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Strong approximation and generalized Zygmund class

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1. In a previous paper [6] we generalized some results of imbedding type in connection with the strong approximation of Fourier series. In the definition of the enlarged Lipschitz class given in [6] we restricted ourselves to functions being moduli of continuity. It seems to be more useful to omit this restriction; therefore in the present work we give a modified definition of this class, which can also be extended to the generalization of the Zygmund class.

The first aim of this note is to continue the extension of the imbedding relations to the cases according to the class Lip 1, where there exists a certain gap comparing the new results of [6] to the known ones. In order to achieve our goal we shall define the concept of the *enlarged Zygmund class* to be an analogue of the modified concept of the enlarged Lipschitz class.

2. Before formulating the new results we give some definitions, notations and theorems.

Let $f(x)$ be a continuous and 2π-periodic function and let

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \tag{2.1}$$

be its Fourier series. Denote by $s_n = s_n(x) = s_n(f; x)$ the n-th partial sum of (2.1) and let $f^{(r)}$ denote the r-th derivative of f. For any positive β and p we define the following strong mean

$$h_n(f, \beta, p) := \left\| \left\{ \frac{1}{(n+1)^\beta} \sum_{k=0}^{n} (k+1)^{\beta-1} |s_k - f|^p \right\}^{1/p} \right\|,$$

where $\|\cdot\|$ denotes the usual maximum norm.

Let $\omega(\delta)$ be a modulus of continuity, i.e. a nondecreasing continuous function on the interval $[0, 2\pi]$ having the properties: $\omega(0)=0$, $\omega(\delta_1+\delta_2) \leqq \omega(\delta_1)+\omega(\delta_2)$ for any $0 \leqq \delta_1 \leqq \delta_2 \leqq \delta_1+\delta_2 \leqq 2\pi$.

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Let $E_n(f)$ denote the best approximation of f by trigonometric polynomials of order at most n.

We define the following classes of functions:

$$\begin{aligned} H(\beta, p, r, \omega) &:= \left\{f\colon h_n(f, \beta, p) = O\left(n^{-r}\omega\left(\frac{1}{n}\right)\right)\right\}, \\ W^r H^\omega &:= \{f\colon \omega(f^{(r)}; \delta) = O(\omega(\delta))\}, \\ W^r H^\omega \ln H &:= \{f\colon \omega(f^{(r)}; \delta) = O(\omega(\delta) \ln 1/\delta)\}, \qquad (2.2) \\ W^r H^* &:= \{f\colon f^{(r)} \in Z\}, \\ W^r (H^\omega)^* &:= \{f\colon |f^{(r)}(x+h)+f^{(r)}(x-h)-2f^{(r)}(x)| = O(\omega(h))\}, \end{aligned}$$

where Z denotes the Zygmund class (see [9], p. 43), and $\omega(f; \delta)$ is the modulus of continuity of f. In the case $\omega(\delta)=\delta^\alpha$ we write $W^r H^\alpha$ and $H(\beta, p, r, \alpha)$ instead of $W^r H^{\delta^\alpha}$ and $H(\beta, p, r, \delta^\alpha)$, respectively; and if $r=0$ H^ω stays for $W^0 H^\omega$.

Let Ω_α $(0 \leqq \alpha \leqq 1)$ denote the set of the moduli of continuity $\omega(\delta)=\omega_\alpha(\delta)$ having the following properties:

(i) for any $\alpha' > \alpha$ there exists a natural number $\mu=\mu(\alpha')$ such that

$$2^{\mu\alpha'}\omega_\alpha(2^{-n-\mu}) > 2\omega_\alpha(2^{-n}) \quad \text{holds for all } n(\geqq 1); \tag{2.3}$$

(ii) for every natural number ν there exists a natural number $N(\nu)$ such that

$$2^{\nu\alpha}\omega_\alpha(2^{-n-\nu}) \leqq 2\omega_\alpha(2^{-n}) \quad \text{if} \quad n > N(\nu). \tag{2.4}$$

