Analytical Methods in Probability Theory. Proceedings, Oberwolfach, Germany, 1980. Edited by D. Dugué, E. Lukács and V. K. Rohatgi (Lecture Notes in Mathematics, 861), X+183 pages, Springer-Verlag, Berlin-Heidelberg-New York, 1981.

This volume contains most of the papers read at the conference "Analytical Methods in Probability Theory", Oberwolfach, June 9—14, 1980. Characterizations of distributions (unimodal, Poisson, Gamma, unimodality of infinitely divisible distributions) are investigated in nine papers. Ten papers deal with asymptotic properties of stochastic processes and their applications in statistics (tests for exponentiality and independence, local limit theorem for sample extremes, local time and invariance, rate of convergence in the central limit theorem, weak convergence of point processes).

Lajos Horváth (Szeged)

S. Burris—H. P. Sankappanavar, A Course in Universal Algebra (Graduate Texts in Mathematics, vol. 78), XVI+276 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1981.

The book gives a high-level course in modern methods and results of universal algebra. It is divided into 5 numbered chapters, contains a large bibliography, author and subject index and a list of some open problems and applications.

Chapter I contains the neccessary definitions and theorems from lattice theory. Chapter II (The elements of universal algebra) describes the most important concepts and notions. Here we can find the most commonly used methods to construct algebraic structures. Chapter III discusses several topics, e. g., how universal algebra can be related to combinatorics and to regular languages. Chapter IV — starting from the notion of a Boolean algebra — presents results of the last years. It deals with primal algebras, quasi-primal algebras, functionally complete algebras and some interesting classes of varieties. Chapter V discusses connections between universal algebra and model theory. Each chapter is divided into sections, and each section ends with references and exercises.

The book is very elegantly and clearly written and can be recommended for all students and mathematicians interested in universal algebra.

László Hannák (Budapest)

W. K. Bühler, Gauss. A Biographical Study, VIII+208 pages, Springer-Verlag, Berlin-Heidelberg-New York, 1981.

Im Jahre 1977 wurde der 200. Geburtstag von Gauss gewürdigt; 1980 sein 125. Todestag. Die Lebenszeit des Princeps mathematicorum liegt also nicht allzu weit zurück; dennoch wird es mit jedem Jahr schwieriger, eine Gauss-Biography zu schreiben. Wie der Verfasser selbst schreibt, enthält sein Werk kaum Dinge, die dem Spezialisten nicht bereits bekannt sein dürften: fast alle Informationen können in bereits veröffentlichten Quellen gefunden werden; darüberhinaus stehen die Gesammelten Abhandlungen von Gauss einigermaßen vollständig zur Verfügung.

Ziel des Buches ist eine Darstellung des Lebens und Wirkens von Carl Friedrich Gauss, der — sogar mit den Maßstäben unserer schnellebigen Zeit — in einer Periode außerordentlicher politischer und sozialer Veränderungen lebte. Der Verfasser vertritt die Auffassung, daß Gauss nicht in die intellektuelle Szene dieser Zeit "paßte" und daß sein Lebensweg ein ungewöhnlich geradliniger gewesen ist.

. Mit seinem Werk wendet sich der Autor an Mathematiker und andere Wissenschaftler unserer. Zeit, wobei aber die Wissenschaftshistoriker und die Psychologen, welche "die Skalps großer Männer sammeln" ausgeschlossen werden. Der Verfasser ist — wie er in seinem Vorwort selbst zugibt — bei seiner Darstellungsweise gleichzeitig bescheiden und unbescheiden. Bescheiden ist er deswegen, weil er nicht den Versuch unternimmt, das "Leben von Gauss" in definitiver Weise niederzuschreiben. Als unbescheiden, wenn nicht gar grandios, sieht er seinen Versuch an, aus dem Leben und Wirken von Gauss diejenigen Gesichtspunkte wenigstens teilweise hervorzuheben, die einerseits von zeitgemäßem Interesse sind und sich andererseits an einen nicht primär historische motivierten Leser werden.

Dieses Vorhaben des Verfassers ist als gelungen einzuschätzen; sein Werk ist eine anspruchsvolle Lektüre, die zu lesen es sich lohnt.

Abschließend ein Überblick über den inhaltlichen Aufbau des Buches: Kindheit und Jugend, 1777---1795; Die zeitgenössische politische und soziale Lage; Studentenzeit in Göttingen, 1795---1798; Zahlentheoretische Arbeiten; Der Einfluss von Gauss' arithmetischen Arbeiten; Rückkehr nach Braunschweig. Dissertation. Die Umlaufbahn der Ceres; Heirat, spätere Jahre in Braunschweig; Die politische Szene in Deutschland, 1789---1848; Familienleben, Umzug nach Göttingen; Tod von Johanna und zweite Ehe. Die ersten Jahre als Professor in Göttingen; Der Stil von Gauss; Astronomische Arbeiten. Elliptische Funktionen; Modulare Formen. Die Hypergeometrische Funktion; Geodäsie und Geometry; Der Ruf nach Berlin und Gauss' soziale Rolle. Das Ende der zweiten Ehe; Physik; Persönliche Interessen nach dem Tode der zweiten Frau; die Göttinger Sieben; Die Methode der kleinsten Quadrate; Numerische Arbeiten; Die Jahre 1838---1855; Gauss' Tod.

In drei Appendizes werden die Organisation der Gesammelten Werke von Gauss, ein Überblick über die Sekundärliteratur und ein Index der Arbeiten von Gauss gegeben. Jedes Kapitel ist von zahlreichen erläuternden Fußnoten begleitet, die zu einem noch besseren Verständnis beitragen.

Manfred Stern (Halle)

J. T. Cannon—S. Dostrovsky, The Evolution of Dynamics: Vibration Theory from 1687 to 1742 (Studies in the History of Mathematics and Physical Sciences, 6), IX+184 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1981.

