserting that under some reasonable hypotheses on the system of axioms all linear operators are bounded.

In the study of the automatic continuity of a linear operator S from a Banach space X into a Banach space Y the separating space $\mathfrak{S}(S)$ of S plays an important rôle. By definition $\mathfrak{S}(S)=\{y\in Y$: there is a sequence $\{x_n\}$ in X with $x_n\to 0$ and $Sx_n\to y\}$. A very important result concerning

$\mathfrak{S}(S)$ is that if $\{T_n\}$ and $\{R_n\}$ are sequences from the sets $B(X)$ and $B(Y)$ of all bounded linear operators on X and Y, respectively, and if $ST_n=R_nS$ $(n=1, 2, \ldots)$, then there exists an index N such that the closure of $R_1\ldots R_n\mathfrak{S}(S)$ equals the closure of $R_1\ldots R_N\mathfrak{S}(S)$ for $n\geqq N$. This result implies necessary and sufficient conditions on the pair (T, R), $T\in B(X)$, $R\in B(Y)$ in order that the operator S satisfying $ST=RS$ be continuous, provided that R has countable spectrum. Another application of the above result on $\mathfrak{S}(S)$ reveals some properties of discontinuous homomorphisms from $C_0(\Omega)$ into a radical Banach algebra, where Ω is a locally compact Hausdorff space. It is also proved in the book that under additional hypotheses on the (bounded) operators T and R there exist discontinuous linear operators S satisfying the relation $ST=RS$. In case X and Y are Hilbert spaces, T and R are normal and again we have the intertwining relation $ST=RS$, then S is decomposed into continuous and highly discontinuous parts. The uniqueness of the complete norm topology of a semisimple Banach algebra is proved in a way that emphasizes its relation to other automatic continuity theorems. By using the properties of the separating space of a homomorphism the continuity of a homomorphism from a Banach algebra onto a dense subalgebra of a strongly semi-simple unital Banach algebra is proved. A new proof for the existence of a discontinuous derivation from the disc algebra into a Banach module over it is given. Bade and Curtis's theorem on the decomposition of a homomorphism from $C(\Omega)$, where Ω is a compact Hausdorff space, into continuous and discontinuous parts is proved. If a unital C^*-algebra has no closed cofinite ideals, then it is shown that any homomorphism from it into a Banach algebra is continuous.

A Bibliography and an Index facilitate the reading of the book. As each member of the London Mathematical Society Lecture Note Series, this work, too, is recommended to postgraduate students and to research workers.

József Szűcs (Szeged)

E. L. Stiefel—G. Scheifele, Linear and Regular Celestial Mechanics (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 174), IX+301 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1971.

The modifier "linear" in the title refers to that the pure two body problem is treated in the book by means of linear differential equations which turn out to have constant coefficients. More precisely, the description of the pure resp. perturbed Keplerian motion reduces to that of a pure resp. perturbed harmonic oscillator. By harmonic oscillator the book means any physical system the behavior of which is described by the equations of the harmonic oscillation regardless the signs of the eigenvalues (i.e., an oscillator does not necessarily oscillate!). The adjective "regular" in the title is used in the usual sense: the differential equations describing the motion are regular, i.e., the highest order derivatives of the unknown functions are expressible in terms of regular functions of the lower order derivatives. The classical Newtonian equations are singular at the place where the attracting central mass is placed, and therefore they are not sufficient if collision occurs. On the other hand, during collision the mutual velocity is infinite. Consequently, from the point of view of numerical integration the Newtonian equations behave badly not only if the two particles collide but also when the two particles move too close with high speed. Such "nearcentre" cases are very important in space science for example when "a space vehicle is parked on an orbit about the earth and is then injected into its interplanetary orbit by a thrust of the engines". As the pure elliptic Kepler motion is described by the differential equations of a harmonic oscillator with negative eigenvalues (the one that really oscillates), the differential equations of the pure motion are stable in the sense of Ljapunov, a feature not occurring if we use the classical equations of Newton. Stability is of great significance if one wants to solve differential equations by numerical methods. This is not to say that one should solve the equations of the pure two body problem by numerical methods, since explicit formulae are available for that purpose. However, one cannot expect stability in the case of slight perturbations if the unperturbed equations are not stable.