For any $\omega_\alpha \in \Omega_\alpha$ the class H^{ω_α} will be called an *enlarged Lipschitz class*, and it will be denoted by $\operatorname{Lip}\omega_\alpha$; furthermore for any $\omega_1 \in \Omega_1$ the class $(H^{\omega_1})^*$ will be called an *enlarged Zygmund class* and denoted by $Z(\omega_1)$; i.e.

$$\operatorname{Lip}\omega_\alpha := \{f\colon |f(x+h)-f(x)| \leqq K\omega_\alpha(h) \text{ with } \omega_\alpha \in \Omega_\alpha\}, \tag{2.5}$$

$$Z(\omega_1) := \{f\colon |f(x+h)+f(x-h)-2f(x)| \leqq K\omega_1(h) \text{ with } \omega_1 \in \Omega_1\}, \tag{2.6}$$

where $K=K(f)$ is a constant.

In [2] we proved the following equivalence and imbedding relations: *If p and α positive numbers, r a nonnegative integer then*

$$\begin{array}{ll} (2.7) & \left.\begin{array}{rl} H(\beta, p, r, \alpha) \equiv W^r H^\alpha & \textit{for} \quad \alpha < 1 \\ W^r H^1 \subset H(\beta, p, r, 1) \equiv W^r H^* & (\alpha = 1) \end{array}\right\} \textit{if } \beta > (r+\alpha)p, \\ (2.8) & \end{array}$$

$$\begin{array}{ll} (2.9) & \left.\begin{array}{rl} H(\beta, p, r, \alpha) \subset W^r H^\alpha & \textit{for} \quad \alpha < 1 \\ H(\beta, p, r, 1) \subset W^r H^* & (\alpha = 1) \end{array}\right\} \textit{if } \beta = (r+\alpha)p. \\ (2.10) & \end{array}$$

These statements were generalized in [6] as follows:

Let p, α and r be as before and let $\omega_\alpha \in \Omega_\alpha$. *Then*

$$\left.\begin{array}{lll} (2.7') & H(\beta, p, r, \omega_\alpha) \equiv W^r H^{\omega_\alpha} & \text{for} \quad \alpha < 1 \\ (2.8') & W^r H^{\omega_1} \subset H(\beta, p, r, \omega_1) & (\alpha = 1) \end{array}\right\} \text{ if } \beta > (r+\alpha)p,$$

$$\left.\begin{array}{lll} (2.9') & H(\beta, p, r, \omega_\alpha) \subset W^r H^{\omega_\alpha} & \text{for} \quad \alpha < 1 \\ (2.10') & H(\beta, p, r, \omega_1) \subset W^r H^{\omega_1} \ln H & (\alpha = 1) \end{array}\right\} \text{ if } \beta = (r+\alpha)p.$$

Comparing these results we see the perfect analogies for $\alpha < 1$, but for $\alpha = 1$ there are some differences; e.g. in (2.8′) the analogy of the statement $H(\beta, p, r, 1) \equiv \equiv W^r H^*$ is missing, furthermore (2.10) and (2.10′) have different shape.

Next, using the concept of the enlarged Zygmund class, we fill up these gaps. More precisely we prove the following

Theorem. *Let β and p be positive numbers, r be a nonnegative integer and let* $\omega_1 = \omega_1(\delta) \in \Omega_1$.

Then

$$H(\beta, p, r, \omega_1) \equiv W^r (H^{\omega_1})^* \quad \text{for} \quad \beta > (r+1)p, \tag{2.11}$$

and

$$H(\beta, p, r, \omega_1) \subset W^r (H^{\omega_1})^* \quad \text{for} \quad \beta \leqq (r+1)p. \tag{2.12}$$

3. To prove our theorem we require some known results and lemmas.

Proposition 1. *For any positive β and p*

$$h_n(f, \beta, p) \leqq K\left\{n^{-\beta} \sum_{k=0}^{n} (k+1)^{\beta-1} E_k^p(f)\right\}^{1/p}. \ ^{*)} \tag{3.1}$$

This is a consequence of Theorem 1 in [1].

