Bibliographie

Joseph Bak and Donald J. Newman, Complex Analysis. Undergraduate Texts in Mathematics, X+244 pages with 69 illustrations, Springer-Verlag, New York—Heidelberg—Berlin, 1982

This is a fascinating little book. It provides an efficient and clear introduction into the theory of complex functions. The arrangement of the material is unusual, the reader need not be familiar even with the definition of complex numbers or of convergence and yet, starting at the very beginning, the authors were able to achieve a level in complex function theory where the proof of the prime number theorem is possible. Of course the book contains the standard topics such as the Cauchy—Riemann equations, line integrals, entire and meromorphic functions, singularities, Laurent series, residues, conformal mappings etc., but besides them some other less elementary results are incorporated, as well, e.g. the Phragmén—Lindelöf-method, natural boundaries, open mapping theorem etc. The last, 19th chapter illustrates the wide range of applicability of complex methods. The first question here is if the set of the positive integers can be partitioned into a finite number of artihmetic progressions such that these have no common differences (try it!). The second problem asserts the unicity of the solution to the system of equations

$$a_n+\binom{n}{1}a_{n-1}b_1+\binom{n}{2}a_{n-2}b_2+\ldots+b_n=2^n \quad (n=1,2,\ldots)$$

$a_n, b_n \geqq 0$. In section 3 it is shown that the total variation of $\sin^2 x/x^2$ over $(-\infty, \infty)$ is e^2-5; in section 4 the Fourier unequeness theorem and, finally, in section 5 the prime number theorem is treated.

The book contains a lot of exercises together with hints for the hardest ones. Sometimes, e.g. at the Riemann-mapping-theorem, physical analogues illustrate the main ideas. Index and 69 illustrations help reading the book. We recommend Bak and Newman's "Complex Analysis" to lecturers and to every student with or without any skill in complex methods.

V. Totik (Szeged)

H. J. Baues, Commutator Calculus and Groups of Homotopy Classes (London Mathematical Society Lecture Note Series 50), 226 pages, Cambridge University Press, Cambridge—London, New York—New Rochelle—Melbourne—Sidney, 1981.

This book is divided into two parts consisting of four and three chapters, respectively. Part A is devoted to homotopy operations, nilpotent group theory and nilpotent Lie algebra theory. Starting with commutator calculus, the text contains a study of distributivity laws in homotopy theory, homotopy operations on spheres and concludes in an investigation into higher order Hopf invariants on spheres. Part B deals with homotopy theory over a subring of rationals. In this part the theory of the homotopy Lie algebra and spherical cohomotopy algebra, theory of groups of homotopy classes and finally the Hilton—Milnor theorem and its dual can be found.

László Gehér (Szeged)

Aldo Bressan, Relativistic Theories of Materials, (Springer Tracts in Natural Philosophy, Vol. 29), XIV+290 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1978.

After Einstein's fundamental work in 1905 on special relativity the reletivistic developments of thermodynamics and elasticity were created as early as in 1911. When trying to include gravitation in relativity Einstein was forced to develop radical changes in his earlier spacetime concept in 1916. These new ideas of general relativity had given a new aspect for the study of the material. In spite of this fact the first general relativistic theory of thermodynamics and fluids and of finitely deformed materials were published only after 1955. Even more recently relativistic theories incorporating finite deformations for polarizable and magnetizable materials and those in which couple stresses are considered have been formulated.

The present book describes the foundation of this theory of general relativistic material. Furthermore it containes some applications of this theory, mainly to elastic waves.

After an introductory chapter the book is divided into two parts. The first part deals with the basis equation of gravitation, thermodinamics and electromagnetism, and constitutive equations from the Eulerian point of view. The second part contains the theory of material from the Lagrangian point of view. In this part the reader can find chapters on subjects such as kinematics and stresses, elasticity, accelaration waves, pieso-elasticity and magnetoelastic waves, couple stresses and more general stresses.

The book is not of an introductory character. It is assumed that the reader is familiar with the classical continuum mechanics and with the general relativity. The main definitions and theorems of these subjects are collected — without proofs — in Appendix A.

Z. I. Szabó (Szeged)

A. J. Chorin and J. E. Marsden, A Mathematical Introduction to Fluid Mechanics (Universitext), V+205 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1979.

A good introduction to fluid mechanics is given in this book. The reader can get acquainted with the principal ideas, the equations of fluid motion under several hypotheses (e.g. the fluid is ideal, homogeneous, isentropic, stationary, viscous, compressible), the discussion of potential flows, vortex motions, boundary layers, one-dimensional gas flows. The starting principles of equations, demonstrations are the physical laws. The material in the book can be read easily, the proofs are written with mathematical exaction. The results derived from the models mathematically are always interpreted. There are many illustrations in the book making the material clear.

The book gives a good base to continue the study of fluid mechanics. We recommend it to mathematicians, engineers and students who want to know the basic ideas of this subject in a mathematically attractive manner.

J. Terjéki (Szeged)

Shui-Nee Chow—Jack K. Hale, Methods of Bifurcation Theory (Grundlehren der matematischen Wissenschaften, 251), XV+515 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

As experience shows, many physical phenomena, biological processes etc. can be modelled by differential equations containing parameters. As we change these parameters, the behaviour of the flow changes. This change is essential when the structure of the phase portrait is modified. This happens when the topology of the non-wandering set changes. Each time this occurs, we say there is a bifurcation.

The readers following the current papers and monographs in the field of the theory and applications of differential equations with attention can experience that nowadays the bifurcation is one of the fastest developing central topics in the field. There are a great number of difficult problems and, accordingly, a great number of publications in this topic with respect to several types of differential equations such as finite and infinite dimensional systems, ordinary and partial differential equations, functional differential equations etc. Now we get an excellent comprehensive handbook which helps us to inquire in several branches of this theory of wide range.

In the first chapter the authors give a flavour of the problems that occur in bifurcation theory by presenting some examples from the applications. Chapter 2 meets a long felt need in the literature: it gives a systematic, self-contained introduction to Nonlinear Analysis. One can find here in detail much of the relevant background material to the modern theory of bifurcation and stability from nonlinear functional analysis and the qualitative theory of differential equations (e.g. local and global implicit function theorem, Malgrange preparation theorem, manifolds and transversality, Sard's theorem, topological degree, Ljusternik—Schnirelman theory). The third chapter contains some applications of the implicit function theorem.

The authors distinguish two aspects of bifurcation theory: static and dynamic. The first one investigates the change of the structure of the set of zeros of a function as parameters in the function are varied. Dynamic bifurcation theory is concerned with the changes that occur in the qualitative behaviour of solutions of differential equations as parameters of the vector field are varied.

Chapter 4—8 (entitled "Variational Method"; "The Linear Approximation and Bifurcation"; "Bifurcation with One Dimensional Null Space"; "Bifurcation with Higher Dimensional Null Spaces"; "Some Applications") deal with static bifurcation theory. The results of the fourth chapter are applied to Hamiltonian systems, elliptic an hyperbolic problems.

Chapters 9—13 (entitled "Bifurcation Near Equilibrium"; "Bifurcation of Autonomous Planar Equations"; Bifurcation of Periodic Planar Equations"; "Normal Forms and Invariant Manifolds"; "Higher Order Bifurcation Near Equilibrium") are devoted to dynamic bifurcation theory.

