Bibliographie

Tom M. Apostol, Modular Functions and Dirichlet Series in Number Theory (Graduate Texts in Mathematics), X+198 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976.

This textbook is the continuation of a book of the same author, which appeared in the Springer-Verlag series Undergraduate Texts in Mathematics under the title "Introduction to Analytic Number Theory". This second volume presupposes a background in number theory comparable to that provided in the first volume, together with a knowledge of the basic concepts of complex analysis.

The first three chapters provide an introduction to the theory of elliptic modular functions, which play a role in additive number theory analogous to that played by Dirichlet series in multiplicative number theory. Applications to the partition function are given in Ch. 5, while Chs. 4 and 6 contain, among others, Lehner's congruences for the Fourier coefficients of the modular function $j(\tau)$, and Hecke's theory of entire forms with multiplicative Fourier coefficients. Ch. 7 deals with the problem of approximating real numbers by rational numbers, including Kronecker's theorem with applications. The last chapter gives an account of Bohr's theory of equivalence of general Dirichlet series.

There are exercises at the end of each chapter. The book will certainly help the nonspecialist become acquainted with a fascinating part of mathematics and, at the same time, will provide an up-to-date background to every specialist in the field.

F. Móricz (Szeged)

J. Bergh and J. Löfström, Interpolation Spaces (An Introduction) (Grundlehren der mathematischen Wissenschaften, 223), X+207 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1976.

In recent years there has emerged a new field of study in functional analysis: the theory of interpolation spaces. Interpolation theory has been applied to other branches of analysis (e.g. partial differential equations, approximation theory, etc.), but it also has considerable interest in itself. This is the first attempt, as far as we know, to treat interpolation theory fairly comprehensively in book form.

The reader is supposed to be conversant with the elements of real (several variables) and complex (one variable) analysis, of Fourier series, and of functional analysis. Beyond elementary level the authors tried to supply proofs of the statements in the main text. Their general reference for elementary results is the first volume of the widely-known monograph of Dunford—Schwartz "Linear operators".

Ch. 1 presents the classical interpolation theorems of M. Riesz, with Thorin's proof, and of Marcinkiewicz, which provided the main impetus for the study of interpolation. The basic concepts are introduced in Ch. 2, where a few general results are discussed, e.g. the Aronszajn—Gagliardo theorem.

The authors treat two essentially different interpolation methods: the real method and the complex method. These two methods are modelled after the proofs of the Marcinkiewicz theorem and the Riesz—Thorin theorem, resp.. The real method is elaborated following Peetre in Ch. 3, the complex method following Calderon in Ch. 4.

The further three chapters contain applications of the above general methods. Ch. 5: Interpolation of L_p-Spaces, Ch. 6: Interpolation of Sobolev and Besov Spaces, Ch. 7: Applications to Approximation Theory.

In each chapter the penultimate section contains exercises, which extend and complement the results of the previous sections. Moreover, many important results and most of the applications can be found only as exercises. The last section of each chapter is devoted to notes and comments. These include historical sketches, various generalizations, related questions and references without aiming at completeness. There is a bibliography consisting of about 200 items.

The treatise provides a rich and up-to-date account of this fast-growing and important field, and it is warmly recommended to everyone who wants to learn, or do research in, interpolation theory.

F. Móricz (Szeged)

M. Braun, Differential equations and their applications (An introduction to applied mathematics, Applied Mathematical Sciences, Vol. 15) XIV+718 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1975.

Two main motives of the present-day development of the theory of differential equations can be emphasized. More and more interesting problems arise in the theory as in an independent branch of pure mathematics. On the other hand, in applications the number of processes that can be modelled mathematically by differential equations are constantly increasing. Accordingly, in the last years a great number of noticeable books have appeared emphasizing one or the other of the motives mentioned above. This book calls for the interest of both users of mathematics and mathematicians. The author writes in the preface: "the material is presented in a manner which is rigorous enough for our mathematics and applied mathematics majors, but yet intuitive and practical enough for our engineering, biology, economics, physics and geology majors." Numerous examples are given of how researchers have used differential equations to solve real life problems. Especially interesting are: the Van Meegeren art forgeries, population growth of various species, a model for the detection of diabetes, L. F. Richardson's mathematical theory of war, why the percentage of sharks caught in the Mediterranean Sea rose dramatically during World War I, the Tacoma Bridge disaster, and a model for the spread of epidemics.

There are many original interesting exercises at the end of each section, and complete Fortran and APL programs are given for every computer exercise in the text.

The titles of chapters are: First order differential equations, Second order differential equations, Systems of first order equations, Qualitative theory of differential equations, Separation of variables and Fourier series, Appendices.

The well-written self-contained book can be understood by anyone having attended a two-semester course in Calculus.

L. Hatvani—L. Pintér (Szeged)

Colloquio Internazionale sulle Teorie Combinatorie. I—II (Atti dei Convegni Lincei 17), 518+526 pages, Accademia Nazionale dei Lincei, Roma 1976.

This combinatorial colloquium, organized by the Accademia Nazionale dei Lincei with the collaboration of the American Mathematical Society, took place in Rome, September 3—15, 1973. The conference was dedicated to Professor Beniamino Segre on the occasion of his 70th birthday, and was chaired by Professor Segre. This is also reflected by the fact that the majority of papers delivered at the colloquium and reproduced in the volumes deal with the exciting and fast-developing field of finite geometries, block designs and matroids, to which field Professor Segre's contribution has been most important.