The book could go under the subtitle "How should one calculate the motion of his space mobile?" It comprises three parts, the first and third being written by the first author, while the second part by the second author. Throughout the book particular attention is paid to numerical solutions of the problems discussed. Part I starts with the classical Newtonian equations of the two body problem and then these equations are linearized and regularized by means of the Levi-Civitâ transformation $x_1+ix_2=(u_1+iu_2)^2$ (squaring complex numbers), provided that the motion takes place in the (x_1, x_2)-plane. Although we know that pure gravitational motions are plane motions there is a need to extend the Levi-Civitâ theory to the three dimensional space. To explain this need it is enough to mention that the perturbed motion will not be a plane motion in general. The first author raised the problem of this extension of Levi-Civitâ's theory at an Oberwolfach meeting in 1964, where P. Kustaanheimo suggested using the ideas of spinor theory, in other words, employing a pair of complex numbers. In an 1965 paper Stiefel solved the problem in giving the theory of such a transformation, the so called KS-transformation. This transformation reminds us of squaring quaternions, and thus it increases the number of space coordinates by one, which, of course, causes some difficulties; however, the advantage gained turns out to be so valuable that one can allow such an increase of the number of parameters that proves to be harmless anyway. In spite of the close connection with queternions the book prefers the usage of usual real matrices to that of queternion formalism and the authors say that any attempt to use quaternions leads "to failure or at least to a very unwieldy formalism". The first part of the book proceeds with the properties of KS-transformation and the equations of motion in the new coordinates. One has to introduce a new independent variable, the so-called fictitious time s and thus the physical time t becomes a dependent variable (this is necessary in the case of plane motion, too). The relation between s and t is given by the equality $dt=r\,ds$, where r denotes the distance of the moving particle from the central mass. The differential equations of the motion after the transformation have total order ten, since we have to add an equation of order one that describes the variation of the total energy (and the above equation for the physical time). The solutions of this set of equations are discussed in detail in the first part of the book. The pure Kepler motion obtains a uniform treatment, regardless the shape of the orbit (ellipse, hyperbola, parabola), by using the Levi-Civitâ transformation and Stumpff functions. Next comes the initial value problem in *space* using the KS-transformation. The initial values in terms of the u coordinates are given in the critical case of ejection, too. Then the u-coordinates are given as functions of the 6 classical Lagrangian elements. (An element of a differential equation system is a quantity that varies linearly with respect to the independent variable.) Several aspects of the unperturbed and perturbed linear differential equations of motion are discussed. It is shown that in the case of elliptic motion the new equations are stable while the classical ones are not. In the case of elliptic motion a complete set of regular elements is given and the element equations are computed for the perturbed motion. The ten first order element equations describe the change of the elements with respect to the so-called generalized eccentric anomaly. One has his choice to solve the element equations (element method), or to solve the perturbed linear equations referred to above (u-method). The advantages and disadvantages of both methods are discussed. A chapter is devoted to gravitational perturbations (oblateness, third body attraction), with numerical examples. The last chapter of Part I bears the name "Refined Numerical Methods". It studies numerical methods for the solution of the perturbed problem that have the following property: "if the perturbing terms are switched-off at an arbitrary instant of the independent variable t (or s), then the numerical methods at hand should integrate without discretization error the subsequent unperturbed equation." It is mentioned that the classical Runge-Kutta method, the finite difference methods and the method of Encke do not satisfy this requirement if we want to integrate the classical Newtonian equations. Any element method satisfies the above requirement combined with any reasonable numerical method since during the unperturbed motion the elements vary linearly. The element- and the *u*-methods are discussed from several aspects of numerical integration, examples and good advices are given.

Part II is devoted to the canonical theory of the perturbed linear differential equations. In contrast to the classical canonical theory of Hamilton, transformations of the independent variable, the use of redundant variables as well as forces without potential are allowed and the form of the canonical differential equation system is different from the classical one, too. It is established that the differential equations in the first part of the book are all canonical and the basic equations are calculated by means of canonical transformations. Canonical equations are given that are closed in the sense that all the differential equations are incorporated in the canonical system which are needed for computing the motion; this is not an obvious problem, since, for example, if the fictitious time is the independent variable, then the system has to include the physical time as a dependent variable. A whole chapter is devoted also to the classical canonical theory of the perturbations of elements, the perturbation of the Delaunay and the classical Lagrangian elements is computed.