Very few works have produced such a tremendous effect on the development of theoretical physics and matematics as Newton's Principia published in 1687. The book deals with the history of vibration theory during the half century that followed this date. Reading the book one has the feeling that the originality and brilliancy of ideas of Newton, Taylor, Euler and the Bernoulli family can really be appreciated only by studying their original works and correspondence. The reader obtains considerable help to do this from the authors who make the contents of these works readily accessible and make clear the historical connections among many of the pertinent ideas and concepts such as isochronism, simultaneous crossing of the axis, pendulum condition. These concepts were used for

the study of the problems of floating bodies, hanging chains, resonating beams, the vibrating ring and the vibrating string pertaining to dynamics in many degrees of freedom.

The nicely presented book is concluded by the facsimile of Daniel Bernoulli's papers on the hanging chain and the linked pendulum with translations.

L. Hatvani (Szeged)

J. Carr, Applications of Centre Manifold Theory (Applied Mathematical Sciences, 35), XII+142 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1981.

What is the centre manifold theory in the title of the book? In the theory of differential equations the case when the linear part of the right-hand side has pure imaginary eigenvalues and the real parts of the remaining spectrum points are bounded above by a negative constant is called critical. If the proper subspace corresponding to the pure imaginary part of the spectrum is of finite dimension the centre manifold theory guarantees the existence of a locally invariant manifold for the system which is tangent to this finite dimensional subspace at the origin. This gives a machinery to reduce the dimension of the system under investigation in the critical case.

The book is based on a series of lectures given in the Lefshetz Center for Dynamical Systems in the Division of Applied Mathematics at Brown University during the academic year 1978—79. This is what may cause that, as in a good lecture, this introductory book first gives a full account of the key ideas and methods in a simpler but interesting in itself case, illustrates them by examples, and then proceeds toward the necessary generalizations. In the first two chapters the basic theorems of the theory are formulated, illuminated and proved. Chapters 3—5 are devoted to applications such as the Hopf bifurcation theory and its application to a singular perturbation problem which arises in biology. In Chapter 6 the theory is extended to a class of infinite dimensional problems. Finally, the use of the extension in partial differential equations is illustrated by means of some simple examples.

We can recommend these notes both to users of mathematics and to mathematicians interested in bifurcation and stability theory and their applications.

L. Hatvani (Szeged)

The Chern Symposium 1979. Proceedings of the International Symposium on Differential Geometry in Honor of S.-S. Chern, held in Berkeley, California, June 1979. Edited by W. Y. Hsiang, S. Kobayashi, I. M. Singer, A. Weinstein, I. Wolf, H.-H. Wu, VII+259 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1980.

This book contains 12 articles reflecting the connection of modern differential geometry with other subjects in mathematics and physics.

Some of the papers are intended to exploit the influence of such differential geometric notions as fiber bundle, characteristic classes etc, which originated from the work of S.-S. Chern, to modern physics, especially to general relativity and gauge theory (M. P. Atiyah: Real and Complex Geometry in Four Dimensions; Raoul Bott: Equivariant Morse Theory and the Yang—Mills Equations on Riemann Surfaces; Chen Ning Yang: Fibre Bundles and the Physics of the Magnetic Monopole; Shing-Tung Yau: The Total Mass and the Topology of an Asymptotically Flat Space-Time.) Three papers are devoted to the study of problems in algebraic geometry. (Eugenio Calabi: Isometric Families of Kahler Structures; Mark Green and Philip Griffiths: Two Applications of Algebraic Geometry to Entire Holomorphic Mappings; F. Hirzebruch: The Canonical Map for Certain Hilbert

Modular Surfaces.) Two papers are discussing classical problems in geometry (Nicolaas H. Kuiper: Tight Embeddings and Maps. Submanifolds of Geometrical Class Three in E^N ; Robert Osserman: Minimal surfaces, Gauss Maps, Total Curvature, Eigenvalue Estimates, and Stability). Two papers are devoted to the interaction of differential geometry and functional analysis (J. Moser: Geometry of Quadrics and Spectral Theory; Louis Nirenberg: Remarks on Nonlinear Problems). One paper is related to homology theory (Wu Wen-tsün: de Rham-Sullivan Measure of Spaces and Its Calculability).

The reader can trace in these papers the influence of differential geometric ideas in the development of mathematical and physical sciences. The book is worth studying for everyone working in differential geometry or in related topics.

Péter T. Nagy (Szeged)

M. Csörgő—P. Révész, Strong Approximations in Probability and Statistics, 284 pages, Akadémiai Kiadó, Budapest and Academic Press, New York—San Francisco—London, 1981.

When writing about a book of which the brother of the reviewer is one of the authors, the topic of which is in a broad sense almost identical to and constitutes a starting point for a large part of recent research of the reviewer and, thirdly, in which the reviewer's earlier work is cited to such a large extent (perhaps because of the first connection?), there is an unavoidable danger of lack of objectivity. Exposing himself freely to such a charge, but greatly economizing with praising adjectives, the reviewer feels that this is a significant monograph with effects that will influence much wider circles than that what is broadly termed nowadays as the "Hungarian school" of probability and statistics.

Chapter 2 (Strong approximations of partial sums of independent identically distributed random variables by Wiener processes) and Chapter 4 (Strong approximation of empirical processes by Gaussian processes) deal with the basic issues indicated in their titles. Both chapters describe the history of the corresponding developments, concentrating on the evolution of the involved ideas. The first breakthrough of this evolution, after Strassen's and Kiefer's work with the Skorohod embedding technique, was the two authors' invention of the quantile transformation technique, formerly used also by Bártfai, by which they disproved a conjecture of Strassen concerning the approximation of partial sums and were able to extend Kiefer's approximation of the final breakthrough follow, respectively, the celebrated Komlós—Major—Tusnády approximations. The proofs of the latter results are not given in detail, save for a partial result for the empirical process. The fourth chapter also containes the authors' approximation for the quantile process.

When the final approximation results by Komlós, Major and Tusnády emerged, another basic question of the possible applications, other than convergence rates for the distribution of functionals and the law of the iterated logarithm, became important. These depend on what the corresponding properties, to be inherited by strong invariance, of the approximating (uni and multivariate) Wiener, Brownian bridge and Kiefer processes are within the best rates of approximation. This question lead to the two authors' important series of "How big and small" papers concerning the almost sure size of the increments of these processes. A comprehensive account of these results, at least in the univariate and in certain bivariate cases, is given in Chapter 1 together with new constructions of the corresponding processes. This 67 pages first chapter is in itself a significant contribution to the literature of stochastics.