It would be impossible to list all of the 77 papers contained in the two extensive volumes of the Proceedings. First, there are many very useful survey papers: HALL writes about the "Construction of Combinatorial Designs", RICHARD RADO about "Partition Calculus", BENIAMINO SEGRE surveys "Incidence Structures and Galois Geometries", BUEKENHOUT "Characterizations of Semi Quadrics", BACHMAN "Hjelmslev Groups", Seidel "2-graphs", TURÁN "Combinatorics, Partitions, Group Theory", just to mention some. ERDŐS has, as usual, a paper on "Problems and Results in Combinatorial Analysis". Besides, there are many papers which contain very significant new results and ideas, some of which have been known, and whose publication has been looked forward to, since the colloquium.

L. Lovász (Szeged)

E. T. Copson, Partial differential equations, VII+280 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1975.

A good survey on the theory of partial differential equations of the first order and of linear partial differential equations of the second order, using the methods of classical analysis. In spite of the advent of computers and the recent applications of the methods of functional analysis to the theory of partial differential equations, the classical theory retains its relevance in several important respects.

The book is well-organized. At the end of each chapter a number of interesting exercises help understanding. The titles of chapters show the treated topics of the theory: Partial differential equations of the first order, Characteristics of equations of the second order, Boundary value and initial value problems, Equations of hyperbolic type, Riemann's method, The equations of wave motions, Marcel Riesz' method, Potential theory in the plane, Subharmonic functions and the problems of Dirichlet, Equations of elliptic type in the space, The equation of heat.

This text-book will be useful for lecturers and students of mathematics or theoretical physics.

J. Terjéki (Szeged)

R. E. Edwards and G. I. Gaudry, Littlewood-Paley and Multiplier Theory (Ergebnisse der Mathematik und ihrer Grenzgebiete, 90), IX+212 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1977.

The classical Littlewood—Paley theorem asserts that to each p in $(1, \infty)$ there corresponds a pair (A_p, B_p) of positive constants such that $A_p\|f\|_p \leqq \|(\sum_{j\in\mathbf{Z}} |S_j f|^2)^{1/2}\|_p \leqq B_p\|f\|_p$ for every f in L^p, where $S_j f$ is the jth dyadic partial sum of the Fourier series $\sum \hat{f}(n)e^{inx}$ of f, defined by (1) $\sum_{2^{j-1}\leqq n<2^j} \hat{f}(n)e^{inx}$, or $\hat{f}(0)$, or $\sum_{-2^{|j|}<n\leqq -2^{|j|-1}} \hat{f}(n)e^{inx}$ according as $j>0$, $j=0$, or $j<0$.

In other words, we can say that the L^p norm of a function f can be computed, up to equivalence, by breaking up the Fourier series of f into its dyadic partial sums, putting them together in an l^2 fashion, and calculating the L^p norm of the resulting function.

The Littlewood—Paley theorem implies, among others, the following analogue of the Riesz—Fischer theorem: Suppose $p\in(1, \infty)$. A series $\sum_{n\in\mathbf{Z}} c_n e^{inx}$ is the Fourier series of a function, say f, in L^p iff $\|(\sum_{j\in\mathbf{Z}} |s_j|^2)^{1/2}\|_p<\infty$, where s_j denotes the trigonometric polynomial obtained from formula (1) by replacing $\hat{f}(n)$ by c_n. Moreover, the series $\sum_{j\in\mathbf{Z}} s_j$ converges unconditionally in L^p to f.

These results make the Littlewood—Paley theorem one of the fundamental results in L^p harmonic analysis.

The treatment proceeds along two main lines, the first relating to singular integrals on locally compact groups (Chs. 2 and 3), and the second to martingales (Ch. 5). Both (classical and modern) versions of the Littlewood—Paley theorem are dealt with for the classical groups $\mathbf{R}^n$, $\mathbf{Z}^n$, $\mathbf{T}^n$ (Chs. 7 and 8) and for certain classes of discontinuous groups (Ch. 4); $\mathbf{R}$ denoting the set of real numbers, $\mathbf{Z}$ the set of integers, and $\mathbf{T}$ the circle group.

The Littlewood—Paley theorem is then applied to Fourier multiplier theory, for instance to obtain the famous theorems of M. Riesz, Marcinkiewicz and Stečkin (Ch. 6); and to lacunary sets (Ch. 9).

For the reader's convenience there are four appendices containing a number of auxiliary topics at the end of the book. Historical Notes, References, Terminology, Index of Notation, and Index of Authors and Subjects complete the book.

The presentation is self-contained and unified. The book is intended primarily for use by graduate students and mathematicians who wish to begin studies in these areas, poorly served by existing books. This well-written book fills in the gap in the literature and satisfies all needs of a beginner as well as of a "worker in the field".

F. Móricz (Szeged)

Carl Faith, Algebra. II, Ring Theory (Grundlehren der mathematischen Wissenschaften, Band 191), XVIII+302 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1976.

This book is the second volume of the work on rings, modules and categories, Volume I (Parts I—IV) of which was published as Band 190 of the same series (and reviewed in these *Acta*, **38** (1976), 209). The present volume II is devoted entirely to ring theory. To a large extent this volume, except Chapters 18 and 26, illustrates the power of homological methods in ring theory. In contrast to what was announced in Volume I, Part VI on commutative rings, hereditary rings, separable algebras, and the Brauer group is not included in Volume II, thus Part V (Chapters 17—26) comprises all of Volume II.

This is the largest part of the book, too rich in content to list here all the topics covered in it. Summarizing in a few sentences, Chapter 17 is on modules of finite Jordan—Hölder length, while Chapter 18 deals with the Jacobson radical of a ring. In Chapter 19 quasiinjective modules are studied and, among others, the Chevalley—Jacobson density theorem is proved. Chapter 20 is devoted to the direct decompositions of rings and modules. Azuyama diagrams are discussed in Chapter 21 while the aim of Chapter 22 is to study the projective covers of modules and perfect rings. In Chapters 23 and 24 Morita's duality theory and some applications are presented; in particular, quasi-Frobenius rings are also discussed. Chapter 25 is on serial and Σ-cyclic rings and, finally, Chapter 26 is concerned with semiprimitive and semiprime rings, the main result being

Amitsur's theorem on the semiprimitivity of group algebras over transcendental fields of characteristic zero.