The chapters are followed by bibliographical notes with informations and references for the history of the problems and the further study.

This well-written excellent book will be undoubtedly the standard reference in nonlinear analysis and bifurcation theory. It can serve also as a text-book (the authors give suggestions for adapting the material to several types of one semester courses). We recommend it for every mathematician, user and student interested in differential equations and their application.

L. Hatvani (Szeged)

James A. Cohran, Applied Mathematics: Principles, Techniques, and Applications, X+399 pages, Wadsworth International Group, Belmont, California, 1982.

The book is designated to be used as a second course in applied mathematics. The prerequisite knowledge includes a thorough grounding in the calculus through ordinary differential equations. A certain acquaintance with vector analysis, elementary complex variables, Fourier series, Laplace/Fourier transforms and partial differential equations is also taken for granted. (The most important definitions and theorems are collected in appendices at the end of the book.)

The topics presented are selected for their relevance to nonroutine applications encountered in today's and hopefully tomorrow's world. For this reason, advanced topics such as stability theory, conformal mapping, generalized functions (distributions) and integral equations are included as are seemingly more elementary topics such as linear algebra, differential equations and special functions.

The discussion of each major mathematical topic in this book is preceded by consideration of a relevant practical application. Indeed, the modelling of difficult "real-world" problems serves to motivate the mathematics that follows. Occasionally the book contains rather detailed analysis of various computational procedures and techniques of obtaining the "results".

The book consists of eleven chapters, six appendices and a long (author and subject) index. There are references and problems at the end of each chapter. Many of the proofs of theorems can be skipped in the text at first reading, solving an appropriate number of the problems, however, is a must. These problems are carefully chosen to illustrate or amplify various portions of the text and constitute an extremely important component of the learning process.

The table of contents (in the parantheses we pick up a theoretical result and/or a practical application characteristic to the chapter in question): 1. Linear algebra and computation (Ill-conditioning, LR and QR), 2. Eigenvalue problems for differential equation (Sturm—Liouville problems), 3. The special functions of applied mathematics (More on Bessel functions), 4. Optimization and the calculus of variations (Least action and Hamilton's principle in mechanics), 5. Analytic function theory and system stability (A satellite attitude-control system, The Cauchy—Goursat theorem), 6. Conformal mapping (Cavity and jet flows), 7. Integral transforms (The Mellin transform), 8. Green's functions (and partial differential equations, The Dirichlet problem for the n-ball), 9. Generalized functions (Delta functions in optics and electrostatics), 10. Linear integral equations (The Fredholm alternative, The Rayleigh—Ritz procedure), 11. Asymptotics (Order relations O and o, The method of steepest descent.)

The book is warmly recommended to engineering and applied mathematics students who will pursue industrial or business careers. But it will be undoubtedly useful to those who are interested in solving diverse physical problems at research laboratories.

F. Móricz (Szeged)

Combinatorial Mathematics IX, Proceedings of the Ninth Australian Conference on Combinatorial Mathematics Held at the University of Queensland, Brisbane, Australia, August 24—28, 1981. Edited by Elizabeth J. Billington, Sheila Oates-Williams, and Anne Penfold Street (Lecture Notes in Mathematics, Vol. 952), XI+443 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

This volume contains seven invited papers and twenty contributed papers. A number of them is concerned with symmetric combinatorial structures (finite projective and affine planes, block design, perfect covering). The reader can find papers close to applied mathematics in the following topics; economic let scheduling, matroid algorithms, heuristics for determining a maximum weight planar subgraph of a given edge weighted graph, mathematical description of woven structures.

The titles of invited papers are: D. R. Breach, Star gazing in affine planes; P. J. Cameron, Orbits, enumeration and colouring; A. Gardiner, Classifying distance-transitive graphs; W. L. Kocay, Some new methods in reconstruction theory; V. Pless, On the uses of contracted codes; Ch. E. Praeger, When are symmetric graphs characterised by their local properties? and R. G. Stanton, Old and new results on perfect coverings.

L. A. Székely (Szeged)

Combinatorics, Prooceedings of the Eighth British Combinatorial Conference, University College, Swansea 1981. Edited by H.N.V. Temperley (London Mathematical Society Lecture Note Series, Vol. 52) 190 pages, Cambridge University Press, 1981.

This book contains the texts of nine invited lectures held at the Eighth British Combinatorial Conference. The list of these papers is as follows.

L. Babai, On the abstract group of automorphisms. Babai surveys results about the existence and non-existence of graphs with prescribed properties having a prescribed abstract group of automorphisms. Similar results and problems concerning other algebraic and combinatorial structures instead of graphs are mentioned.

L. W. Beineke, A tour through tournaments or bipartite and ordinary tournaments: a comparative survey. A bipartite tournament is an oriented complete bipartite graph. This paper seems to be a germ of the theory of bipartite tournaments. All results are is comparison with similar results concerning ordinary tournaments.

H. Baker and F. Piper, Shift register sequences. Linear and non-linear feedback shift registers are treated. Shift registers can be applied in cryptography so as to mix the statistics of letter frequencies (what may obstruct to discover the 1—1 function between letters and their codes).

B. Bollobás, Random graphs. The chapters of this paper are: the automorphism group, sparse graphs, threshold functions, graphs with many edges, and random regular graphs. The last one contains new results of great importance and includes sketches of the proofs.

F. R. K. Chung and R. L. Graham, Recent results in graph decompositions. This report gives a brief overall view of decomposition problems and treats some topics in which significant progress has been made recently, e.g. decomposition into complete bipartite graphs.

B. Grünbaum and G. C. Shephard, The geometry of planar graphs. This paper surveys the theory of infinite planar graphs. These graphs may occur as edge-graphs of tilings. Euler's Theorem and Kotzig's Theorem are generalized by the authors.

F. J. MacWilliams, Some connections between designs and codes. Author's introduction is: "This paper describes how to get designs from codes".

R. W. Robinson, Counting graphs with a duality property. Robinson surveys the enumeration of graphs and other structures satisfying a duality condition. The main tool is a modification of the Burnside lemma due to de Bruijn. The notion of duality used here includes self-complementarity.

J. G. Thompson, Ovals in a projective plane of order 10. The author investigates the following problem: "does there exist a set S of 99 fixed point free involution on 12 points such that for each involution $(ab)(cd)$ which moves just 4 points, there is a unique s in S which has $\{a, b\}$ and $\{c, d\}$ as orbits?"

L. A. Székely (Szeged)

Constructive Mathematics, Proceedings, New Mexico, 1980, edited by F. Richman, Lecture Notes in Mathematics, 873, VIII+347 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

In the last decade or so one could observe increased interest in constructive mathematics. Surely this has to do with the developing computation/computer science but Bishop's book "Foundations of constructive analysis" also influenced a number of mathematicians, probably because it was able to take off the "dogmatism" of the earlier theories. Several branches (or degrees?) of constructivism can be found in current mathematics from Markov's school admitting only finite strings of symbols to those who accept classical mathematics, only they are interested in effective algorithms rather than just computation in principle. In 1980 at Las Cruces a conference was organized with the intention that the representatives of the several schools should exchange their ideas and thoughts concerning constructive mathematics. These proceedings contain all but five lectures delivered at this conference.