Chapters 3 and 5 utilize the results of Chapter 1 for partial sum and empirical processes, respectively, via the strong approximations in Chapters 2 and 4. The sixth chapter deals with applica-

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tions of the basic approximation results for further empirical processes such as density and regression estimators and the empirical characteristic function, while Chapter 7 with those for randomly indiced partial sum and empirical processes. Each chapter ends with a section of supplementary remarks concerning various side developments.

It is no risk to predict that this book will be a frequent reference in research papers for a longer time to come. It is also appropriate as a textbook for a one-year graduate course.

Sándor Csörgő (Szeged)

L. R. Foulds, Optimization Techniques. An introduction (Undergraduate Texts in Mathematics), XI + 502 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

This book provides an introduction to the main optimization techniques which are presently in use. Chapter 1 contains a brief introduction to the basic terminology of the theory of optimization. Chapter 2 is concerned with linear programming (LP). After describing the basic problem of LP, the author presents different forms of the simplex algorithm such as the primal simplex method, the "big M" method and the two-phase method. Relationships are introduced between dual and primal problems. We can find a short part about the postoptimal analysis. Some special LP problems (transportation problem, assignment problem) are studied and different solutions of them are described. Chapter 3 introduces the techniques developed to solve large scale LP problems (Revised Simplex Method, Dantzig-Wolfe decomposition, dual-primal algorithm). There is a short overview of the parametric programming. Chapter 4 discusses the integer programming problem and its solutions by different methods as enumerative techniques, and cutting plane methods. New formulations and models are presented. Chapter 5 is concerned with network optimization, emphasizing the shortest path problem, the minimal spanning tree problem and different flow problems. Chapter 6 contains a short review about some known dynamic programming problems and their solutions. Chapter 7 - classical optimization — is an introduction to nonlinear programming, which takes place in Chapter 8. This part of the book consists of several methods to solve unconstrained nonlinear problems, and among the methods concerned with the solution of constrained problems, the author discusses some efficient strategies such as the gradient projection method, the penalty function method and linear approximations.

For unexperienced readers, there is an Appendix with linear algebra and basic calculus. The book contains a large number of excercises and their solutions.

This work evolved out from the experience of teaching the material to finishing undergraduates and beginning graduates. So all chapters contain a "real-life" problem to solve, which is modified depending on the aims of the chapter. All problems are examined not from theoretical but from practical point of view, so algorithms are simply explained with illustrative numerical examples. This book is easy to read, it is a very useful material for education.

G. Galambos (Szeged)

From A to Z. Proceedings of a symposium in honour of A. C. Zaanen. Edited by C. B. Huijsmans, M. A. Kaashoek, W. A. J. Luxemburg, W. K. Vietsch (Mathematical Centre Tracts, 149) VII+130 pages, Mathematisch Centrum, Amsterdam, 1982.

The symposium was held on July 5-6, 1982, at the University of Leiden, on the occasion of Professor A. C. Zaanen's retirement.

There were three invited lectures (H. H. SCHAEFER: Some recent results on positive groups and semi-groups; F. SMITHIES: The background to Cauchy's definition of the integral; B. SZ.-NAGY:

Some lattice properties of the space L^2), and nine lectures by former Ph. D. students of Prof. Zaanen (J. L. GROBLER: Orlicz spaces — a survey of certain aspects; C. B. HUUSMANS: Orthomorphisms; K. DE JONGE: Embeddings of Riesz subspaces with an application to mathematical statistics; M. A. KAASHOEK: Symmetrizable operators and minimal factorisation; W. A. J. LUXEMBURG: Orthomorphisms and the Radon-Nikodym theorem revisited; P. MARITZ: On the Radon-Nikodym theorem; B. DE PAGTER: Duality in the theory of Banach lattices; A. R. SCHEP: Integral operators; W. K. VIETSCH: Compact operators). A Curriculum vitae of A. C. Zaanen, a list of his publications, and a list of all his former Ph. D. students close the handy and beautifully presented small volume.

Professor Zaanen's infatigable and successful mathematical activity, which, we hope, he will be able to continue by the same energy and precision through many years to come (he is actually working on his book *Riesz spaces. II*) has won for him a high appreciation from the mathematical community. All who had the chance to meet him or to contact him at least by correspondence, enjoyed his attractive and suggestive, but utmost modest personality: no wonder that his excellent former pupils, although spread out by now on four continents, are still so much attached to him. His fine personality is mirrored also by his nice photo at the beginning of this volume.

As a survey of various modern areas of mathematical analysis, this book also appeals to, and will be an interesting and useful reading for, mathematicians far beyond the circle of those closely attached to the person of Professor Zaanen.

Béla Sz.-Nagy (Szeged)

Geometry and Differential Geometry. Proceedings, Haifa, Israel 1979. Edited by R. Artzy and I. Vaisman (Lecture Notes in Mathematics 792) VI+444 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

The present book contains the text of the lectures presented at a Conference on Geometry and Differential Geometry, which was held at the University of Haifa, Israel, March 18—23, 1979. The conference was divided into two sections, namely Geometry and Differential Geometry, and in both sections the subject matters covered a broad range, and many of the aspects of modern research in the field were discussed. Altogether 42 papers are published in the book, thus it is impossible to give a complete list of the authors' works. The reader can find several papers from the fields of synthetic and axiomatic geometry, theory of matroids, Riemannian manifods, Lie algebras and Lie groups, conformal geometry, foliations and fibrations etc.

The book is structurally well arranged and the single papers are of high-level affording good reading.

Z. I. Szabó (Szeged)

B. D. Hassard—N. D. Kazarinoff—Y.-H. Wan, Theory and Applications of Hopf Bifurcation (London Mathematical Society Lecture Note Series, 41), VIII+311 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1981.