Each chapter ends with a list of related results aiming to help those wishing to specialize in that topic. The book is concluded with a rich bibliography up to 1975.

Á. Szendrei (Szeged)

Robert Fortet, Elements of Probability Theory, XIX+524 pages, Gordon and Breach Science Publishers, London—New York—Paris, 1977.

This book is the translation of the French original "Eléments de la Théorie des Probabilités, Vol. I", published by the Centre National de la Recherche Scientifique in 1960. A few errors of the French original have been corrected, but otherwise the text remained unaltered. The book is introductory, written in the best tradition of French scholarship. Each newly introduced concept is carefully motivated from various aspects. The author is not shy to get into philosophical problems, and discussions of questions from physics, mechanics, genetics, etc., to help the beginner get a real feeling of the subject. He recommends his book to non-mathematical research workers such as physicists, engineers, biologists and operation research workers "to provide users with an exposition of the fundamentals of probability theory, at a level of mathematical sophistications which would not repel the non-specialist reader". Chapter headings: I. Combinatorial analysis and its application to classical and quantum statistics and to the chromosome theory of heredity; II. The concept of probability, Measures or mass distributions, Hilbert spaces, Random elements and probability laws; III. Distribution functions; IV. Random variables, axiom of conditional probability; V. n-dimensional random vectors and variables; VI. Addition of independent random variables; Stochastic convergence, laws of large numbers, ergodic theorems; Convergence to a normal law, convergence to a Poisson law; Generalizations.

Gordon and Breach is to be praised for having made available this valuable book in English.

Sándor Csörgő (Szeged)

Э. Фрид—И. Пастор—И. Рейман—П. Ревес—И. Ружа, Малая математическая энциклопедия, 693 стр. Изд. АН Венгрии, Будапешт, 1976.

Эта книга является энциклопедией в менее привычном смысле слова; именно, она является обзором высшей математики, напоминающим превосходную книгу Куранта и Роббинса «Что такое математика?» В ней представлены важнейшие разделы математики: алгебра, геометрия, математический анализ, теория множеств, теория вероятностей, математическая статистика и математическая логика. Такая классификация, разумеется, отражает и личный интерес авторов, что, в свою очередь, отражает в некоторой мере и главные напривления математических исследований в Венгрии.

Книга может быть использована для первого ознакомления с различными математическими понятиями (например, группы занимают в ней 7 страниц, числовые ряды — 13, начертательная геометрия — 9, а исчисление предикатов — 11). Ее можно использовать также в качестве справочника, поскольку в ней можно найти много простых, важных теорем (без доказательства), как, например, теорему Кэли, критерий Коши, теорему Полке и теорему Гёделя о полноте. Книга в основном написана живо, местами увлекательно. Следует, однако, предупреждать читателя, что переводчики часто упожребляют нестандартную терминологию, особенно в разделах теории чисел и теории множеств.

B. Csákány (Szeged)

Morris W. Hirsch, Differential Topology (Graduate Texts in Mathematics, 33), 221 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976.

The aim of this book is to give an introduction to the problems and results of differential topology, that is of the topology of differentiable manifolds. Although topological questions on manifolds occur in differential geometry and global analysis as well, there is always some extra structure present, e.g. Riemannian metric or a differential equation on the manifold. In differential topology the manifold itself is studied, the extra structures are used only as tools. Some typical questions: Can a given manifold be embedded in another one? If two manifolds are homeomorphic, are they necessarily diffeomorphic? Which manifolds are boundaries of compact manifolds? Do the topological invariants of a manifold have any special properties? Does every manifold admit a non-trivial action of some cyclic group? This book presents some answers to these questions.

The first three chapters are fundamental for the understanding of the book. Definitions are introduced and the basic properties of manifolds, the approximation theorems for the maps of manifolds and the unifying idea in differential topology: the transversality are treated. In Chapter 4 the elementary theory of vector bundles is developed, including the classification theorem: isomorphism classes of vector bundles over the manifold M correspond naturally to homotopy classes of maps from M into a certain Grassmann manifold. Chapter 5 is devoted to the study of the theory of degrees of maps. In this way some results of classical algebraic topology are derived. In Chapter 6 an introduction to the Morse theory is presented. Chapter 7 contains the elementary part of one of the most elegant theories in differential topology: René Thom's theory of cobordisms. (Two manifolds are cobordant if together they form the boundary of a compact manifold.) In chapter 8 the isotopy of embeddings of manifolds is investigated. Chapter 9 deals with the classification of surfaces.

Each chapter contains many interesting exercises and historical remarks.

The book is a rich, up-to-date account of differential topology. It is also very well-written. I can warmly recommend it to everyone interested in the theory of manifolds.

P. T. Nagy (Szeged)

Wu Yi Hsiang, Cohomology Theory of Topological Transformation Groups (Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 85), X+164 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1975.

The pioneering results of P. A. Smith for prime periodic maps on homology spheres, and of L. E. J. Brouwer on periodic transformations suggest a general direction of studying topological transformation groups in the framework of algebraic topology. This book is an excellent summary of these different generalizations. Chapters I and II contain general material on compact Lie groups, G-spaces and on structural and classification theory of compact Lie groups and their representations.

Let G be a compact Lie group and let X be a given G-space. Then the equivariant cohomology $H_G^*(H)$ of the G-space X is the ordinary cohomology of the total space X_G of the universal bundle $X \to X_G \to B_G$, with the given G-space as the typical fibre. In Chapter III some fundamental properties and theorems (such as the localization theorem of Borel-Atiyah-Segal) of this equivariant cohomology theory of A. Borel are formulated and proved.