Since the main treads and ideas of constructive mathematics are unknown for many mathematicians even today, let us present here the table of contents: F. Richman: Seidenberg's condition P; W. Ruitenburg: Field extensions; R. Milnes and F. Richman: Dedekind domains; J. H. Davenport:

Effective mathematics — the computer algebra viewpoint; Y. K. Chan: On some open problems in constructive probability theory; A. Scedrov: Consistency and independence results in intuitionistic set theory; W. A. Howard: Computability of ordinal recursion of type level two; J. P. Seldin: A constructive approach to classical mathematics; D. Isles: On the notion of standard non-isomorphic natural number series; N. D. Goodman: Reflections on Bishop's philosophy of mathematics; M. Beeson: Formalizing constructive mathematics: why and how? J. Lambek and P. J. Scott: Independence of premisses and the free topos; R. Vesley: An intuitionistic infinitesimal calculus; N. Greenleaf: Liberal constructive set theory; D. S. Bridges, A. Calder, W. Julian, R. Mines and F. Richman: Locating metric complements in Euclidean space; J. R. Moschovakis: A disjunctive decomposition theorem for classical theories; D. S. Bridges: Towards a constructive foundation for quantum mechanics; A. S. Yessenin—Volpin: About infinity, finiteness and finitization; M. Gelfond: A class of theorems with valid constructive counterparts; J. R. Geiser: Rational constructive analysis.

Anynone who feels inclined to get acquainted with this "new world" (where it may happen e.g. that every real function is uniformly continuous) is recommended to consult these proceedings since several of their papers are expository or contain the philosophy of the subject.

V. Totik (Szeged)

I. P. Cornfeld, S. V. Fomin and Ya. G. Sinai, Ergodic Theory. Grundlehren der mathematischen Wissenschaften 245, X+482, Springer-Verlag, New York—Heidelberg—Berlin.

At the beginning ergodic theory dealt mainly with averaging problems but now, due to the readical changes in it during the last two decades, "it is a powerful amalgam of methods used for the analysis of statistical properties of dynamical systems". This book is an up-to-date development of the theory written by three outstanding scholars of the discipline. Since the authors' aim was to create a monograph focusing on applications, "Ergodic Theory" deserves the attention of research workers in other sciences as well, such as physics, biology, chemistry etc.

The book consists of four parts. Part I contains the description of several classes of dynamical systems. It begins with the basic definitions: ergodicity, mixing, operators adjoint to dynamical systems etc. and proceeds on to many classical constructions: dynamical systems on smooth manifolds, on torus, on homogeneous spaces; billiard type systems, systems in number theory and probability theory etc. In Part II the authors construct the direct and skew product of DS-s, introduce the important concept of entropy and give a detailed proof for the celebrated theorem of Ornstein on the existence of a stationary code. Part III is devoted to the spectral theory of DS-s. This is the shortest part of the book. Nevertheless, it contains von Neumann's theory of dynamical systems with discrete spectrum and the spectral analysis of DS-s associated to Gaussian stationary random processes. Finally, in Part IV the authors consider the possibility of approximation of dynamical systems by periodic DS-s and give some applications of the theory such as an example of an ergodic automorphism with a spectrum without the group property.

The authors pay much attention to illustrating the general concepts and theorems through concrete examples and these examples help very much in understanding the main ideas perhaps because they arise in very natural context. The bibliography contains more than 150 items. The publisher also did his best, the text is arranged in an especially legible form.

V. Totik (Szeged)

H. S. M. Coxeter, P. Du Val, H. T. Flather, J. F. Petrie, The Fifty-Nine Icosahedra, XX+26 pages with 20 plates and 9 figures, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

An exciting problem of classical geometry was to enumerate and to describe the polyhedra that can be derived from the five Platonic solids by stellation, i.e., by extending or producing the faces until they meet again, always preserving the rotational symmetry of the original solid. The complete enumeration was first published by the above authors in 1938 by the University of Toronto Press. The present booklet is the reprint of this first edition with a new preface by P. Du Val. The text is a classical work of geometry which containes the mathematical explanation of the stellations and plates with pictures of all 59 variations descibing also the transformations between these stellations.

Z. I. Szabó (Szeged)

A. J. Dodd, The Core Model (London Mathematical Society Lecture Note Series 61), XXXVIII+229 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1982.

This book is the first systematic study of the simplest core model K of set theory.

An arbitrary model M of ZFC is called an inner model iff M is transitive and $\mathrm{On}\in M$ (where On is the class of ordinals). An inner model M is said to have the covering principle, CP (M), for short, if for any uncountable $X\subseteq \mathrm{On}$, there exists $Y\in M$, such that $X\subseteq Y\subseteq \mathrm{On}$ and $\overline{\overline{X}}=\overline{\overline{Y}}$ (for a set $A, \overline{\overline{A}}$ denotes the cardinality of A). M is rigid if no elementary embedding of M into M other than identity exists.

The main aim of developing the core models is to obtain a generalization of the covering lemma due to Jensen: if L (the constructible universe of Gödel) is rigid, then CP (L). Core models are such models of set theory which subsumes L, the GCH is true in them and have the covering property under some inner model assumption. It is known, that if $M\models$ GCH and CP (M), then $M\models$ SCH, where SCH stands for the singular cardinal hypothesis, i.e., SCH denotes the assumption:

"for all singular cardinals α, $2^{cf(\alpha)}<\alpha$ implies $\alpha^{cf(\alpha)}=\alpha^{+}$"; hence in any core model SCH is true.

K is the simplest core model which is constructed by using two basic set theoretical tools: the fine structural investigations of the constructible universe developed mostly by Jensen, and the method of iterated ultrapowers due to Kunen, used in the theory of measurable cardinals.

The text is devided into six main parts followed by two appendices and a collection of historical notes (and, of course, a list of references).

The second part treats normal measures and iterated ultrapowers. The concept of a normal measure is used in the following form: U is a normal measure on $\varkappa$, if there are M and j: $V\to M$ such that M is an inner model, j is an elementary embedding with $j\restriction\varkappa=\mathrm{id}\restriction\varkappa$, $j(\varkappa)>\varkappa$, and M is the smallest model with $X\subseteq M$, where X is an elementary submodel of M such that range $(j)\cup\varkappa\subseteq X$ and $X\in U$ iff $\varkappa\in j(X)$. If such a model M exists, then it is unique, and is called the ultrapower of V by the normal measure U. Albeit quantifiers range over proper classes, this definition can be formulated in the language of set theory: let $L[U]$ denote the universe of sets constructible from the normal measure U and suppose $L[U]\models$ "U is a normal measure", moreover, let M be the ultrapower of $L[U]$ by U; then $M\models L[\mathscr{V}]$, where $\mathscr{V}=j(U)$. By a result due to Scott, $L[\mathscr{V}]$ is a proper subclass of $L[U]$. It was shown by Kunen, that $\mathscr{V}$ is the only normal measure in $L[\mathscr{V}]$. The iteration of this construction, also defined by Kunen, taking the ultrapower of $L[\mathscr{V}]$ by $\mathscr{V}$ and using direct limits to get through limit stages yields to the recurrence:

$$M_0 = M, \quad U_0 = U;$$

M_{i+1}=the ultrapower of M_i by U_i with j; $M_i \to M_{i+1}$ and $j_{i+1} = j \cdot j_i$;
$U_{i+1} = j_{i+1}(U)$ for successors and
(M_λ, j_λ) is the direct limit of the sequence $(M_i, j_i)_{i<\lambda}$ for limit λ,
and $U_\lambda = j_\lambda(U)$.