The adventurous history of James Watt's centrifugal governor provides an interesting example of Hopf bifurcation. This device was invented by Watt in about 1782 for the purpose of controlling steam engines. Centrifugal governors worked well for roughly a century after their introduction. Later on, however, when a great number of machines were constructed having different physical parameters, curious behaviour was observed in a large percentage of them: some governors would "hunt" for the right operating speed before settling down; others would oscillate, never attaining a

constant angular velocity. To eliminate "hunting", viscous dampers (dashpots) were added on to the governors. It was proved that there is a minimum amount of damping which must be present in the system in order to guarantee stability. As the damping is decreased past a critical value, the stationary solution becomes unstable; but a stable periodic solution appears and takes its place as the long-term behaviour of the system.

In general, the "Hopf Bifurcation" describes this phenomenon: the birth of a family of oscillations as a controlling parameter is varied.

The authors first present the mathematical theory of Hopf bifurcation (Ch. 1). The reader can find the bifurcation formulae for computing the form of the oscillations, their amplitudes, their periods, and their stability or lack of it. A "Recipe-Summary" also is given making easier to apply the formulae in a particular case. The amount of algebraic manipulation, necessary for analytical evaluation of bifurcation formulae, increases rapidly with the number of state coordinates. For the avoidance of this difficulty a numerical algorithm is presented in FORTRAN codes for the required calculation (Ch. 3); moveorer, a set of computer programs is provided on microfiche, which enables anyone with moderate FORTRAN ability to run Hopf bifurcation computations.

The applications illustrated by the numerous examples worked out are divided into groups according as the basic model is an ordinary differential equation (Ch. 2), differential-difference and integro-differential equation (Ch. 4), or partial differential equation (Ch. 5). In these chapters the reader can find interesting examples such as Watt's steam-engine governor, the Hodgkin—Huxley model nerve conduction equations, the Brusselator, and Dowell's panel flutter model.

Summing up, this book will be very useful for all scientists in whose fields bifurcation phenomena occur.

L. Hatvani (Szeged)

T. Hida, Brownian Motion (Applications of Mathematics, Vol. 11), XVI+325 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1980.

Mentioning the name of Brownian motion provokes a lively reaction from most mathematicians, and their attitude reflects mostly memories of their former professor of probability. But before one would decide to punish a sometime teacher by neglecting his subject, it is worth of weighing who else will be hit by this decision. We warn: The Brownian motion is not a privilege of probabilists! On the contrary, by its internal wealth it forms an everywhere dense set in mathematics. Even if it is not a primary object of someone's investigation, the connaissance of Brownian motion can offer a different, often equivalent view-point to the matter or at least it can provide the scientist with essential, non-trivial examples.

The author of the present book had K. Yosida and K. Ito as professors, so it wasn't difficult for him to get engaged to Brownian motion. But for him the fundamental structure is less a random process with its sample paths (as for Ito) or a semi-group of operators on a Banach space (as for Yosida) but rather a measure on a space of generalized functions. Taking advantage of the possibilities offered by Gaussian measures, he presents essentially a Hilbert space theory of Brownian motion and of white noise. Fourier transforms, orthogonal expansions and infinite-dimensional transformation groups play key role in the book. At the same time as he investigates one of the infinitely many aspects of Brownian motion the author presents a new, interesting functional analytic theory.

Besides being indispensable for specialists of stochastic processes, the book is recommended to any mathematician feeling enough force to get acquired with a new non-trivial field. The reviewer would like to call special attention of collegues bearing a deep sympathy towards Hilbert space methods. Brownian motion can not only give much more motivation to mathematicians as say quantum mechanics, but being a natural, internal creature of mathematics with its infinitely many interrelations, it can promote cohesion and mutual understanding in the divergent family of mathematicians of today.

D. Vermes (Szeged)

J. Kevorkian—J. D. Cole, Perturbation Methods in Applied Mathematics (Applied Mathematical Sciences, 34), X+558 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1981.

This is a revised and updated version, including a substantial portion of new material, of J. D. Cole's book under the same title published by Ginn-Blaisdell in 1968.

Perturbation methods are very often used by applied mathematicians and physicians when attempting to solve physical problems. In essence, a perturbation procedure consists of constructing the solution for a problem involving a small parameter ε , either in the differential equation or the boundary conditions or both, when the solution is known for the limiting case $\varepsilon = 0$.

Traditional regular perturbation problems, for example, the problem of calculating the perturbed eigenvalues and eigenfunctions of a selfadjoint differential operator, are omitted; they are discussed in most texts on differential equations. Rather, the present book concentrates on the so-called singular perturbation problems. Such problems are, among others, layer type problems and cumulative perturbation problems.

In a layer type problem the small parameter multiplies a term in the differential equation which becomes large in a thin layer. Often this is the highest derivative in the differential equation and the $\varepsilon = 0$ approximation is therefore governed by a lower order equation which cannot satisfy all the initial or boundary conditions prescribed. In a cumulative perturbation problem the small parameter multiplies a term which never becomes large. However, its cumulative effect becomes important for large values of the independent variable.

The book consists of five chapters, and ends with Bibliography, Author and Subject Index. No particular attempt is made to have a complete list of references.

Chapter 1 contains some background on asymptotic expansions. Chapter 2 gives a deeper exposition of limit process expansions through a sequence of examples for ordinary differential equations. Chapter 3 is devoted to cumulative perturbation problems using the so-called multiple variable expansion procedure. Applications to nonlinear oscillations, celestial mechanics are discussed in detail. In Chapter 4 the procedures of the preceding chapters are applied to partial differential equations. Finally, Chapter 5 deals with typical examples from fluid mechanics: linearized and transonic aerodynamics, shallow water theory etc.

The book is written from the point of view of an applied mathematician. Sometimes less attention is paid to mathematical rigour. Instead, physical reasoning is often used as an aid to understanding a problem and to formulating an appropriate approximation procedure.

To sum up, this well-written book contains a unified account on the methods of current researches in Perturbation Theory. The authors present the "state of the art" in a systematic manner. The book under review will certainly serve as a textbook both for advanced undergraduate students and for practicing scientists dealing with truly complicated problems of mathematical physics.