In Chapter IV the relationship between the geometric structure of a given G-space X and the algebraic structure of its equivariant cohomology $H_G^*(H)$ is investigated. The reader obtains in this Chapter an answer for the following problems: How much of the cohomology structure of the fixed point set, $H_G^*(F)$, is determined by the equivariant cohomology $H_G^*(X)$? Is it possible to give a criterion for the existence of fixed points purely in terms of the equivariant cohomology $H_G^*(H)$? Suppose $F(G, X)=\emptyset$. How to determine the set of maximal isotropy subgroups from the algebraic structure of $H_G^*(X)$?

The structural splitting theorem for linear tori actions can be generalized to various structural splitting theorems of the equivariant cohomology, and combining the structural splitting theorems with the maximal tori theorem, a geometric weight system for topological transformation groups can be defined.

Such a program is carried out in Chapters IV, VI and for the special cases of acyclic manifolds and cohomology spheres in Chapter V. In Chapter VII the cohomology method is applied to study transformation groups on compact homogeneous spaces.

This book comprises a very large material. To read it certain knowledge on differential manifolds, Lie groups and cohomology theory is necessary.

Z. Szabó (Szeged)

John G. Kemeny—J. Laurie Snell—Anthony W. Knapp, Denumerable Markov Chains (Graduate Texts in Mathematics, 40), XII+484 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976.

After the second edition of "Finite Markov Chains" by the first two authors in the series "Undergraduate Texts in Mathematics", Springer-Verlag has provided us with the second edition of another success-book, which treats discrete parameter Markov chains having countable state space and stationary transition probabilities, with special emphasis on the context of potential theory. The original was published by Van Nostrand, Princeton, N. J., 1966, in the University Series in Higher Mathematics, and has been reviewed by J. L. Doob in detail (MR 34 (1967)#6858). Doob wrote that "the potential-theoretic point of view should have a strong influence on future research", and his prediction has already been proved to be true. The book has served as a source of inspiration in the past ten years, occurring often in the reference list of research papers in the field of probabilistic potential and boundary theory. An erroneus theorem is corrected, but aside from this change, the text of the first eleven chapters of the first edition is left intact. This new edition contains a new twelfth chapter on Markov random fields, written by David Griffeath (pp. 425—428). In addition to this, it also contains a new section of Additional Notes (pp. 465—470), covering some of the developments of the past ten years, which is accompanied by a section of Additional References, listing 58 items.

Sándor Csörgő (Szeged)

A. A. Kirillov, Elements of the Theory of Representations (Grundlehren der mathematischen Wissenschaften, 220), XI+315 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1976.

This is a translation, by Edwin Hewitt, of the original Russian text. The translation is faithful except for some corrections supplied by Professor Kirillov himself. The bibliography, for an obvious reason, has been considerably modified from the original.

The material of the book grew out of courses given and seminars directed by the author at Moscow State University. The first part of the book (§§ 1—6) is not directly related to representations, it contains the facts needed from other parts of mathematics. Those topics that are not included in elementary university courses are treated here in detail. A reader familiar with this material may skip the first part of the book. The second part (§§ 7—15) is devoted to the principal concepts and methods of the theory of representations. In the third part (§§ 16—19) the general ideas of the second one are illustrated by concrete examples.

The book includes a large number of exercises playing an essential rôle in the text proper. "A majority of the proofs are given in the form of a cycle of mutually connected problems". Most problems are supplied with remarks to help the reader solve them.

Little attention has been paid to finite dimensional representations of semisimple Lie groups and Lie algebras as there exist good expositions of this subject (in both the Russian and English literature, many books being mutually translated). For the same reason the applications of the theory of group representations in the theory of special functions as well as in mathematical physics have been completely ignored in this book. However, a large space is devoted to the method of orbits, which has not yet been included in any textbook. The author hopes that some of the readers of this book will contribute to the development of the rapidly growing and important theory of orbits.

The paragraph headings are as follows. § 1. Sets, categories, topology, § 2. Groups and homogeneous spaces, § 3. Ring and modules, § 4. Elements of functional analysis, § 5. Analysis on manifolds, § 6. Lie groups and Lie algebras, § 7. Representations of groups, § 8. Decomposition of representation, § 9. Invariant integration, § 10. Group algebras, § 11. Characters, § 12. Fourier transforms and duality, § 13. Induced representations, § 14. Projective representations, § 15. The method of orbits, § 16. Finite groups, § 17. Compact groups, § 18. Lie groups and Lie algebras, § 19. Examples of wild Lie groups.

"A short historical sketch and a guide to the literature", and a "Bibliography" complete the book.

This masterly written and translated book may be mainly recommended to those wanting to begin the study of the vast field of representations.

József Szűcs (Szeged)

S. Lefschetz, Applications of Algebraic Topology (Applied Mathematical Sciences, 16), viii+189 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1975.

Solomon Lefschetz's book published posthumously consists of two independent monographs.

In Part I the author first gives a short résumé of the algebraic topology up to dimension 2 (Chapters I—V). Except for the theorem of Jordan—Schoenflies all results are presented here together with proofs. In Chapter VI Kirchoff's Laws are formulated in terms of the co-theory and a system of differential equations is deduced from them. In Chapters VII and VIII the elements of the theory of 2-dimensional complexes and surfaces are presented. They are applied in Chapter IX to the problem of planar graphs and dual networks. Maclane's and Kuratowski's characterization theorems are proved.

Part II is devoted to the demonstration of the connection between the Picard—Lefschetz theory and the theory of Feynman integrals. After a short algebraic and topological résumé with almost no proofs (Chapter I) the author treats a special phase of Picard's program: he investigates the behaviour of the abelian integral of a rational function on a complex irreducible algebraic surface near an isolated singularity (Chapter II). Chapter III deals with the extension of this theory to higher varieties.