Then, for all $\alpha \in \mathrm{On}$, $M_\alpha = L[U_\alpha]$.

Part II is a detailed exposition of this ultrapower construction.

The first part of the volume is devoted to developing the fine structural apparatus needed in constructing K. This is based mostly on the J hierarchy due to Jensen. The J hierarchy, as a whole is the same as the constructible universe L, the levels, however, are rearranged in the following way. Rudimentary functions, generalizing primitive recursive functions to arbitrary sets, are finitely generated by the initial functions consisting exhaustively of all projections, complementations, pairings, compositions and recursions of the form

$$f(y, \vec{x}) = \bigcup_{z \in y} g(z, \vec{x})$$

where $\vec{x}$ stands for a list of arguments. Let X be a set and put

$$R(X) = \{f(\vec{x}) \mid f \text{ is rudimentary and } \vec{x} \in X\}.$$

Then let $J_0 = \emptyset$, $J_{i+1} = R(J_i \cup \{J_i\})$ for successor i and $J_\lambda = \bigcup_{i<\lambda} J_i$ for limit λ. Clearly, $\bigcup_{\alpha \in \mathrm{On}} J_\alpha$ is the constructible universe of Gödel. Let U be a normal measure in an inner model M and suppose that for some α, $M = J_\alpha^U$. The basic fine structural tool, the projectum ϱ_M of M is the least γ such that there exists $A \subsetneqq \gamma$ such that A is Σ_1-definable over M and $A \notin M$. The main fine structural result, central for the construction of K states that if $\varrho_M \leqq \varkappa$, then M can be "coded" by a subset A_M of $\varkappa$; more precisely, if $\varrho_M \leqq \varkappa$, where U is a normal measure on $\varkappa$, then there is a surjective function from a subset A_M of $\varkappa$ onto J_α^U, such that A_M is Σ_1-definable over J_α^U. If $L[\mathscr{V}]$ is the ultrapower of $L[U]$ by U and j is an elementary embedding of $L[U]$ in $L[\mathscr{V}]$ such that $j \restriction \varkappa = \mathrm{id} \restriction \varkappa$, then $j(A_M) \cap \varkappa = A_M$, so $A_M \in L[\mathscr{V}]$ and hence $M \in L[\mathscr{V}]$.

Let $T = \{M \mid M = J_\alpha^{U_i}$ for some normal measure U_i on $\varkappa_i$ and $\varrho_M \leqq \varkappa_i\}$. Then the core model K is defined by $K = \bigcup T$. By a mouse, an element of T is meant. Mice are studied in details in Part III. It is shown for example, that the dependence of the definition of T on the normal measures U_i can be eliminated by allowing any normal measure instead of U_i. This process yields to the concept of a premouse: M is a premouse at $\varkappa$ if $M = J_\alpha^U$ for some U and $M \models$ "U is a normal measure on $\varkappa$". A premouse M is iterable iff the model $(M, \in_M)$ defined just as in the definition of a normal measure (ultrapower), with the only difference: "elementary" is replaced by "Σ_1-definable", is well-founded. Indeed, the well-foundedness property of M_i is inherited by the iteration of ultrapowers. Let $T' = \{M \mid M$ is an iterable premouse at some $\varkappa$ and $\varrho_M \leqq \varkappa\}$.

Then $K = \bigcup T' \cup L$. Part IV is devoted to the investigation of K. In particular, an important internal characterization of K is proved. If there exists an inner model $L[U]$ with $L[U] \models$ "U is a normal measure", then $K = \bigcap_{i<\infty} L[U_i]$. It is also shown, that $K \models$ ZFC and $K \models$ GCH. Moreover, in Part V, a generalization of the covering lemma is obtained: if there is no inner model with a measurable cardinal, then CP(K). As an application, one has: if there is no inner model with a measurable cardinal, then $K \models$ SCH. It is also shown that several combinatorial principles such as $\diamondsuit$ and $\square$ hold in K. Part VI collects some recent results on core models larger than K. In particular, a few properties of supercompact and superstrong cardinals are established. Appendices relate core models to the forcing construction and to some absoluteness results for models of ZFC.

The volume is selfcontained, clearly written and gives a full exposition of the state of the art concerning core models. It is sure that this book will become a basic reference for researchers in the fields of large cardinals as well as for graduate students.

P. Ecsedi-Tóth (Szeged)

Burton Dreben and Warren D. Goldfarb, the Decison Problem, Solvable Classes of Quantificational Formulas, XII+271 pages, Addison-Wesley Publ. Comp. Inc., Advanced Book Program, Reading, Mass., 1979.

The classical decision problem (called "the fundamental problem of mathematical logic" by Hilbert) asked for an algorithm to decide for any formula if it is satisfiable. Since the work of Gödel and Church it is well known that there can be no such algorithm. Special eases with restricted classes of formulae having a decision procedure have been investigated intensively during the past decades. These classes are defined by syntactic restrictions e.g. on the form of quantifiers, a basic decidable case being the so-called Gödel—Kalmár—Schütte class of formulae with quantifiers $\exists \ldots \exists \forall \forall \exists \ldots \exists$.

This book gives a comprehensive description of the known solvable classes including the complete list of solvable prefix classes. A unified treatment is given to the subject by the use of the Herbrand expansion method.

The book is written in a very clear style and gives a good picture of the current state of this side of the decision problem, providing a deep knowledge of the important Herbrand expansion method and indicating some interesting open problems as well. It can be recommended to logicians and computer scientists. (A good complementary reading is given by the book Unsolvable Classes of Quantificational Formulas of H. R. Lewis [Addison—Wesley Publ. Comp. Inc. Adwanced Book Program, Reading, Mass., 1979, 214 pages], and a recent branch of the topic is described in the paper Complexity Results for Classes of Quantificational Formulas by H. R. Lewis [J. of Computer and System Sciences 21, No. 3. Dec. 1980, pp. 317—353].)

György Turán (Szeged)

C. H. Edwards, Jr., The Historical Development of the Calculus, XII+351 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1979.

A scientific concept cannot be understood completely without knowing its development. Calculus has become the language of Western science for three centuries, so all the students in science have to know something about its history. Lecturers are to take a general view of this subject and they need a handbook in this topic.

Edwards' book is suitable for the above mentioned purpose. It begins calculus with Eudoxus' definition on proportionality of ratios and the method of exhaustion based on that definition. The method culminates in Archimedes' works to whom a chapter is devoted in the book. He used a double reductio ad absurdum rather than limits and his "geometric calculus" could not have been continued.

Edwards emphasizes the influence of medieval speculations on motion, variability and infinity to medieval mathematics to break up the Greek horror of infinity. A number of early tangent constructions are shown and the difference between them and the calculus according to Newton and Leibniz is elucidated.

The classicals of the calculus are treated circumstantially until Weierstrass.