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F. Móricz (Szeged)

Oldrich Kowalski, Generalized Symmetric Spaces (Lecture Notes in Mathematics, 805) XII+187 pages, Springer-Verlag, Berlin—Heidelberg-New York, 1980.

In the last 15 years the theory of "s-manifolds", which is a very succesful generalization of symmetric spaces, has been largely built up. This type of spaces is defined similarly as symmetric spaces by postulating the existence of a geodesic symmetry at each point of the space without requiring the involutiveness of these symmetries. The author of this Lecture Note is one of the developers of the theory of these spaces. He presents in this book a self-contained treatment of the geometric theory of Riemannian and affine "s-manifolds". Must of the results contained in this lecture note were avaible earlier only in journal articles, and some of them are published here the first time. The reader is supposed to be familiar with the basic notions and methods of modern differential geometry and Lie group theory.

The list of the chapter headings gives a glimpse of the content: Generalized symmetric Riemannian spaces, Reductive spaces, Differentiable *s*-manifolds, Locally regular *s*-manifolds, Operations with *s*-manifolds, Distinguished *s*-structures on generalized symmetric spaces, The classification of generalized symmetric Riemannian spaces of low dimensions, The classification of generalized affine symmetric spaces in low dimensions.

The Lecture Note is very clearly and well written. It is recommended to everyone interested in the geometric theory of spaces with a transformation group.

Péter T. Nagy (Szeged)

Measure Theory. Proceedings, Oberwolfach 1979. Edited by D. Közlow (Lecture Notes in Mathematics, 794), XV+573 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

Judging from its proceedings, this must have been a significant conference on measure theory. There are 10 papers on what is classified as general measure theory, 8 on measurable selections and 4 on liftings. Two papers deal with differentiation of measures and integrals and 5 are on vector and group valued measures and their probabilistic applications. Stochastic analysis and various abstract probabilistic topics are the theme for 8 more articles, L^p -spaces and related topics are dealt with in 2 papers. Integral representations and transforms figure in 3 papers and there are 4 papers classified miscellaneous. This comes to altogether 46 articles, and the collection is closed by a record of the problem session of the conference with 8 problems from 7 authors.

Sándor Csörgő (Szeged)

Richard M. Meyer, Essential Mathematics for Applied Fields (Universitext), XVI+555 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1979.

The purpose of this work is to provide a wide spectrum of "essential" mathematics for workers in the variety of applied fields. Much of the material covered here either too widely scattered or too advanced as presented in the literature to those who need it. The treatment in this book requires only the calculus through differential equation. There is no need for measure theoretic background.

This bulky volume consists of twenty sections, arranged into six units. The first unit comprises Basic Real Analysis beginning with the notions of sets, (single and multiple) sequences, series and functions, and ending with some Abelian and Tauberian theorems. The second unit is devoted to (the 1- and *n*-dimensional) Riemann—Stieltjes Integration. The third unit treats Finite Calculus, i.e., the theory of finite differences and difference equations. The fourth unit contains Basic Complex Analysis, including Laurent's theorem and expansion, and residue theorem. The fifth unit gives an account of Applied Linear Algebra, among others, of the generalized inverse and characteristic roots. Finally, the sixth unit collects Miscellaneous things such as convex sets and functions, max-min problems and some basic inequalities.

Each unit develops its topic rigorously based upon material previously established. Throughout the text are found solved Examples and Exercises requiring solution, both being essential parts of the development. Complete hints or answers are provided for the exercises. There are References to additional and related material, too.

This self-contained textbook is warmly recommended to everyone who is going to get acquainted with the background of applied mathematics.

F. Móricz (Szeged)

Physics in One Dimension. Proceedings, Fribourg 1980. Edited by J. Bernasconi, and T. Schneider (Springer Series in Solid-State Sciences, Vol. 23) IX + 368 pages. Springer-Verlag, New York—Heidelberg—Berlin, 1981.

Mathematics in one dimension is a commonly accepted thing, but physics is essentially threedimensional. Nevertheless in certain cases some problems of 3D mathematical physics can be reduced to one dimension and then solved in a simpler way. Ideas of this type of transformations are described by the introductory lecture of this volume, which contains the Proceedings of an International Conference on Physics in One Dimension, held in Fribourg in 1980. The rapid development of experimental physics of long polymer chains in the last decade has shown however that one dimensional physics itself is more than speculation.

This book is a clever mixing of recent theoretical and experimental works in this field. Part II is devoted to the theory of soliton type excitations, Part III deals with magnetic properties. Several points of views on solitons in the simplest polymer, the polyacetilene are outlined in Part IV. The last three parts contain papers on metallic conductivity, disorder, localization, excitons and other interesting questions relevant to one dimensional systems. The wide range and the great number of the authors yield a good overview on the whole subject which will attract an even growing interest in the near future.

M. G. Benedict (Szeged)

J. H. Pollard, A Handbook of Numerical and Statistical Techniques with Examples Mainly from the Life Sciences, XII + 349 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1979.

This is a well and carefully compiled easy-to-use handbook of basic numerical and statistical techniques designed to aid the practising statistician in solving day-to-day problems on a small programmable desk calculator or mini-computer.

Part I (Basic numerical techniques) consists of seven chapters. After listing the modest amount of prerequisites from calculus and linear algebra and the usual techniques of reducing truncation and round-off errors, it describes methods for smoothing data, for numerical integration and differentation, interpolation and some related topics.

Part II (Basic statistical techniques) consists of equally seven chapters but covers, of course, nearly the 60% of the text. After supplying the elements of probabilistic and statistical theory very briefly, the essential characteristics of the most commonly used continuous and discrete distributions are given. Following then a short chapter on fitting a Pearson curve, two rich chapters come on

hypothesis testing and estimation. The final one is on random numbers, data transformation and on the simplest techniques with randomly of deterministically censored samples. Part III is fully devoted to the method of least squares in (simple and multiple) linear, curvilinear and non-linear regression.

There are 15 tables, 105 references and good author and subject indices. Each of the amazingly large number of techniques covered is demonstrated by at least one numerical example according to the title and there is a wealth of good computational and programming advices. All in all, this book is way above the usual level of the many books on "basic statistics" published nowadays. Experimental scientists, particularly those in life sciences, will find it very useful.