Proposition 2 (Corollary 2 in [3]). *For any positive β and p*

$$E_n(f) \leqq K h_n(f, \beta, p). \tag{3.2}$$

Proposition 3 ([7, pp. 59 and 61]). *For any nonnegative r*

$$\omega\left(f^{(r)}; \frac{1}{n}\right) \leqq K\left\{n^{-1} \sum_{k=1}^{n} k^r E_k(f) + \sum_{k=n+1}^{\infty} k^{r-1} E_k(f)\right\}. \tag{3.3}$$

Lemma 1 (Lemma 3 of [4]). *For any nonnegative sequence $\{a_n\}$ the inequality*

$$\sum_{n=1}^{m} a_n \leqq K a_m \quad (m = 1, 2, \ldots; K > 0) \tag{3.4}$$

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*) $K, K_1, \ldots$ denote positive constants not necessarily the same at each occurrence.

holds if and only if there exist a positive number c and a natural number μ such that for any n

$$a_{n+1} > ca_n \quad \text{and} \quad a_{n+\mu} > 2a_n.$$

Lemma 2 (Lemma 2 of [6]). *Condition* (3.4) *implies that for any positive p*

$$\sum_{n=1}^{m} a_n^p \leqq K_1 a_m^p \tag{3.5}$$

also holds.

4. Proof of Theorem. First we prove that for any positive β, p and for any nonnegative r the relation

$$H(\beta, p, r, \omega_1) \subset W^r(H^{\omega_1})^* \tag{4.1}$$

holds.

Assuming that $f \in H(\beta, p, r, \omega_1)$ we get that

$$h_n(f, \beta, p) = Kn^{-r}\omega_1(1/n).$$

Hence, by (3.2), the estimate

$$E_n(f) \leqq K_1 n^{-r}\omega_1(1/n) \tag{4.2}$$

also holds.

Setting

$$V_n(x) = \frac{1}{n}\sum_{k=n+1}^{2n} s_k(x)$$

and

$$U_n(x) = V_{2^n}(x) - V_{2^{n-1}}(x) \quad (n = 0, 1, \dots; \ W_{2^{-1}}(x) \equiv 0),$$

then, by (2.4) and (4.2), we obtain that

$$f(x) = \sum_{n=0}^{\infty} U_n(x) \quad \text{and} \quad f^{(r)}(x) = \sum_{n=0}^{\infty} U_n^{(r)}(x); \tag{4.3}$$

in the proof of the last statement we also used the following well-known inequalities

$$|U_n(x)| \leqq KE_{2^{n-1}}(f) \quad \text{and} \quad |U_n^{(r)}(x)| \leqq K2^{nr}\max|U_n(x)|. \tag{4.4}$$

By (4.3) we obtain that

$$f^{(r)}(x+h)+f^{(r)}(x-h)-2f^{(r)}(x) = \sum_{n=0}^{\infty}\{U_n^{(r)}(x+h)+U_n^{(r)}(x-h)-2U_n^{(r)}(x)\} \equiv \sum\nolimits_0.$$

We split the sum $\sum_0$ into two parts by the index μ given by the inequalities $2^{-\mu}<h\leqq 2^{-\mu+1}$, i.e.

$$\sum\nolimits_0 = \sum_{n=0}^{\mu} + \sum_{n=\mu+1}^{\infty} \equiv \sum\nolimits_1 + \sum\nolimits_2.$$

The terms of $\sum_2$ do not exceed $4 \cdot \max |U_n^{(r)}(x)|$, so, by (2.4), (4.2) and (4.4), the following inequalities

$$|\sum_2| \leqq K \sum_{n=\mu+1}^{\infty} \omega_1(2^{-n}) \leqq K_1 \omega_1(2^{-\mu})$$

hold.