A short postcript is devoted to two results of the twentieth century: the Lebesgue integral and the non-standard analysis. Edwards does not think non-standard analysis to be a correct reformulation of infinitesimals, he states "Leibniz seems not to have committed himself on the question of actual existence of infinitesimals, and he certainly expressed doubts on occasion".

The author does prove the importance of adequate concepts and notations in mathematics. It is clear all over the book that calculus is for calculations.

The reader can take part in the work of the classicals: there are exercises interspersed throughout the text and the reader is invited to solve them using the tools of that time.

The book is offered to lecturers, students and to the wide mathematical community.

L. A. Székely (Szeged)

Functional Differential Equations and Bifurcation, Proceedings of a Conference Held at São Carlos, Brazil, July 2—7, 1979, edited by A. F. Izé (Lecture Notes in Mathematics, 799), XXII+409 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

This book contains contributions presented at the conference by authors from Brasil, Iceland, Italy, Japan, U.S.A. and South Africa. It gives a good survey on some present topics of the theory of differential equations and some related fields. The reader can find papers on subjects such as control theory, boundary value problems, periodic solutions, stability theory, structural stability, bifurcation theory, dissipative processes, existence type results, asymptotic equivalence of solutions, linear difference equations, Volterra—Stieltjes-integral equations, Hartree type equation, Levin—Nohel equation on the torus, almost periodic functional differential equations.

J. Terjéki (Szeged)

A. Gardiner, Infinite Processes (Background to Analysis), IX+306 pages with 182 illustrations, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

The book provides a well-rounded picture of the basic material of Analysis. Its main goal is to show why the concepts such as infinite decimals, length, area, volume, functions are handled as they are in mathematics.

The text is divided into four parts. Part I is short and largely descriptive. It indicates how, around 1800, mathematicians began to realize that the lack of precision in their manipulation of the infinite processes involved in the naive calculus was a source of error and confusion.

Part II is the longest one of the book. It examines in detail infinite processes arising in arithmetic of the real numbers. Most of the text is devoted to the analysis of specific examples.

Part III explores that any attempt to invest the familiar geometric notions of length, area and volume with precision involves the fundamental properties of real numbers. It also points out to the fact that modern mathematics is not so much the study of numbers and space as the study of functions.

In Part IV the author outlines some of the basic questions which result from the differential and integral calculus. In particular, the following crucial question is considered: What exactly is a function?

A lot of exercises are included, which constitute an integral part of the text. They arise directly out of the text and need to be understood in context.

The book is described as a stimulus for thinking about the role of infinite processes in mathematics. The presentation is clear and precise, the ideas are illuminated by consideration of historical developments. The better understanding is helped by 182 figures. The book ends with an (author and subject) index.

This good overview is especially suited to mathematical history and review courses, as well as for math teachers and for nonspecialists who have mastered the calculus.

F. Móricz (Szeged)

Bernard Gelbaum, Problems in Analysis. Problem Books in Mathematics, VII+228 pages with 9 illustrations, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

"The major part of every meaningful life is the solution of problems; a considerable part of the professional life of technicians, engineers, scientists etc. is the solution of mathematical problems. It is the duty of all techers, and of teachers of mathematics in particular, to expose their students to problems much more than to facts." These are the words of Paul Halmos about mathematical problems and he adds that we have to "...train our students to be better problem — posers and problem — solvers than we are". Surely it is not accidental that P. R. Halmos is the series editor of the new Springer-series "Problem Books in Mathematics". Anyone agreeing with Halmos on the role of problems will greet this fascinating idea: supply the students, teachers and mathematicians with books focusing on problems which may range from elementary exercises to unsolved research problems. As prototypes Pólya and Szegő's "Problems and Theorems in Analysis" and Hilbert's famous 23 problems are marked. However, it is an almost impossible task to give such a comprehensive selection as Pólya and Szegő's in any branch of mathematics, therefore the author's taste and "intelligence" play enormous role in writing these problem books. For a newly launched series the succesful start is vital and "Problem Books in Mathematics" accomplished this task excellently: the first two exemplares: Gelbaum's book and Kirillov and Gvishiani's "Theorems and Problems in Functional Analysis" are really worth for beginning the series with them.

Gelbaum's book contains 518 problems and their solutions. The topic is real analysis and the elements of functional analysis. The standard exercises of this theme are mostly left out, almost every problem requires some thinking — some of them may be very puzzling for a beginner. The proofs are short but sometimes incaccurate or lenghty, e.g. the solution of problem 248: "$\sum_{1}^{\infty} k\lambda(G_k)=\Sigma_n \lambda(A_n)$ where $G_k=\{x \mid x\in A_n$ for exactly k distinct values of $n\}$ $(A_n \neq A_m)$" presented in the book is not complete and at the same time the problem itself is easy if we use characteristic functions. Unfortunately there are false problems and solutions (!). For example for problem 126: "Let f be in $C(R, R)$ and assume $\lim \sup_{h\to 0+0} (f(x+h)-f(x))/h \geqq 0$ a.e. Show f is monotone increasing" any descreasing continuous singular function provides a counterexample. Problem 175" Give an example of a measure space (X, S, μ), a sequence $\{E_n\}$ of measurable sets of finite measure, and a sequence $\{f_n\}$ of functions such that f_n and $1-f_n$ are integrable, $0\leqq f_n \leqq 1$, $f_n=1$ on E_n, $\lim_{n\to\infty} f_n(x)=1$ a.e. and $\int_X (1-f)\,d\mu \not\to$ as $n\to\infty$ "asks for a non-existing construction. In some cases the formulation of the problem is clumsy since by the same method a much nicer problem could be solved, e.g. in problem 57 if $f_0\in C([0, 1], R)$ and $f_n(x)=\int_0^x f_{n-1}(t)\,dt$ then $f_0\equiv 0$ provided for every $x\in[0, 1]$ there is an n with $f_n(x)=0$. Nevertheless these faults must not be exaggerated since the majority of the problems and solutions are indeed very nice.

An undergraduate or graduate student should have enough knowledge to solve most of the problems although the author freely uses harder results from real analysis. For example, to solving problem 316 "If $f\in L^\infty(R, \lambda)$ and $\int_k \exp(-(x-y)^2)f(y)\,dy=0$ for all $x\in R$ then $f=0$ a.e." one has to know Wiener's tauberian theorem. A special merit of the book is that besides general abstract results and theorems it contains several "concrete" problems. For instance problem 473 states that if $\nu(n)=2^{-n!}$ $(n\geqq 2)$ and $\nu(1)=1-\sum_{1}^{\infty} 2^{-n!}$ then there are no nonconstant functions f and g independent with respect to ν.

Gelbaum's book may be recommended to students, teachers and research workers, as well, who may get fun and make progress while reading and solving these non-trivial excellent problems.