Sándor Csörgő (Szeged)

Probability in Banach Spaces III. Proceedings, Medford, USA, 1980. Edited by. A. Beck (Lecture Notes in Mathematics, 860), VI + 329 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

Four papers of this volume give surveys on the developments of probability theory in infinitedimensional vector spaces in the last two-three years. Generalized domains of attraction, empirical processes, central limit theorems and inequalites in Banach spaces are studied. Twenty research papers present various directions of this subject, well-known results of finite- dimensional spaces are extended to the case of Banach spaces (martingales, strong, and weak laws of large numbers, laws of iterated logarithm, selfdecomposability and stability of measures, stability of linear and quadratic forms). Properties of Gaussian measures in function spaces are also investigated.

Lajos Horváth (Szeged)

Robert D. Richtmyer, Principles of Advanced Mathematical Physics, Vol. II. (Texts and Monographs in Physics) XI + 322 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1981.

There are many good books on the mathematical methods of physics, and it seems quite difficult to select just the principles out of this enormous subject, but this book of Richtmyer has successfully solved this problem, enabling the student or the non-expert research worker to grasp some modern views on contemporary mathematical physics.

This second volume begins with chapter 18, which together with the subsequent four, cover the traditional material of applications of group theory in physics. On the other hand, the next three chapters approach continuous groups from a less conventional viewpoint. It is the concept of manifold that leads us to Lie groups and then through chapters 26-28 to the apparatus of general relativity.

A topic, which has never been considered so far in books of this kind, is outlined in the last three chapters. Starting from the problem of hydrodynamical turbulence, the first steps of the theory of bifurcations, attractors and chaos are presented.

The main virtue of this book is that it gives much information about newly developed mathematical concepts on a language, adopted usually in books on physics. There are a lot of examples and several problems, challenging the conscientious reader.

M. G. Benedict (Szeged)

Charles E. Rickart, Natural Function Algebras (Universitext) XIII+240 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1979.

The term "function algebra" in the title refers to a uniformly closed algebra of complex-valued continuous functions defined on a compact Hausdorff space. Such Banach algebras have been intensively studied recently. Since the most important examples of these algebras are built up from analytic functions, the majority of the papers in question was earlier dominated by problems of analyti-

city. The present author is concerned, however, with another facet of the subject based on the observation that very general algebras of continuous functions may exhibit certain properties that are reminiscent of analyticity. The most striking one of them is a local maximum principle proved by Hugo Rossi in 1960. This deep result plays a key role throughout the discussion. Although the main body of the material presented here was published in a series of papers during the last 10 or 15 years, the book contains numerous improvements on old results as well as unpublished results.

To be more definite, the classical holomorphy theory, based on the *n*-dimensional complex space C^n , is ultimately determined by the algebra \mathscr{P} of all polynomials in C^n . In the abstract situation the space C^n is replaced by a Hausdorff space Σ and the algebra \mathscr{P} by a given algebra \mathscr{A} of continuous complex-valued functions on Σ . In order to obtain interesting results, one must impose some rather general conditions on the pair $[\Sigma, \mathscr{A}]$. In the first place, \mathscr{A} is assumed to determine the topology of Σ in the sense that the given topology is equivalent to the weakest one under which the elements of \mathscr{A} are continuous. Secondly, it is assumed that every homomorphism of \mathscr{A} onto the complex field C, which is continuous relative to the compact-open topology in \mathscr{A} , is a point evaluation in the space Σ . Then \mathscr{A} determines an " \mathscr{A} -holomorphy" theory based on Σ roughly analogous to the way \mathscr{P} determines the classical theory.

This fairly general setting makes it possible to establish a variety of results, many of which are full or partial generalizations of results in Several Complex Variables. The \mathscr{A} -holomorphy theory might also be considered as an approach to the Infinite Dimensional Holomorphy theory. The latter subject, which already has an extensive literature, involves the study of functions on infinite dimensional linear topological spaces.

The material is divided into fourteen chapters: 1. The category of pairs, 2. Convexity and naturality, 3. The Šilov boundary and local maximum principle, 4. Holomorphic functions, 5. Maximum properties of holomorphic functions, 6. Subharmonic functions, 7. Varieties, 8. Holomorphic and subharmonic convexity, 9. $[\Sigma, \mathcal{A}]$ -domains, 10. Holomorphic extensions of $[\Sigma, \mathcal{A}]$ -domains, 11. Holomorphy theory for dual pairs of vector spaces, 12. $\langle E, F \rangle$ -domains of holomorphy, 13. Dual pair theory applied to $[\Sigma, \mathcal{A}]$ -domains, 14. Holomorphic extensions of \mathcal{A} domains. The text is supplemented with Bibliography containing 75 items, Index of Symbols, and General Index.

This self-contained textbook is addressed to graduate students and warmly recommended to everyone who wants to keep pace with up-to-date developments in Holomorphy Theory and Banach Algebras.

F. Móricz (Szeged)

Séminaire de Probabilités XIV, 1978/79. Edité par J. Azéma et M. Yor (Lecture Notes in Mathematics, 784), VII+546 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

This is the first volume of this series of seminar notes after that the old Strasbourg seminar has been "decentralised to Paris". Nevertheless the authors and the names in the titles of the papers are basically the same as before, completed by some distinguished overseas visitors (10 of the 49 papers are in English). Thus, with a very few exceptions, the topic continues to be what is broadly described the general theory of stochastic processes. Although, naturally, some of the shorter communications might prove important, it is interesting to note that if we take out the 14 longer or middle size papers then the average lenght of the remaining 35 notes is 6 typed pages. Many of them (commenting, correcting, or exposing already existing developments by other or the same authors) could have never been published elsewhere. This is of course the very feature of these seminar notes which reflect a vivid but somewhat closed research activity.

Sándor Csörgő (Szeged)

Solitons. Edited by R. K. Bullough and P. J. Caudrey (Topics in Current Physics, Vol. 17) XVIII + 389 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1980.