Feynman's problem, which is treated in Chapters IV and V can be outlined as follows:

Set $x=\{x_1, \dots, x_n\}$, $y=\{y_1, \dots, y_n\}$, where x_k are real or complex coordinates and y_k are real or complex parameters. Let $Q_h(x, y)$ denote real quadratic polynomials. The Feynman problem consists of the study of the analytical character of

$$I(y) = \int_{\Gamma} \frac{dx_1 \dots dx_n}{\prod\limits_h Q_h(x, y)},$$

as function of y, where Γ is the whole admissible part of the x-space. In Part II the author explains only the crucial points of proofs for a reader well versed in classical function theory.

A. Krámli (Budapest)

Edwin E. Moise, Geometric topology in dimensions 2 and 3 (Graduate Texts in Mathematics), VII+262 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1977.

The manuscript of this book was used in 1975—76 at Texas University and earlier at the University of Wisconsin in seminars conducted by R. H. Bing. It is intended to be a textbook; the text is divided into 37 sections and contains all of the most important classical and new results concerning this branch of topology. The traditional material of plane topology has been reformulated in such a way that, bringing three-dimensional ideas in sharper focus, it serves as an introduction to the methods to be used in dimension 3. The proofs of the triangulation theorem and the *Hauptvermutung* are largely new. So is the proof of Schoenflies' theorem.

At the end of each section there are sets of problems, which are composed in an unusual way. Most of the problems state true theorems, extending or elucidating the preceding section of the text. But in a large number of them false propositions are also stated as if they were true. Here it is the reader's job to discover that they are false and find counterexamples.

The book is highly recommended to anyone interested in topology and mature enough to understand abstract mathematical thinking.

L. Gehér (Szeged)

R. Narasimhan, Analysis on Real and Complex Manifolds (Advanced Studies in Pure Mathematics), X+246 pages, Masson & Cie, Paris, and North-Holland Publishing Company, Amsterdam, 1968.

This book contains the basic material for the study of differential equations on manifolds. It has three chapters.

In Chapter 1 some theorems on differentiable functions in $\mathbf{R}^n$ are proved such as the implicit function theorem, Sard's theorem and Whitneys' approximation theorem. Chapter 2 is an excellent introduction to the study of real and complex manifolds. This chapter contains, among others, the theorem of Frobenius, the lemmata of Poincaré and Grothendieck, the imbedding theorem of Whitney and Thom's transversality theorem. In chapter 3 properties of linear elliptic differential operators are formulated. This chapter deals with Peetre's and Hörmander's characterizations of linear differential operators, the inequalities of Gårding and of Friedrichs on elliptic operators, and finally with the approximation theorem of Malgrange-Lax. The Runge theorem on open Riemann surfaces is also proved.

The book is written in a very elegant style. It is an excellent graduate textbook.

Z. Szabó (Szeged)

M. S. Raghunathan, Discrete Subgroups of Lie Groups (Ergebnisse der Mathematik und ihre Grenzgebiete, 68), 226 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1972.

The theory of discrete subgroups of Lie groups, which originated among others from geometrical crystallography, has become a separate discipline as a consequence of the work of A. D. Malcev, A. Selberg, A Weil, A. Borel, L. Auslander and others in the last 20 years.

This book is a fundamental monography on this subject. Its aim is to present a detailed account of recent work on the theory of discrete subgroups of a Lie group from the geometric point of view.

Chapters I—V contain a fairly complete study of lattices in nilpotent, solvable and semisimple Lie groups, where a "lattice" in a locally compact group G means a discrete subgroup H in G such that the homogeneous space G/H carries a finite G-invariant measure. Chapter VI presents some general theorems on finitely generated subgroups of a Lie group. In Chapter VII results on the cohomology of solv-manifolds and compact symmetric spaces are treated. Chapter VIII plays a central role. Among other results it is proved here that, at least up to a point, the study of lattices in general Lie groups can be split into the study of those in solvable and semisimple groups separately. Chapters IX—XIV are devoted to further interesting results on discrete subgroups (results of Kazdan—Margolis, arithmetic groups, existence of arithmetic lattices, etc.).

It is assumed that the reader has considerable familiarity with Lie groups and algebraic groups. Most of the results used frequently in the book are summarized in the "Preliminaries"; this chapter may be useful as a reference as well.

P. T. Nagy (Szeged)

Alfréd Rényi, Selected Papers, I—III, edited by Pál Turán, co-editors: P. Bártfai, I. Csiszár, J. Fritz, G. Halász, Gy. Katona, P. Révész, D. Szász, E. Szemerédi, I. Vincze; technical editor: G. Székely, 628+646+667 pages, Akadémiai Kiadó, Budapest, 1976.

Alfréd Rényi (1921—1970) was one of the most outstanding mathematicians of the new Hungarian generation. He inventively commanded a wide area of interest. He published about 350 papers and books.

The Selected Papers includes 156 articles. The papers published in English, German, or French are kept in their original form, and those published in Hungarian, Russian or Chinese are here translated into English. The papers follow in chronological order. In the first volume 52 works, written between 1948 and 1956, in the second 48 works, written between 1956 and 1961, and in the third 56 works, written between 1962 and 1970, are reproduced. Each volume contains a general introduction by the editor, a biography of Alfréd Rényi and a list of his scientific works. Besides, the first volume contains a photograph of Alfréd Rényi.