By the mean-value theorem and arguing as before

$$|\sum_1| \leqq Kh^2 \sum_{n=0}^{\mu} \max |U_n^{(r+2)}(x)| = K_1 h^2 \sum_{n=0}^{\mu} 2^{2n} \omega_1(2^{-n}).$$

Here the last sum, by (2.3) and Lemma 1, has the same magnitude as its last term, whence

$$|\sum_1| \leqq K_2 h^2 \cdot 2^{2\mu} \omega_1(2^{-\mu}) \leqq K_3 \omega_1(2^{-\mu}).$$

Collecting our partial results we obtain that

$$|f^{(r)}(x+h)+f^{(r)}(x-h)-2f^{(r)}(x)| \leqq K_4 \omega_1(h),$$

which proves that $f \in W^r(H^{\omega_1})^*$; and this verifies (4.1) and (2.12).

In order to prove (2.11), in respect to (4.1), it is enough to show that if $\beta > (r+1)p$ then

$$W^r(H^{\omega_1})^* \subset H(\beta, p, r, \omega_1) \tag{4.5}$$

or equivalently that any $f \in W^r(H^{\omega_1})^*$ also belongs to $H(\beta, p, r, \omega_1)$.

The proof of this statement will be similar to that of the first result proved by Zygmund for the original Zygmund's class.

Let us define the moving average of f as usual:

$$f_\delta(x) = \frac{1}{2\delta} \int_{-\delta}^{\delta} f(x+t)\, dt.$$

It is known (see e.g. [9] pp. 117—119) that if f has k continuous derivatives then f_δ has $k+1$ such derivatives, furthermore

$$f_\delta^{(k+1)}(x) = \frac{f^{(k)}(x+\delta) - f^{(k)}(x-\delta)}{2\delta}. \tag{4.6}$$

Let $f_{\delta\delta}$ denote the moving average of f_δ, and let

$$g(x) = f(x) - f_{\delta\delta}(x).$$

Then, by (4.6) and $f \in W^r(H^{\omega_1})^*$, we have the following statements:

$$|f_{\delta\delta}^{(r+2)}(x)| = (2\delta)^{-1} |f_\delta^{(r+1)}(x+\delta) - f_\delta^{(r+1)}(x-\delta)| = (4\delta^2)^{-1} |f^{(r)}(x+2\delta) + \\ + f^{(r)}(x-2\delta) - 2f^{(r)}(x)| \leqq K\delta^{-2} \omega_1(\delta),$$

whence, using the following well-known inequality

$$E_n(\varphi) = A_k \max_x |\varphi^{(k)}(x)| n^{-k}, \tag{4.7}$$

we obtain that

$$E_n(f_{\delta\delta}) = K_1 n^{-r-2} \delta^{-2} \omega_1(\delta). \tag{4.8}$$

A standard but not quite short calculation gives

(4.9)

$$f_{\delta\delta}(x) = \frac{1}{4\delta^2} \int_{-\delta}^{\delta} \int_{-\delta}^{\delta} f(x+u+v)\, du\, dv = \frac{1}{4\delta^2} \int_{0}^{2\delta} \{f(x+t)+f(x-t)\}(2\delta-t)\, dt.$$

Since the operation δ commutes with differentiation, by (4.9) and $f \in W^r(H^{\omega_1})^*$, we get

$$|g^{(r)}(x)| = |f_{\delta\delta}^{(r)}(x) - f^{(r)}(x)| = (4\delta^2)^{-1} \int_{0}^{2\delta} \{f^{(r)}(x+t)+f^{(r)}(x-t)-2f^{(r)}(x)\} \times$$

$$\times (2\delta - t)\, dt| \leqq K_2 \omega_1(\delta).$$

Hence, using (4.7), we obtain that

$$E_n(g) \leqq K_3 n^{-r} \omega_1(\delta); \tag{4.10}$$

and setting $\delta = n^{-1}$ (4.8) and (4.10) give that

$$E_n(f) \leqq E_n(f_{\delta\delta}) + E_n(g) \leqq K_4 n^{-r} \omega_1\left(\frac{1}{n}\right). \tag{4.11}$$

