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On intertwining dilations. V
(Letter to the Editor)

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1. In the paper [3] the last two theorems (Lemma 5.1 and Proposition 5.1) are incorrect. Namely, the mapping constructed in the proof of Lemma 5.1 (yielded by the formula (5.6)) is not injective (as asserted at the end of the proof of Lemma 5.1). The error consists in the assumption which is implicitly made in § 5, that for any Ando dilation $\{U_1, U_2\}$ on $\mathfrak{K}$, U_2 is the minimal isometric dilation of

$$\hat{A} = P_K U_2|K \quad \text{where} \quad K = \bigvee_{n=0}^{\infty} U_1^n \mathfrak{H}$$

(the terminology and the notation are that of [3]). Here is a counterexample:

Set

$$\mathfrak{H} \doteq \mathbf{C}\oplus\{0\}, \quad K = H^2\oplus\{0\}, \quad \mathfrak{K} = H^2\oplus S_+H^2$$

$$T_1 = T_2 = 0_{\mathfrak{H}}, \quad U_1 = S_+\oplus S_+, \quad U_2 = \begin{pmatrix} 0 & S_+ \\ S_+ & 0 \end{pmatrix}$$

where S_+ denotes the canonical multiplication unilateral shift on the classical Hardy space H^2. Then $\{U_1, U_2\}$ is an Ando dilation of $\{T_1, T_2\}$ but U_2 is not the minimal isometric dilation of $\hat{A}=P_K U_2|K=O_K$. Moreover, changing the role of U_1 and U_2 does not improve the situation since, if we set

$$K' = \bigvee_{n=0}^{\infty} U_2^n \mathfrak{H}, \quad \hat{A}' = P_{K'} U_1|K',$$

then analogously U_1 is not the minimal isometric dilation of $\hat{A}'=O_{K'}$.

2. Therefore one cannot range the present Ando dilation $\{U_1, U_2\}$ of $\{O_{\mathfrak{H}}, O_{\mathfrak{H}}\}$ in the frame considered in [3], § 5. Consequently, we must withdraw Lemma 5.1 and Proposition 5.1 from our paper [3]. However we take this opportunity to state

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that one can give a similar, but more complicated labeling of all Ando dilations by referring besides the paper [3] also to our next paper [4]. Since this correct form of Lemma 5.1 and Proposition 5.1 of [3] needs a much longer discussion, it will be given in a subsequent paper.

3. We should like to indicate a simple case in which Lemma 5.1 of [3] conserves its validity, namely *if the factorization* $T_1 \cdot T_2$ *is regular* (in the sense of [5], Ch. VII). Indeed the only fact we have to prove is that for any Ando dilation $\{U_1, U_2\}$, U_2 is the minimal isometric dilation of $\hat{A}$. In the present case this is equivalent to the relation

$$U_2(\mathfrak{K} \ominus K) \subset \mathfrak{H} \ominus K. \tag{1}$$

In proving (1) we firstly notice that for any $l \in \mathfrak{L} = ((U_1 - T_1)\mathfrak{H})^-$ there exists (because of the regularity of $T_1 \cdot T_2$) a sequence $\{h_j\}_{j=1}^\infty \subset \mathfrak{H}$ such that

$$D_{T_2} h_j \to 0 \quad \text{and} \quad (U_1 - T_1) T_2 h_j \to l. \tag{2}$$

From the first relation (2) we obtain

$$\|(U_2 - T_2) h_j\|^2 = \|D_{T_2} h_j\|^2 \to 0 \tag{3}$$

so that the second relation (2) becomes $(I-P)U_1 U_2 h_j \to l$. Therefore, setting $\mathfrak{H}_0 = \mathfrak{H}$ and $\mathfrak{H}_n = \mathfrak{H} \vee U_1 \mathfrak{H} \vee \ldots \vee U_1^n \mathfrak{H}$ $(n = 1, 2, \ldots)$ as in [3], § 1 we have

$$(U_2 \mathfrak{H}_1 + \mathfrak{H})^- \ni U_2 U_1 h_j - T_1 T_2 h_j \to l,$$

whence $(U_2 \mathfrak{H}_1 + \mathfrak{H})^- \supset \mathfrak{L}$, and consequently,

$$(U_2 \mathfrak{H}_1 + \mathfrak{H})^- \supset \mathfrak{H}_1. \tag*{(4)$_1$}$$

We can apply $(4)_1$ to the compressions $(T_1)_n$ and $\hat{A}_n$ to $\mathfrak{H}_n$ of U_1 and U_2, respectively (since by [1], $(T_1)_n \cdot \hat{A}_n$ is also regular) and obtain

$$(U_2 \mathfrak{H}_n + \mathfrak{H}_{n-1})^- \supset \mathfrak{H}_n \tag*{(4)$_n$}$$

for all $n \geqq 1$. By iterating $(4)_n$ we finally obtain

$$(U_2 K + \mathfrak{H})^- \supset K. \tag{5}$$

Now

$$U_2(\mathfrak{K} \ominus K) \subset U_2(\mathfrak{K} \ominus \mathfrak{H}) \subset \mathfrak{K} \ominus \mathfrak{H} \perp \mathfrak{H} \tag{6}$$

(because of formula (5.8) of [3]) and

$$U_2(\mathfrak{K} \ominus K) \perp U_2 K. \tag{7}$$

Relation (1) follows now directly from Relations (5), (6), and (7).

The validity of Lemma 5.1 of [3] under the supplementary condition of regularity completes also the proof of Theorem 6.1 (3) and Corollary 6.1 of [2].

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