The papers are selected very carefully. Thanks to this the present selection offers a good survey of Alfréd Rényi's main fields of interest and shows in what areas Alfréd Rényi contributed most significantly to the development of mathematics. The most important papers in the first volume are those dealing with the generalization and application of the Linnik large sample method, those concerning the Poisson process and the generalization of Kolmogorov's inequality as well as those devoted to the foundations of probability theory and to the applications of probability theory in chemistry and biology. In the second and third volumes the articles about the applications of probability theory in graph theory, number theory, group theory, and information theory are the most valuable.

The selection is greatly enhanced by the remarks of the editors after each paper. These professional remarks inform the reader about the international influence of the results and problems included in each paper: who developed and in what direction the problems in question and in what sense the problems were solved. On the basis of the papers followed by the remarks the reader obtains a picture of a widely ranging mathematical cooperation in the centre of which stood Alfréd Rényi. His untimely death was a great loss to mathematics.

Károly Tandori (Szeged)

T. G. Room—P. B. Kirkpatrick, Miniquaternion geometry. An introduction to the study of projective planes, Cambridge Tracts in Math. 60, viii+176 pages, Cambridge University Press, Cambridge, 1971.

This book is of a rather unusual nature. Its primary aim is the study of four concrete mathematical objects: the four projective planes (known at present) of order 9. It is the authors' aim that the study of these planes should also serve as an introduction to the study of projective planes in general (9 being the smallest order for which non-desarguesian projective planes exist).

The first chapter describes the relevant algebraic objects: the finite field of 9 elements: $GF(9)$ and the other near-field of order 9: Q, which is called by the authors the miniquaternion system because of its common features with the skew-field of ordinary quaternions. The automorphism group of Q is also determined, and solution of equations in Q is discussed.

Chapter 2 gives a rapid introduction to projective planes (including Bruck's theorem on the possible orders of subplanes) and collineations. The standard procedure of constructing a plane $\Pi(K)$ from a field K is then desribed and projectivities, correlations and conics of $\Pi(K)$ are discussed. In Chapter 3 these investigations are carried through in much greater detail in the case of the plane $\Pi(GF(9))$, after a brief study of $\Pi(GF(3))$.

Chapters 4 and 5 are devoted to the study of the 3 non-desarguesian planes of order 9, discovered by Veblen and Wedderburn in 1907. In Chapter 4 the translation plane Ω is defined with the aid of Q and an appropriate coordinatization. The collineation group of Ω is completely determined. Ω has subplanes of order 2 and 3; both types of subplanes are studied in detail. Then the Rodriguez oval in Ω is introduced. Finally it is proved that the dual plane of Ω: Ω^D is a plane of order 9 which is not isomorphic to Ω.

In Chapter 5 the plane Ψ is defined using Q in a different coordinatization. (Ψ is the smallest Hughes plane.) The collineation group of Ψ is determined. It is proved that Ψ is self-dual and polarities of Ψ are studied. The subplanes of Ψ are investigated and another definition of Ψ (essentially the original definition of Veblen and Wedderburn) is also given.

Throughout the book great emphasis is laid on concreteness. Points, lines etc. are given names and there are lots of tables enumerating all the objects of a certain type in the plane under study. The numerous exercises also aim usually at checking certain concrete statements rather than prove theorems.

This book seems to be particularly useful to people who want to see (or want to show their students) what certain concepts, constructions, theorems concerning projective planes, usually presented in an abstract setting, actually mean. This type of book is particularly welcome since in the modern literature there is a tendency toward the most general possible formulation without giving examples illuminating the motivations behind the various investigations. At the same time the book is a really good starting point for a further study of projective planes.

József Pelikán (Budapest)

G. B. Seligman, Modular Lie Algebras (Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 40), IX+165 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1967.

The theory of classical Lie algebras over a base field of characteristic zero was developed by Lie, Killing, Elie Cartan, and Weyl. The first papers studying the structure of Lie algebras over arbitrary fields were those of Jacobson (Rational methods in the theory of Lie algebras) and Landherr (Über einfache Liesche Ringe). On this stage of generalization it was already clear that it is not a too straight-forward problem to work out new methods to establish the analogues of characteristic-zero theorems.

A modular Lie algebra (such are the algebras to which the title of this book refers) is a Lie algebra over a field of positive characteristic. The study of these structures is now more than forty years old, and this book is the first general treatment on this active field.

In Chapter I (Fundamentals) generalities, such as restricted Lie-algebras, Iwasawa theorem, Cartan subalgebras, are formulated and proved. Chapter II (Classical semi-simple Lie Algebras) contains the Cartan decompositions of algebras with non-degenerate trace form, and the classification of the classical algebras. In Chapter III (Automorphisms of the Classical Algebras) the automorphism groups of the classical algebras are determined, and Chapter IV (Forms of the Classical Lie Algebras) is motivated by the problem of determining all Lie algebras with non-singular Killing form over an arbitrary field of characteristic $\neq 2,3$. Chapter V (Comparison of the Modular and Non-modular Cases) deals with a number of analogues of fundamental classical theorems. Chapter VI (Related Topics) is an indication of some ways in which Lie algebras, especially those of prime characteristic, have arisen in other areas of mathematics.

The book is clearly written and could serve as an excellent textbook for a graduate course in Lie algebras.

Z. Szabó (Szeged)

I. M. Singer—S. A. Thorpe, Lecture Notes on Elementary Topology and Geometry (Undergraduate Texts in Mathematics), VIII+232 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1967.

This book presenst an introduction in modern topology and modern global differential geometry. The text consists of 8 chapters. The first two chapters give a short glimpse into point set topology. Chapter 3 treats homotopy, fundamental group, and covering spaces. In Chapter 4 the concept of simplicial complexes is introduced, geometry and barycentric subdivisions of simplicial complexes, the simplicial approximation theorem and fundamental group of simplicial complexes are treated. After some preliminaries in Chapter 5 concerning the theory of differentiable manifolds and differential forms, Chapter 6 deals with simplicial homology and the de Rham theorem. The two final chapters are devoted to studying intrinsic Riemannian geometry of surfaces and imbedded manifolds in R^n.