V. Totik (Szeged)

G. Gierz, K. H. Hoffmann, K. Keimel, J. D. Lawson, M. Mislove, D. S. Scott, A Compendium on Continuous Lattices, XX+371 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

In lattice theory, the seventies brought a considerable development of (and interest in) the study of continuous lattices. Historically, the most important stimulation was Dana Scott's research on the problems of syntax and semantics of computer languages and his interpretation of computer programs (including cyclic ones) by means of continuous lattices, which led, among other things, to Scott's model in set theory of the type free λ-calculus. These results became at once well-known also outside algebra, among specialists of computer science and logic. But, at about the same time, important contributions were made by others, most of them on the list of authors of the book under review. Especially, the results of Hoffmann, Lawson, Mislove and Stralka on compact semilattices and those of Keimel and Gierz on the topological representation theory and spectral theory of non-distributive lattices are to be mentioned. This led to a collaboration of the authors of the book, resulting in what now may be called the theory of continuous lattices. The present book is the first monograph on the subject addressed to the general mathematical public.

To describe the subject, one first has to define the "way-below" relation, which is basic for the entire theory. We say that, for elements x, y of the complete lattice L, x is way below y, in symbols, $x \ll y$ iff for directed subsets $D \subseteq L$ the relation $y \leqq \sup D$ always implies the existence of a $d \in D$ with $x \leqq d$. (Elements satisfying $x \ll x$ are exactly the compact elements.) A lattice L is called a continuous lattice if L is complete and satisfies the axiom of approximation: $x = \sup \{u \in L : u \ll x\}$ for all $x \in L$. (In particular, all algebraic lattices are continuous.) Intuitively, thinking of the realization in computability theory, x way below y can be interpreted as x is a "(finite) approximation" of y. Then the axiom of approximation says that each element is the limit of its finite approximations. The motivation for the study of continuous lattices comes not only from computer science and logic. Other fields where such lattices appear quite naturally (sometimes in disguised forms) are, for example, general topology, functional analysis, category theory, and, of course, algebra.

Chapter I introduces continuous lattices from an order theoretic point of view. In Section 1 the way-below relation is discussed. Section 2 gives an equational chracterization of continuous lattices. Section 3 deals with irreducible and prime elements. Section 4 considers the important special case of algebraic lattices. Chapter II defines the Scott topology and develops its applications to continuous lattices. The second important topology for continuous lattices, the Lawson topology, is discussed in Chapter III. Chapter IV considers various important categories of continuous lattices together with certain categorical constructions. Section 1 presents important duality theorems for the study of continuous lattices. The last two sections give general categorical constructions for obtaining continuous lattices which are fixed points with respect to some self-functor of the category. This process is needed for the construction of set-theoretic models of the λ-calculus. Chapter V deals with spectral theory. The most important result of Chapter VI is the Fundamental Theorem of Compact Semilattices, establishing the equivalence between the category of compact semilattices with small semilattices and the category of continuous lattices. Chapter VII completes the study of connections between topological algebra and continuous lattice theory with methods coming from topological algebra rather than from lattice theory.

This book is warmly recommended to anyone wishing to become acquainted with the subject.

A. P. Huhn (Szeged)

Franklin A. Graybill, Matrices with Applications in Statistics, Second edition (Wadsworth Statistics/Pribability Series), XII+461 pages, Wadsworth International Group, Belmont, California, 1983.

Matrices are used so extensively in the theory and applications of statistics that a firm knowledge of matrix and linear algebra is required from a student who wants to study the theory of linear statistical models. A number of topics in matrix algebra that are useful in a study of multivariate analysis is generally not available in an elementary course or in textbooks in linear algebra. On the other hand, the most part of monographs and advanced textbooks in matrix algebra has a too hard algebraical emphasis and some topics that are important for a statistician are mentioned only briefly. Graybill wanted to write a book which is "useful for any-one who takes courses in regression and correlation, analysis of variance, least squares, linear statistical models, multivariate analysis, or econometrics; and it could serve as a resource book for many other subjects". As the second edition of his book shows, the author achieved his purpose.

The book assumes that the reader has had a course that includes the most important and fundamental theorems in linear algebra. An introduction and summary are given in the first three chapters, the theorems are stated without proofs. Some geometric interpretations of vectors and the elementary theory of analytic geometry are discussed briefly in Chapters 4 and 5. Chapter 6 is devoted to the general inverse and conditional inverse. The author states some additional theorems on the inverses of special matrices and proves some theorems that can be used to compute the generalized inverse of a matrix. Chapter 7 deals with the existence and number of solutions of systems of linear equations. Approximate solutions of inconsistent systems including least squares are in the focus of this chapter. Chapter 8 contains theorems on the patterned and other special matrices (partitioned, triangular, dominant diagonal, Vandermonde, Fourier, permutation and Toeplietz matrices). The following chapter treats the many applications in which the sum of diagonal elements (trace) of a matrix plays an important role. Chapter 10 demonstrates how matrices and vectors can be used in transforming random variables, in evaluating multiple integrals and in differentation. These methods are useful in the study of multivariate normal distributions. The author briefly discusses some important general types of matrices (positive, non-negative, idempotent, tripotent matrices) in the last two chapters. Each chapter contains a lot of examples and exercises that can help the reader in understanding the presented material.

The book is very elegently and clearly written, it can be recommended to all students and statisticians interested in linear algebra from a statistical point of view.

Lajos Horváth (Szeged)

Richard K. Guy, Unsolved Problems in Number Theory (Unsolved Problems in Intuitive Mathematics, Vol. I), XVIII+161 pages with 17 figures, Springer-Verlag, New York—Heidelberg—Berlin, 1981.

This book lists 178 challenging open problems (or group of problems) to stimulate beginning researchers. No matter how easily one can understand them, none of us lives as long as to see the proof or counterexample for all the listed problems. The touched topics are prime numbers, divisibility, additive number theory, diophantine equations, sequences of integers, and others.

This volume is dedicated to Erdős Pál, whose influence to number theory can be observed everywhere in the book, as follows: "Among his several greatnesses are an ability to ask the right questions and to ask it of the right person." The reader is supplied with plentiful references. Many prizes are set by Erdős and some by Graham.

L. A. Székely (Szeged)

Frank C. Hoppensteadt, Mathematical Methods of Population Biology (Cambridge Studies in Mathematical Biology, 4), VIIII+149 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1982.

According to the publisher, "this introduction to mathematical methods that are useful for studying population phenomena is intended for advanced undergraduate and graduate students, and will be accessible to scientists who do not have a strong mathematics background." The first two chapters introduce the usual deterministic models of total population and population age structure (Malthus, Verhulst, the predator pit, chaos, synchronisation, fisheries, Fibonacci's reproduction, McKendrick's model etc.), the third chapter deals with random models of bacterial and human genetics (urn, Fisher—Wright, and branching process models) and of epidemics (Reed—Frost model) based on Markov chains, and the last two chapters describe very shortly perturbation methods and diffusion approximations.

Matheatical notions are used without definitions. It is not that the reviewer would like to seee mention of the Radon—Nikodym theorem on p. 63, for example, where conditional expectations are used (and an embarrassing misprint is left in line 6 from bottom), but he feels that students and scientists "who do not have a strong mathematical background" will not learn the "mathematical methods" from this book. The reviewer agrees that "mathematical details" should not "obscure biological relevance" but such non-technical and non-sensical descriptions of the central limit theorem as the one on p. 97, that "(it) states that *any* random variable, discrete or continuous (?!), is in a definite sense approximated by a normally distributed random variable", will not help anybody to understand neither population biology, nor mathematics.