The very short third part is concerned with those geometrical properties of the KS-transformation that were not treated in the first two parts of the book.

The work of Stiefel and Schiefele is a very interesting reading. As it reflects some of the newest developments of the perturbed two body problem and pays special attention to the actual, numerical solution of the problem, it can and has to be recommended to anyone whose work is concerned with the calculation of the motion of artificial celestial bodies. Only basic calculus and familiarity with the elementary concepts of physics are supposed on the part of the reader (for example, the classical canonical theory is *not* a prerequisite). Thus graduate students of mathematics may enjoy its reading, too.

József Szűcs (Szeged)

Götz Uebe, Produktionstheorie (Unter Mitwirkung von Joachim Fischer) (Lecture Notes in Economics and Mathematical Systems, Bd. 114), X+301 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1976.

Dieses Buch ist aus der Mitschrift einer Vorlesung und eines Seminars entstanden, die an der Universität Bonn gehalten wurden. Der Autor schreibt in dem Vorwort: "Hauptziel dieses Buches ist eine strenge Grundlegung zu geben. Die zweite Zielsetzung ist, die Produktionstheorie als Problem der Konkaven Programmierung zu sehen. ...Ein drittes Anliegen schliesslich ist, einige natürliche Erweiterungen aus der Theorie der Produktionsfunktion zu bringen." Das Buch verwirklicht diese Zielsetzungen. Diese Arbeit ist von einer modernen Betrachtungsweise durchgedrungen. Sie enthält Kenntnisse, die für die Leser die neuesten Ergebnisse der Literatur der Produktionstheorie zu verstehen erleichtern. Ein ausführliches Literaturverzeichnis ist beigefügt.

L. Megyesi (Szeged)

M. M. Wainberg und W. A. Trenogin, Theorie der Lösungsverzweigung bei nichtlinearen Gleichungen, XII+408 pages, Berlin, Akademie Verlag, 1973.

Summing in a few words the subject of the book it is the analysis of branching points of certain nonlinear functional equations depending on parameters. There are given methods for the description of all solutions of the equation which bifurcate from a known solution by the change of the parameters. Among others the book discusses the question of the theory of implicit functions (both in the finite dimensional case and in Banach space) the branching theory of periodic solutions of ordinary differential equations and contains an extensive account of certain classes of nonlinear integral and integro-differential equations, as well as applications of the theory to a number of practical problems. There is a valuable bibliography.

The topic is of fundamental importance for applied mathematics, especially in problems represented by a nonlinear system of parameters. It has been researched for a long time and numerous articles have been published. The present book gives a good summary of the researches up to the recent ones. If will be useful both for mathematicians as a monograph and for students as an introduction to bifurcation theory.

The original Russian work was published in 1969.

J. Terjéki (Szeged)

Garth Warner, Harmonic Analysis on Semi-Simple Lie Groups I, II (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 188), XVI+529, VIII+461 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1972.

This two volume monograph is devoted to the representation theory of semi-simple Lie groups which, to a great extent, is the work of one single man: Harish-Chandra. Harmonic analysis on locally compact groups has been vigorously developed in the past 30 years. Although little is known about representations of a general locally compact group, two extreme cases have been studied in detail: Dixmier, Kirillov and Pukánszky have made great progress in the theory of nilpotent groups and, on the other hand, Harish-Chandra has created the representation theory of semi-simple Lie groups, the book under review is the first systematic exposition of his results. The author made a successful effort to give complete proofs of the basic theorems and thus to provide a reasonably self-contained introduction to Harish-Chandra's theory. The prerequisites vary in the following way: the part that is an introduction to general group representation theory and spherical functions (chapters 4, 5, 6, 7) requires a general knowledge of functional analysis, some distribution theory and elementary abstract harmonic analysis. The reader, if he wishes to, can start with this part of the book without the danger of encountering references to the first three chapters. The part on the structure theory of semi-simple Lie groups and algebras (chapters 1 and 2) requires a background equivalent to JACOBSON's *Lie algebras*. Interscience, New York, 1962. The part on the finite dimensional representations of semi-simple Lie groups (chapter 3) assumes a little sheaf theory, while the rest of the treatise (chapters 8, 9, 10) deals exclusively with semi-simple Lie groups and needs all the prerequisites mentioned above.