The book is highly recommanded to anybody interested in modern differential topology.

L. Gehér (Szeged)

Ernst Specker—Volker Strassen, Komplexität von Entscheidungsproblemen: ein Seminar (Lecture Notes in Computer Science, 43), 217 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1976.

The volume contains eleven lectures of a seminar on the complexity of logical and combinatorial decision problems held at the University of Zürich, in 1973—74. Besides some lectures giving algorithms to some nontrivial decision problems of rather general art, an overview is given of the main methods of finding lower bounds on the complexity of decision problems and a systematic exposition of the underlying concepts.

The Introduction gives the definition of the classes of sets decidable (P) or verifiable (NP) on a Turning machine in polynomial time. One is further told about the problems concerning various relations between these classes (e.g. $P=NP$?), and the NP-complete-problems discovered by Cook and Karp.

The lectures are the following:

I. Time-bounded Turning machines and polynomial reduction. (W. Baur)

II. Polynomially bounded nondeterministic Turning machines and the completeness of the satisfiability problem of propositional logic. (A. Häussler)

III. Problems equivalent to the satisfiability problem of propositional logic. (P. Schuster)

IV. Further combinatorial problems equivalent to the satisfiability problem. (J. von zur Gathen and M. Sieveking)

(II—IV expose the Cook-Karp theory of NP-completeness: the part of Computer Science which contains the simplest and hardest open mathematical problems and is best known for its far-going consequences in applications.)

V. A polynomial algorithm for finding systems of independent representatives. (E. Specker) (The existence of this algorithm can be considered as a warning that even if first an exponential search intrudes itself upon us, sometimes a more detailed analysis leads to a polynomial algorithm.)

VI. Polynomial transformations and the Axiom of Choice. (M. Fürer) (The transformations (reductions) dealt with in Lectures III and IV have an immediate analogy with the transformations used in axiomatic set theory to prove the equivalence of various weakened forms of the Axiom of Choice. The lecture works out this analogy and constructs transformations suitable for both equivalences.)

VII. The spectral problem and complexity theory. (C.-A. Christen) (The spectrum of a logical formula is the set of cardinalities of its finite models. The generalized spectrum (called here projective class) in the set of structures with relations $(R_1, \ldots, R_k)$ (as defined by R. Fagin, whose work does not seem to be known by the author) of a formula Φ in the first order language with relation symbols $(R_1, \ldots, R_k, S_1, \ldots, S_n)$ is the set of structures which are restrictions to $(R_1, \ldots, R_k)$ of models of Φ. Spectra and generalized spectra were realized to be in a one-to-one correspondence with the sets recognizable nondeterministically in exponential resp. polynomial time. In this way old unsolved problems of spectral theory correspond to problems of complexity theory.)

VIII. Lower bounds on the complexity of logical decision problems. (J. Heintz) (Fischer and Rabin, to whose work this lecture is devoted, showed that any first order theory which has groups as models and allows for an element of infinite order, has exponential complexity of decision. Examples are the theory of real numbers and the Presburger arithmetic.)

IX. A decision method for the theory of real-closed fields due to Collins and Moenck—Soloway. (H. R. Wüthrich)

X. Simulation of Turning machines, by logical networks. (M. Fürer) (The theorem of Fischer—Pippenger is proved, which yields an efficient representation of Boolean mappings by logical networks, provided they are rapidly computable on many-tape Turning machines.)

XI. Lengths and formulas. (E. Specker and G. Wick) (Which "inner" properties of Boolean functions imply that the minimum of lengths of formulas representing them, is large? The first method, that of Neciporuk, estimates this by the help of the number of subfunctions; the second one, that of Hodes—Specker, shows that any function representable by a short formula contains some particularly simple subfunction.)

The papers can be read almost independently of each other. Although everything used is defined, knowledge of the elementary Turning machine and recursion theory is presupposed. The book will be useful for those interested in Computer Science and for combinatorists or algebraists with a taste for logic.

P. Gács (Budapest)

Universale Algebren und Theorie der Radikale (Studien zur Algebra und ihre Anwendungen, Band 1), Herausgegeben von Hans-J. Hoehnke, 85 Seiten, Akademie-Verlag, Berlin, 1976.

This is essentially the proceedings of the International Winter School for Universal Algebra and Radical Theory, held at Reinhardsbrunn, GDR, from January 26 to February 9, 1974. It contains quite detailed abstracts of 17 lectures, given by the following authors: V. A. Andrunakievič (with Yu. M. Ryabuhin, K. K. Kračilov, and E. I. Tebyrce), L. Budach, N. Jacobson, K. Keimel (with H. Werner), J. Lambek (with B. A. Rattray), P. Němec, L. Bican, T. Kepka, J. Rosický, B. M. Schein, D. Simson, L. A. Skornjakov, Bo Stenström, R. Strecker, A. V. Tiščenko, R. Wiegandt and B. Davey. The book, having a definite categorical flavor, acquaintes the reader with several modern directions and results in radical theory.

B. Csákány (Szeged)

John Wermer, Banach Algebras and Several Complex Variables, Second Edition (Graduate Texts in Mathematics, 35), IX+161 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976.

The relationships between the theory of functions of one or more complex variables and that of commutative Banach algebras have been studied extensively during the past twenty years. Function theoretic methods have been applied to solve Banach algebra problems, for example, the question of the existence of idempotents in a Banach algebra. On the other hand, the concepts of the theory of Banach algebras such as the maximal ideal space, the Silov boundary, Gleason parts have fertilized function theory. About one third of the book is devoted to the most important applications of function theory in several complex variables to Banach algebras. No knowledge of function theory in several complex variables is assumed on the part of the reader. The rest of the book studies uniform approximation on compact subsets of the space of n complex variables.