Next we prove that (4.11) implies that $f \in H(\beta, p, r, \omega_1)$ assuming $\beta > (r+1)p$. (3.1) and (4.11) give that

$$h_n(f, \beta, p) \leqq K\left\{n^{-\beta} \sum_{k=1}^{n} (k+1)^{\beta-1} k^{-rp} \omega_1^p\left(\frac{1}{k}\right)\right\}^{1/p} \leqq$$

$$\leqq K_1\left\{n^{-\beta} \sum_{m=1}^{\log n} 2^{m(\beta-rp)} \omega_1^p(2^{-m})\right\}^{1/p}.$$

Using Lemma 1 and 2, on account of (2.3) and $\beta > (r+1)p$, the sum above has the same magnitude as its last term, consequently

$$h_n(f, \beta, p) \leqq K_2 n^{-r} \omega_1\left(\frac{1}{n}\right)$$

holds, and this means that $f \in H(\beta, p, r, \omega_1)$.

Thus the proof is complete.

5. Finally we make some remarks in connection with the following two classes of functions:

$$E_r^{\omega} := \left\{f\colon E_n(f) = O\left(n^{-r}\omega\left(\frac{1}{n}\right)\right)\right\}, \quad W^r E^{\omega} := \left\{f\colon E_n(f^{(r)}) = O\left(\omega\left(\frac{1}{n}\right)\right)\right\}.$$

These classes have already been investigated in [5]. Now we show that if $\omega=\omega_\alpha\in\Omega_\alpha$, i.e. if $H^{\omega_\alpha}=\mathrm{Lip}\,\omega_\alpha$ is an enlarged Lipschitz class, then for $0<\alpha<1$ these classes coincide with the class $W^rH^{\omega_\alpha}$, and if $\alpha=1$ then they coincide with $W^r(H^{\omega_1})^*$.

By the following known estimates (see [8], p. 308)

$$E_n(f)\leqq K\omega\left(f;\frac{1}{n}\right)\quad\text{and}\quad E_n(f)\leqq Kn^{-r}E_n(f^{(r)})$$

the imbedding relations

$$W^rH^\omega\subset W^rE^\omega\subset E_r^\omega \tag{5.1}$$

obviously hold for any modulus of continuity.

In order to prove our first statement it is enough to show that if $\omega_\alpha\in\Omega_\alpha$ and $0<\alpha<1$ then

$$E_r^{\omega_\alpha}\subset W^rH^{\omega_\alpha}, \tag{5.2}$$

or equivalently, that

$$E_n(f)=O\left(n^{-r}\omega_\alpha\left(\frac{1}{n}\right)\right) \tag{5.3}$$

implies $f\in W^rH^{\omega_\alpha}$.

To prove (5.2) we use Proposition 3 and conditions (2.3) and (2.4). Then we obtain that

$$\omega\left(f^{(r)};\frac{1}{n}\right)\leqq K\left\{\frac{1}{n}\sum_{k=1}^{n}\omega_\alpha\left(\frac{1}{k}\right)+\sum_{k=n+1}^{\infty}\frac{1}{k}\omega_\alpha\left(\frac{1}{n}\right)\right\}\leqq$$
$$\leqq K_1\left\{\frac{1}{n}\sum_{m=1}^{\log n}2^m\omega_\alpha(2^{-m})+\sum_{m=\log n}^{\infty}\omega_\alpha(2^{-m})\right\}\leqq$$
$$\leqq K_2\omega_\alpha\left(\frac{1}{n}\right).$$

The last estimate shows that $f\in W^rH^{\omega_\alpha}$, i.e. (5.2) is proved.

The imbedding relations (5.1) and (5.2) immediately yield the following

Proposition 4. *Let $\omega_\alpha\in\Omega_\alpha$ and $0<\alpha<1$. Then the following function classes $E_r^{\omega_\alpha}$, $W^rE^{\omega_\alpha}$ and $W^rH^{\omega_\alpha}$ coincide, i.e.*

$$E_r^{\omega_\alpha}\equiv W^rE^{\omega_\alpha}\equiv W^rH^{\omega_\alpha}. \tag{5.4}$$