It is not a contradiction in terms, however, that this is a good book. Good for those who do have a stronger mathematics background and are interested in applications. These people will enjoy the numerous interesting examples and exercises from population biology.

Sándor Csörgő (Szeged)

B. Huppert—N. Blackburn, Finite Groups II—III (Grundlehren der mathematischen Wissenschaften, Band 242—243), XIII+531 and IX+454 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

This is the continuation, awaited for 15 years, of the classic book on finite groups:
B. Huppert, Endliche Gruppen I (Grundlehren der mathematischen Wissenschaften, Band 134), XII+796 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1967; reprinted 1979.

While Volume I presents a large part of what was known about the structure of finite groups at the time of its writing (started in 1958), the same goal is evidently not attainable now. During the past two decades the subject has made a tremendeous progress, with a lot of new branches and powerful methods coming into existence which, combined together, produced a number of extremely deep results. Few accomplishments ever reached in mathematics are comparable with the recent completion of the classification of finite simple groups. In view of this great development it is no surprise that the authors of Volumes II and III had to be contended with selecting several important topics and even within those topics no attempt on completeness was made.

All three chapters of Volume II are devoted to discussing the role of linear methods in finite group theory. Representation theory is presented first (Chapter VII: Elements of General Representation Theory), the emphasis being put on the modular case, as the classical one is studied in Volume I, Chapter V. Next (Chapter VIII: Linear Methods in Nilpotent Groups) some ways of "translating" commutator calculations into calculations with linear structures are shown along with several theorems illustrating the power of these methods. For example, bilinear forms are used to determine the Suzuki 2-groups, and the Lie-ring method is applied to prove that for prime exponent the answer to the restricted Burnside problem is affirmative. The last chapter (IX: Linear Methods in Soluble Groups) gives an introduction to the Hall—Highman methods and numerous applications to obtain upper bounds for the p-length of a p-soluble group in terms of various invariants of its Sylow p-subgroups.

Volume III also consists of three chapters. The first one (Chapter X: Local Finite Group Theory) is concerned with deriving properties of the whole group from hypotheses involving only its p-subgroups and their normalizers (which are regarded as local properties of the group). Such results turned out to be important for example in proving the solubility of groups of odd order. The book ends with two chapters on permutation groups, including also several important characterization theorems, i.e., descriptions of specific groups solely in terms of group-theoretical properties. One of the earliest instances of such results was given for Zassenhaus groups, which is presented in full detail (Chapter XI: Zassenhaus Groups). The last chapter (XII: Multiply Transitive Permutation Groups) is a collection of some of the most interesting investigations on multiply transitive and sharply multiply transitive permutation groups.

No doubt, these volumes will soon become as indispensable reference books for group theorists, as Volume I. Besides, by giving a systematic treatment of a number of results which, up to now, were available in research papers only, they will be an immense help for those wishing to specialize in the subject.

Ágnes Szendrei (Szeged)

Ching-Lai Hwang, Abu Syed Md. Masud in collaboration with **Sudhakar R. Paidy and Kuangsun Yoon, Multiple Objective Decision Making—Methods and Applications,** A State-of-the-Art Survey, (Lecture Notes in Economics and Mathematical Systems, 164) XII+351 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1979.

This is a good guide through the literature of the Multiple Objective Decision Making (MODM) methods. The authors present the existing methods, their characteristics, and their applicability to analysis of MODM problems. The book contains a good classification of about two dozen MODM methods. The first level of the classification is the information available for the decision maker. The second level is the type of information, and the lowest level contains the major classes of methods. Most of these methods have been proposed by various researchers in the last few years, and the main usefulness of this work is the unified discussion. This is the first time they are presented together. The literature of these methods is identified and classified systematically. All procedures of each method have been illustrated by a simple numerical example in detail. This helps the reader understand the basic concept and the characteristic of each method.

Since most methods have not been tested by real-world problems yet, the authors cannot discuss in depth the advantages, disadvantages, computational complexity and difficulty of each method. The appendix contains a bibliography of more than 400 books, journal articles, technical reports and theses on this field of mathematics.

G. Galambos (Szeged)

D. L. Iglehart and G. S. Shedler, Regenerative Simulation of Response Times in Networks of Queues (Lecture Notes in Control and Information Sciences 26). XII+204 pages. Springer—Verlag. New York—Heidelberg—Berlin. 1980.

Discrete event digital simulation of stochastic models is one of the most important practical tools of systems analysis. The real systems are so complex that we are unable to study them analytically and we must, therefore, use computer simulation. This monograph deals with probabilistic and statistical methods for discrete event simulation of networks of queues.

The initial section provides some motivation for study of simulation methods for passage times in networks of queues. Section 2 gives a review of the regenerative method. The authors deal with a specification of the class of closed networks of queues in Section 3 and describe the marked job method in Section 4. Applications of the marked job method can be found in the next section and an extension of this method is the subject of Section 6. Further estimations for the first passage times are described in Sections 7 and 8. The statistical efficiency of the marked job and decomposition methods are studied in the next section. The estimation of passage times in closed networks of queues is the focus of Section 10. The last section is devoted to the algorithms for random number generation.

The presentation is selfcontained, some knowledge of elementary probability theory and stochastic processes is the only requirement from the reader.

Lajos Horváth (Szeged)

Kenneth Ireland and Michael Rosen, A Classical Introduction to Modern Number Theory, Graduate Texts in Mathematics Vol. 84, XIII+341 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

This book is a revised and greatly expanded version of the authors' Elements of Number Theory published in 1972 by Bogden and Quigley. The well selected topics and treatments bridge the gap between elementary number theory and the systematic study of advanced topics. The reader must be familiar with the material in a standard undergraduate course in abstract algebra, but a large portion of the first eleven chapters is understandable with a small amount of suplementary reading. The later chapters assume some knowledge of Galois theory and in the last ones an acquaintance with the theory of complex variables is necessary.

The authors' focus is on topics which point in the direction of algebraic number theory and arithmetic algebraic geometry, without requiring very much technical background. The major themes are the following: Unique factorizations and its applications; reciprocity laws which lead from the quadratic reciprocity to the Artin reciprocity law, one of the major achievements of algebraic number theory; the theory of Gauss and Jacobi sums and its generalizations; diophantic equations over finite fields and over the rational numbers; the Riemann zeta function.

There are also several hundreds of exercises, some routine, some challenging. Some of them supplement the text. In the last chapters a number of exercises is adopted from the recent research literature. Throughout the book there are considerable emphasis on the history of the subject.

This book with its particulary extensive bibliography is highly recommended to research students and to anyone who wants to be familiar with some of the themes and subjects currently under investigations in algebraic number theory and arithmetic algebraic geometry.