The exposition is elementary and self-contained. The emphasis is put on the easy understanding of the main ideas and not on generality and completeness.

The connections between the theory of functions of one complex variable and Banach algebras are only "touched on", as this topic is well treated in other monographs.

This second edition contains the following sections: 1. Preliminaries and notations, 2. Classical approximation theorems, 3. Operational calculus in one variable, 4. Differential forms, 5. The $\bar{\partial}$-operator, 6. The equation $\bar{\partial}u=f$, 7. The Oka-Weil theorem, 8. Operational calculus in several variables, 9. The Silov boundary, 10. Maximality and Radó's theorem, 11. Analytic structure, 12. Algebra of analytic functions, 13. Appromixation on curves in C^n, 14. Uniform approximation on disks in C^n, 15. The first cohomology group of a maximal ideal space, 16. The $\bar{\partial}$-operator in smoothly bounded domains, 17. Manifolds without complex tangents, 18. Submanifolds of high dimension, 19. Generators, 20. The fibers over a plane domain, 21. Examples of hulls, 22. Solutions to some exercises. Sections 18—21 are new, Section 11 has been revised. Exercises of varying degrees of difficulty are offered, the starred exercises are solved in Section 22.

This excellent book may be recommended mainly to specialists or to those wanting to become specialists in the subject matter treated in the book.

József Szűcs (Szeged)

O. Zariski—P. Samuel, Commutative algebra, Vols. I, II, (Graduate Texts in Mathematics. 28, 29), Springer-Verlag, New York—Heidelberg—Berlin, 1975.

This is a new and essentially unchanged edition of a great classic in commutative algebra. The original was published with Van Nostrand, Princeton, N. J. in 1958 (vol. I) and 1960 (vol. II).

Since that time several excellent textbooks on commutative algebra have been written. To mention just some of them: N. BOURBAKI, *Algèbre commutative* (Hermann, Paris, 1961—1965); M. F. ATIYAH—I. G. MACDONALD, *Introduction to commutative algebra* (Addison—Wesley, 1969); I. KAPLANSKY, *Commutative rings* (Allyn and Bacon, 1970). Important new developments, like the work of Grothendieck have also taken place in commutative algebra. None the less the book of Zariski and Samuel still constitutes an excellent and thorough introduction to those classical parts of the theory which can be handled without the use of homological methods.

Let us describe the contents of the book. The first volume consists of 5 chapters while the second one contains 3 chapters and 7 appendices.

Chapter 1 deals with fundamental concepts: groups, rings, fields, unique factorization, and euclidean domains, polynomial rings, vector specas. Quotient rings are also introduced. The material in this chapter is much the same as in most textbooks of algebra (except that non-commutative structures are not studied at all).

Chapter 2 deals with the theory of fields: algebraic extensions (separable and inseparable), normal extensions and splitting fields, the elements of Galois theory, finite fields, norms, traces and the discriminant. Next come transcendental extensions, with a discussion of the transcendence degree. Algebraically closed fields are considered and algebraic function fields and derivations are discussed.

Chapter 3 contains classical material on ideals and modules. Prime, primary and maximal ideals are considered and the chain conditions introduced. A discussion of direct sums follows. Tensor products of rings and free joins of integral domains are defined and studied.

Chapter 4 discusses noetherian rings. After the Hilbert basis theorem comes a thorough presentation of the Lasker—Noether decomposition theory. Quotient rings are then studied, especially the relations between the ideals of a ring and its quotient ring. Prime ideals in noetherian and in particular principal ideal rings are discussed. There is an appendix on primary representation in noetherian modules.

Chapter 5 starts with a discussion of integral dependence and integral closure. Dedekind domains are then thoroughly discussed as well as finite algebraic extensions of quotient fields of Dedekind domains. Some sections deal with ramification theory after which some applications to quadratic and cyclotomic fields are given.

Chapter 6 discusses valuation theory. Places are introduced, the valuation ring, residue field and dimension of a place are defined. Next comes a discussion of specializations and the existence of places. The behaviour of places under field extensions is considered. Valuations are then introduced and their connection with places analyzed. The rank of a valuation is considered together with the behaviour of valuations under field extensions. Ramification theory of general valuations is presented as a generalisation of the ramification theory of Chap. 5 (which turns out to have dealt with the case of a discrete, rank 1 valuation). After a discussion of prime divisors in function fields, the abstract Riemann surface of a field is introduced with a discussion of the topological aspects. Finally normal and derived normal models are considered.

Throughout the whole chapter there is a strong evidence of the algebro-geometric motivation of the authors.

Chapter 7 deals with polynomial and power-series rings. More generally graded rings are introduced together with a study of homogeneous ideals. Algebraic varieties in affine and projective spaces are considered, the Nullstellensatz is proved. Then comes the dimension theory in finite integral domains, especially polynomial rings and then the dimension theory in power-series rings. The chapter closes with a study of characteristic functions (with a proof of the Hilbert—Serre theorem) and chains of syzygies.

The subject matter of Chapter 8 is local algebra. After an introduction to topological rings, modules and completions, Zariski rings (the term was introduced by Samuel in 1953) are considered. Hensel's lemma is proved. After a section on dimension theory in semi-local rings comes a discussion of the theory of multiplicities. Regular local rings are then discussed and a structure theorem of I. S. Cohen on certain complete local rings is proved. The final topic is analytical irreducibility and analytical normality of normal varieties.

The appendices deal with some special but interesting questions and form a valuable part of the book.

The present edition contains one alteration worth mentioning compared with the original edition: the new, stronger formulation (and modified proof) of Theorem 29 on pp. 303—305 of volume I.

J. Pelikán (Budapest)