If $\alpha=1$ then $E_r^{\omega_1}$ and $W^rE^{\omega_1}$ coincide with $W^r(H^{\omega_1})^*$. Namely $f\in E_r^{\omega_1}$ implies, by the following known estimate (see [8], p. 303)

$$E_n(f^{(r)})\leqq K\left(n^rE_n(f)+\sum_{\nu=n+1}^{\infty}\nu^{r-1}E_\nu(f)\right)$$

and by (2.4), that

$$E_n(f^{(r)}) \leqq K_1 \omega_1\left(\frac{1}{n}\right),$$

i.e. f also belongs to $W^r E^{\omega_1}$, consequently

$$E_r^{\omega_1} \subset W^r E^{\omega_1}. \tag{5.5}$$

To prove the coincidence of the classes $E_r^{\omega_1}$ and $W^r(H^{\omega_1})^*$ we use our new theorem. Namely if $\beta > (r+1)p$ then $W^r(H^{\omega_1})^*$ coincides with $H(\beta, p, r, \omega_1)$, so it is enough to show that $E_r^{\omega_1} \equiv H(\beta, p, r, \omega_1)$ $(\beta > (r+1)p)$. In virtue of Proposition 2 the imbedding relation

$$H(\beta, p, r, \omega) \subset E_r^{\omega} \tag{5.6}$$

is obvious for any modulus of continuity. Therefore we have to verify

$$E_r^{\omega_1} \subset H(\beta, p, r, \omega_1) \quad \text{for} \quad \beta > (r+1)p. \tag{5.7}$$

The assumption $f \in E_r^{\omega_1}$ implies that

$$E_n(f) \leqq K n^{-r} \omega_1\left(\frac{1}{n}\right),$$

whence, by (3.1), (3.5), (2.3) and $\beta > (r+1)p$, arguing as at the end of the proof of Theorem, we obtain that

$$h_n(f, \beta, p) \leqq K_1\left\{n^{-\beta} \sum_{k=1}^{n} (k+1)^{\beta-1} k^{-rp} \omega_1^p\left(\frac{1}{k}\right)\right\} \leqq$$

$$\leqq K_2\left\{n^{-\beta} \sum_{m=0}^{\log n} 2^{m(\beta-rp)} \omega_1^p(2^{-m})\right\} \leqq K_3 n^{-r} \omega_1\left(\frac{1}{n}\right).$$

This shows that $f \in H(\beta, p, r, \omega_1)$, i.e. (5.7) is verified.

Summing up, by (2.11), (5.1), (5.5), (5.6) and (5.7), we get

Proposition 5. *If $\omega_1 \in \Omega_1$ then*

$$E_r^{\omega_1} \equiv W^r E^{\omega_1} \equiv W^r (H^{\omega_1})^*. \tag{5.8}$$

We mention that Proposition 4 and 5 have a certain intersection with Corollary of [5].

Considering all of the imbedding relations proved or mentioned in this paper we obtain the following

Summarization. *Let p and α be positive numbers and r be a nonnegative integer. Then*

$$\left.\begin{aligned} &(5.9) && H(\beta, p, r, \omega_\alpha) \equiv W^r H^{\omega_\alpha} \equiv W^r E^{\omega_\alpha} \equiv E_r^{\omega_\alpha} && \text{for} \quad \alpha < 1 \\ &(5.10) && W^r H^{\omega_1} \subset H(\beta, p, r, \omega_1) \equiv W^r (H^{\omega_1})^* \equiv W^r E^{\omega_1} \equiv E_r^{\omega_1} && \text{for} \quad \alpha = 1 \end{aligned}\right\}$$

$$\text{and} \quad \beta > (r+\alpha)\, p,$$

$$\left.\begin{aligned} &(5.11) && H(\beta, p, r, \omega_\alpha) \subset W^r H^{\omega_\alpha} \equiv W^r E^{\omega_\alpha} \equiv E_r^{\omega_\alpha} && \text{for} \quad \alpha < 1 \\ &(5.12) && H(\beta, p, r, \omega_1) \subset W^r (H^{\omega_1})^* \equiv W^r E^{\omega_1} \equiv E_r^{\omega_1} && \text{for} \quad \alpha = 1 \end{aligned}\right\} \text{and} \quad \beta = (r+\alpha)p.$$

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