Frontiers of the two closely related sciences, mathematics and physics have moved away quite a distance from each other, after the mathematical foundations of quantum mechanics have been established.

Now again some important questions of physics have become of fundamental interest in mathematics and vice versa. Namely the problem of solving nonlinear partial differential equations has arisen in many fields of physics and this "white land" then attracted the mathematicians too.

The soliton is a very special solution of a nonlinear wave equation, it propagates e.g. like a single, bell shaped wavelet, preserving its shape during the propagation and even after "interaction" with similar "objects". It behaves much like a physical particle, and that is why it has attracted most attention among other possible solutions. Some of them were known for a long time, but a systematic study has become possible only after the discovery of the so-called inverse scattering method in the late sixties and early seventies. This method reduces the nonlinear problem to the solution of a system of linear integral equations by a process resembling the Fourier transformation. Besides the editors' review on the history and the present status of this field, the volume contains 11 invited studies on the subject, all of them written by the establishers of the theory of solitons. Almost the half of the contributions deals with the inverse scattering method, namely the works of Newell, Zakharov, Wadati, Faddeev, Calgero and Degasperis. A direct method, transforming the nonlinear equation into a bilinear one is outlined by Hirota. Physical aspects are treated in the contributions of Lamb and Maclaughlin and also of Bullough, Caudrey and Gibbs. The famous Toda lattice is dealt by Toda. Novikov investigates equations with periodic boundary conditions. Possible quantization procedures are treated by Luther.

Each work is clearly written, emphasizing the underlying principles and giving many examples.

This book is highly recommended to all who work in the field of nonlinear analysis and partial differential equations, and without doubt, it must be found in every physics library.

M. G. Benedict (Szeged)

Superspace and Supergravity. Proceedings of the Nuffield Workshop, Cambridge, June 16— July 12, 1980. Edited by S. W. Hawking and M. Rocek, XII+527 pages, Cambridge University Press, London—New York—New Rochelle—Melbourne—Sydney, 1981.

10—15 years ago the considerable part of physicists was convinced on the "in principle" impossibility of the unified field theory. But in the last decade the research has been boomed in this direction. In contrast with the classical unified theories of Weyl, Einstein, Cartan, Calusa, Klein and others the new theories are not restricted only to the gravitation and electromagnetism and are quantized. The possibility of building of unified theories of this type turned to be promising by the large development of mathematical tools as differential geometry and topology, functional analysis and representation theory.

The new unifying theory bears the name of supergravity theory: the classical space-time of relativity theory has been extended by commuting and anticommuting variables to a higher dimensional manifold, the so called superspace. The symmetry groups have a graded structure corresponding to the bundle structure of the superspace, these are the "supersymmetries", their infinitesimal versions are the "superalgebras".

The present book contains the proceedings of the Workshop on Supergravity held in the Department of Applied Mathematics and Theoretical Physics at the University of Cambridge from

16 June to 12 July, 1980, and supported by the Nuffield Foundation. The aim of this meeting was to give a survey on the present state of the very recent and rapidly developing supergravity theory.

This book is a collection of lectures devided into six parts: Introduction to supergravity, Quantization, Extended supergravity, N=8 supergravity, Kähler spaces and supersymmetry, Other aspects of supergravity.

The supergravity theory is far from being worked out. One of the outstanding problems at the present time is "The construction of a complete formulation of extended supergravity with auxiliary fields, preferably in superspace" formulated by the aditors in the introduction.

The book gives for the reader a very recent survey on the present state of the very interesting subject of supergravity theory.

Péter T. Nagy (Szeged)

Michio Suzuki, Group Theory I (Grundlehren der mathematischen Wissenschaften, 247), XIV + 434 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

The theory of finite groups developed extremely rapidly during the past twenty five years or so, reaching its zenith in 1981, when the full classification of finite simple groups was completed. The book under review is the first part of a two-volume work first published in Japanese by Iwanami Shoten in 1977—78, and now translated into English. For the translation several mistakes of the original version were corrected, and, more importantly, to reflect the progress achieved in the meantime, a few paragraphs were added to the survey of finite simple groups in Chapter 3, and a few items were added to the bibliography as well.

According to the Preface, one of the main aims of the author was to present an introduction to the theory of finite simple groups. Of course, taking into account the tremendous diversity of the branches of group theory involved, it is understandable that even such important topics as permutation groups, or representation theory could not be discussed in detail.

Volume I contains three of the six chapters constituting the book. In Chapter 1 the basic ideas of group theory are discussed, including permutation groups, operator groups, semidirect products, and general linear groups. Chapter 2 is devoted to the most fundamental theorems and methods of group theory, for example, some results on *p*-groups, Sylow's Theorems, Schreier's Refinement Theorem, the Krull—Remak—Schmidt Theorem, the fundamental theorem on finitely generated abelian groups, Schur's Lemma, cohomology theory, the Schur-Zassenhaus Theorem, and wreath products. Chapter 3 starts the discussion of more specific branches of group theory with considering several particular classes of groups, namely torsion-free abelian groups, symmetric and alternating grcups, linear groups, and Coxeter groups. The volume ends with a survey of finite simple groups, and the proof of Dickson's theorem on the subgroups of 2-dimensional special linear groups over finite fields.

As a rule, each section is followed by a collection of interesting exercises, most of them supplemented with a "Hint" to a solution. The exercises mainly serve to introduce the reader to important concepts and theorems not discussed in the text. However, the book can also be read without solving a single exercise, as no reference is made in the text to results of earlier exercises.

The book is written in a very clear style. The only prerequisite for its reading is some basic knowledge in linear algebra (matrices, determinants) and elementary number theory. This excellent book is warmly recommended to students intending to specialize in group theory, as well as to other non-specialists who are interested in the recent advances of group theory.

Ágnes Szendrei (Szeged)

W. Törnig, Numerische Mathematik für Ingenieure und Physiker, Band 2: Eigenwertprobleme und numerische Methoden der Analysis, XIII+350 pages, Springer-Verlag, Berlin—Heidelberg— New York, 1979.