Let us say a few words about the subject of the book in a way understandable to every mathematician. To this end let G be a locally compact unimodular group satisfying the second axiom of countability. Assume, moreover, that G is postliminaire, i.e., all continuous representations of G are of type I (the von Neumann algebras generated by the images of G via the representations are discrete). Let us denote by G the topological space of unitary equivalence classes of irreducible (continuous) unitary representations of G equipped with the so-called hull-kernel topology. Take a Haar measure dx on G. Then a famous theorem of I. E. Segal asserts the existence of a unique positive measure μ defined on the Borel structure of $\hat{G}$ such that $\int_G |f(x)|^2\,dx=\int_{\hat{G}} \operatorname{tr}\left(U(f)\,U(f)^*\right)\,d\mu(\hat{U})$ for all $f\in L^1(G)\cap L^2(G)$. On the right side in the above equality tr means "trace of an operator" and for the representant U of any element $\hat{U}$ of $\hat{G}$ the operator $U(f)$ is defined as $\int_G f(x)\,U(x)\,dx$. The measure μ is called the Plancherel measure (associated with dx). One basic problem of harmonic analysis is the explicit determination of μ. It is the subject of the book to do this in the case of a semi-simple Lie group.

For those that have a solid background in abstract harmonic analysis we are now going to give more insight into the contents of the book by drawing freely form the chapter and section headings: The structure of real semi-simple algebras: Bruhat decomposition, parabolic subgroups, Cartan subalgebras and subgroups; The universal enveloping algebra of a semi-simple Lie algebra: invariant theory, reductive Lie algebras, representations of reductive Lie algebras, representations on cohomology groups; Finite dimensional representations of a semi-simple Lie groups; Infinite dimensional group representation theory: representations on a locally convex, Banach or Hilbert space, differentiable and analytic vectors, large compact subgroups; Induced representations: unitarily induced representations, quasi-invariant distributions, irreducibility of unitarily induced representations systems of imprimitivity ; The general theory of spherical functions; Topology on the dual, Plancherel measure; Analysis on a semi-simple Lie group: differential operators, central eigendistributions and invariant integral on reductive Lie groups and algebras; Spherical functions on a semi-simple Lie group: asymptotic behavior of μ-spherical functions and zonal spherical functions on a semi-simple Lie group, spherical functions and differential equations; the discrete series for a semi-simple Lie group — existence and exhaustion.

There are examples throughout the book that help the reader understand the abstract theory and acquire some knowledge of the applications. Altogether there are three appendices in the two volumes that compile notions and results concerning quasi-invariant measures, distributions on a manifold, and the theory of differential equations.

While the work cannot be recommended as a first introduction, those who have a solid background in the theory can learn from it most of what is presently known about semisimple Lie groups.

József Szűcs (Szeged)

J. H. Wells, L. R. William, Embeddings and Extensions in Analysis (Ergebnisse der Math., Band 84, VI+108 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1975.

The object of this book is a presentation of the main results of two geometrically inspired problems in analysis. The first is the problem of embedding metric spaces into Hilbert space or more generally into L^p spaces, the second is the problem of extending of continuous maps.

Chapter I deals with isometric embeddings into Hilbert space and characterization of subspaces of L^p; Chapter II is devoted to integral representations of functions of positive definite and negative type. Chapter III contains the main results of extension problems for contractions and isometries of Banach spaces. Chapter IV gives a glimpse on interpolation and L^p inequalities. The theme of Chapter V is the extension problem for Lipschitz—Hölder maps between L^p spaces.

The book is recommended to all mathematicians, who are interested in extension and embedding problems.

L. Gehér (Szeged)