The development of large-scale computers have formed a basis for algorithmic constructions and extensive mathematical experiments in many areas of science and technology, thereby attracting a new generation of scientists to problems of numerical mathematics. This is the second volume of a textbook. The first volume published also in 1979 contains the following three parts. Part I: Auxiliary results, Computation of zeros of a function; Part II: Solution of linear systems of equations; Part III: Solution of nonlinear systems of equations.

This volume consists of four parts. Part IV: The eigenvalue problem for matrices. It contains, among others, the iteration procedure of Mises, the inverse iteration, the algorithms of Jacobi and Givens, and the LR algorithm. Part V: Interpolation, approximation and numerical integration. It treats the usual interpolating polynomials, approximation by orthogonal polynomials as well as the cubic spline interpolation, the well-known quadrature and some cubature formulae, the Romberg integration procedure etc. Part VI: Numerical solution of ordinary differential equations. It deals with methods for initial value problems (mainly one-step methods, in particular Runge-Kutta ones), for boundary and eigenvalue problems (difference methods, variational and Ritz methods). The notions of consistency and convergence are discussed in detail, but multi-step methods (in particular, predictor-corrector ones) are not presented. Part VII: Numerical solution of partial differential equations. One section contains the method of finite differences for the numerical solution of initial value and initial-boundary value problems of hyperbolic and parabolic differential equations. Another section is devoted to hyperbolic systems of first order and the last section to the boundary value problem of elliptic differential equations of second order, including the method of differences, the Ritz method and the method of finite elements.

There are examples throughout the text. Each section ends with exercises and some of them with a FORTRAN programme. The reader can find references to additional and related material, too.

This accurately written textbook is warmly recommended to every undergraduate student in applied mathematics who is going to acquire a firm basis of Numerical Analysis.

F. Móricz (Szeged)

G. Whyburn-E. Duda, Dynamic topology (Undergraduate Texts in Mathematics), Springer-Verlag, New York-Heidelberg-Berlin, 1979.

Everybody who learned mathematics is convinced that the knowledge acquired by means of active learning is the deepest and most fruitful. This can be carried out by developing one's own proofs for the theorems. Both learning and teaching a subject in this way are conditioned upon reducing to individual steps of the proofs. In this book the reader finds an excellent realization of this method in topology. As we can learn from J. L. Kelly's foreword, G. Whyburn was a master of this manner of theaching. The book is based on a set of his notes, which have been completed and arranged by E. Duda. Each of the short sections, which can serve as the subject-matter of one lesson, consists of a preparing part including the necessary definitions, exercises and, finally, their solutions. The problems in the first sections are rather simple but later they become more complicated. The first exercises of the book is: "Prove that the union of the elements of any countable collection of countable sets results in a countable set", and the last one is proving the Jordan Curve Theorem. The materials of the sections are collected from the field of dynamic topology, developed origonally

by G. Whyburn. It concerns those topological objects which center around the function concept. The book is concluded by Whyburn's article entitled "Dynamic Topology" which appeared in 1970 in the American Mathematical Monthly.

J. L. Kelley writes in the Foreword: "Constructing a proof for a good known theorem is next best to finding and proving a good theorem. It gives one hope of eventually creating mathematics". Therefore, we recommend this book to every student and teacher interested in analysis, especially in topology.

L. Hatvani (Szeged)

Livres reçus par la rédaction

- S. S. Abhyankar, Weighted expansions for canonical desingularization (Lecture Notes in Mathematics, Vol. 910), VII + 236 pages, Springer-Verlag, Berlin—Heidelberg— NewYork, 1982. — DM 25,—.
- Analytic Theory of Continued Fractions. Proceedings, Loen, Norway 1981. Edited by W. B. Jones, W. J. Thron, and H. Waadeland (Lecture Notes in Mathematics, Vol. 932), VI+240 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982. — DM 28,—.
- D. M. Arnold, Finite rank torsion free Abelian groups and rings (Lecture Notes in Mathematics, Vol. 931), VII+191 pages, Springer-Verlag, Berlin-Heidelberg-New York, 1982. DM 24,--.
- Automorphic Forms, Representation Theory and Arithmetic. Papers presented at the Bombay Colloquium 1979 (Tata Institute Studies in Mathematics), VII+355 pages, Tata Institute of Fundamental Research, Bombay—Springer-Verlag, Berlin—Heidelberg—New York, 1981. DM 39,—.
- J. Back—D. J. Newman, Complex analysis (Undergraduate Texts in Mathematics), X+244 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982. — DM 54,—.
- P. Berthelot—L. Breen—W. Messing, Théorie de Dieudonné cristalline II (Lecture Notes in Mathematics, Vol. 930), XI+261 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.
 DM 33,50.
- J. M. Bismut—L. Gross—K. Krickeberg, Ecole d'été de Probabilités de Saint-Flour X—1980. Edité par P. L. Hennequin (Lecture Notes in Mathematics, Vol. 929), X+313 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982. — DM 39,—.
- P. M. C. de Boer, Price effects in input-output relations: A theoretical and empirical study for the Netherlands 1949—1967 (Lecture Notes in Economics and Mathematical Systems, Vol. 201), X+140 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982. — DM 24,—.
- R. Bott—L. W. Tu, Differential forms in algebraic topology (Graduate Texts in Mathematics, Vol. 82), XIV+331 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982. — DM 74,—.
- Brauer Groups in Ring Theory and Algebraic Geometry. Proceedings, University of Antwerp, Belgium, August 17-28, 1981. Edited by F. van Oystaeyen and A. Verschoren (Lecture Notes in Mathematics, Vol. 918), VIII+300 pages, Springer-Verlag, Berlin-Heidelberg-New York, 1982. — DM 34,50.
- Categorical Aspects of Topology and Analysis. Proceedings of an International Conference held at Carleton University, Ottawa, August 11–15, 1981. Edited by B. Banaschewski (Lecture Notes
- in Mathematics, Vol. 915), XI+385 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982. — DM 39,—.
- K. L. Chung, Lectures from Markov processes to Brownian motion (Grundlehren der mathematischen Wissenschaften, Band 249), VIII+239 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982. — DM 79,—.