Lajos Klukovits (Szeged)

Thierry Jeulin, Semi-Martingales et Groississement d'une Filtration (Lecture Notes in Mathematics, 833), IX+142 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

As the author writes, "probabilists have now fully accepted Doob's idea that the adequate structure for the study of a stochastic process is that of a probability space $(\Omega, \mathcal{A}, P)$ filtered by an increasing family $\mathcal{F}=\{\mathcal{F}_t, t\geqq 0\}$ of σ-fields. While $\mathcal{A}$ represents the whole universe, $\mathcal{F}_t$ consists of events whose outcome is known to the observer at time t, and predictions at time t are conditional expectations $E(\cdot \mid \mathcal{F}_t)$". However, the description of a partially observable random system requires a pair $\mathcal{F}=\{\mathcal{F}_t, t\geqq 0\}$, $\mathcal{G}=\{\mathcal{G}_t, t\geqq 0\}$ of filtrations such that $\mathcal{F}_t\subset\mathcal{G}_t$, $t\geqq 0$. The purpose of this monograph is to construct $\mathcal{G}$ from $\mathcal{F}$ "by forcing information into $\mathcal{F}$" and then to measure "how much the prediction processes relative to $\mathcal{F}$ have been distorted by the new information". Thus the two basic problems dealt with are: 1) Does an $\mathcal{F}$-martingale X remain a $\mathcal{G}$-semi-martingale? 2) If yes then give an explicit decomposition of X into a $\mathcal{G}$-local martingale and a process of bounded variation. After giving the necessary preliminaries in the short first chapter, Chapter 2 is devoted to the discussion of the most general results concerning the first problem. The next three chapters deal, respectively, with initial enlargement of $\mathcal{F}$ (at time 0), with progressive enlargement (when additional random variables are added to the set of stopping times as the time goes on), and with enlargement by adding a single "honest" random variable to $\mathcal{F}$ as an extra stopping time. The sixth chapter deals with concrete applications to Markov processes in general and to the Brownian notion, Brownian excursions and Bessel processes in particular. The book can be recommended to martingale theorists and perhaps also to experts in advanced engineering applications of filtration theory.

Sándor Csörgő (Szeged)

Ole G. J⌐rsboe and Leif Mejlbro: The Carleson-Hunt Theorem on Fourier Series, Lecture Notes in Mathematics 911, IV+123, pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

The Carleson—Hunt theorem: The Fourier series of every function $f\in L^p[-\pi, \pi]$, $1<p\leqq\infty$ converges almost everywhere.

This is the book that should have been published many years ago. Besides the importance of the Carleson—Hunt theorem there are at least two reasons for publishing a book that contains nothing else but the proof of the above theorem. First of all the recent books on Fourier series only quote the theorem but leave out its proof. Since the original articles of Carleson and Hunt were written for specialists, it is necessary to have a treatment available also for mathematicians and students without much knowledge in harmonic analysis. According to this Jørsboe and Mejlbro's book assumes only some rudiments of measure theory and every other concept such as maximal function, Hilbert transform, interpolation of operators etc. and their properties needed in the proof is given in full detail. The second reason is connected with the first one, namely the Carleson—Hunt theorem can hardly be the topic of a regural or special course because of the fine and often very technical details of its proof, therefore it is desirable to have a work which may substitute these courses.

These lecture notes realize the above aims in a perfect way. The presentation is extraordinarily clear, the proof is built up from small steps each of which is proved very carefully. These small steps are united into four main chapters each preceded by a short but very useful description of the chapter's content. At almost every delicate step one can find a remark enlightening the necessity of the given concept or consideration. Nevertheless a warning is in order at this point: the Carleson—Hunt theorem is far from being trivial, so its proof is very hard, especially the material in Chapter 4 is very difficult to read.

In Chapter 1 the authors introduce the Hardy—Littlewood maximal operator and prove a special form of the Marcinkiewicz interpolation theorem. Finally, they prove the Carleson—Hunt theorem under the assumption

$$(*) \qquad \|(\sup_n |S_n(f)|)\|_{L^p} \leqq K_p \|f\|_{L^p} \quad (1<p<\infty,\ f\in L^p[-\pi,\pi])$$

($S_n(f)$ denotes the n th partial sum of the Fourier series of f). The rest one hundred pages is devoted to the proof of $(*)$. Chapter 2 contains some basic facts about the Hilbert transform. In Chapter 3 the necessary technique is introduced: dyadic intervals, modified Hilbert transform, generalized Fourier coefficients. The proof is completed in Chapter 4 by constructing in several steps a set of measure zero such that on the complement the Fourier series is "not very large"

We recommend the book to everybody working in related fields of mathematics as well as to students interested in the subject.

V. Totik (Szeged)

Hua Loo Keng, Introduction to Number Theory, XVII+572 pages with 14 figures, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

This is the English edition of the famous Chinese original first published in 1957. The book is an excellent and broad introduction to the subject and will soon prove itself a very good successor of the classical introductory texbook "An Introduction to the Theory of Numbers" by G. M. Hardy and E. M. Wright. Several recent results in number theory appear in such a form as to make this textbook suitable for teaching purposes. This English edition contains additional notes compiled by Wang Yuan and Peter Shiu (the translator) at the end of nearly all chapters. These enable the reader to acquint himself with the current research literature. In the running text there are several examples and exercises to help the deeper understanding.

A great value of this book is that the author tries to highlight certain connections of elemantary number theory to other branches of mathematics. For example: the relationship between the prime number theorem and Fourier series; the partition problem, the four squares problem and their relationship to modular functions; the theory of quadratic forms, modular transformations and their connections to Lobachevskian geometry, etc.

The book, which serve not only as a textbook but a fundamental reference work, contains the following main topics: The elementary proof of the prime number theorem due to Erdős and Selberg; Roth's theorem; Gelfond's solution to Hilbert's seventh problem; Siegel's theorem on the class number of binary quadratic forms; Linnik's proof of the Hilbert—Waring theorem; Selberg's sieve method and Schnirelman's theorem on the Goldbach problem; Vinogradov's result concerning least quadratic non-residues. In addition, some of the author's own work is represented, too.

Lajos Klukovits (Szeged)

S. M. Khaleelulla, Counterexamples in Topological Vector Spaces, Lecture Notes in Mathematics 936, XXI+179 pp, Springer-Verlag, Berlin—Heidelberg—New York, 1982

"During the last three decades much progress has been made in the field of topological vector spaces. Many generalizations have been introduced... To justify that a class C_1 of topological vector spaces is a proper generalization of another class C_2..., it is necessary to construct an example of a topological vector space belonging to C_1 but not to C_2; such an example is called a counterexample". The book contains more than two hundred examples of this kind. Very often the same one (e.g. L^p $(0<p<1$ or $p>1)$, $C[0, 1], l^\infty, l^1, C_0$ etc.) works in different situations by which several interesting properties are displayed for the most frequently used Banach spaces and topological linear spaces. The examples treated in the book range from perfectly trivial ones (e.g. "A bounded sequence in a topological vector space which is not convergent") to more sophisticated constructions. The hardest counterexamples are only recorded (with a reference) without proof or construction (e.g. Enflo's separable Banach space without basis).

The material is arranged in a clear way. It was a good idea to name the examples fully in the "Contents"; this helps in finding the needed constructions. The book is divided into eight chapters. Each chapter begins with definitions and some basic theorems, there is always a reference pointing to the source of the quoted results. The examples themselves are presented in a legible form, although the author very often leaves out the verification that they do work, and in many cases this constitutes the hardest part of the job. Detailed index and bibliography help the reader in remembering the concepts and in further study. The content of the chapters are as follows:

1) Topological vector spaces (general properties), 2) Locally convex spaces, 3) Special classes of locally convex spaces, 4) Special classes of topological vector spaces, 5) Ordered topological vector spaces, 6) Hereditary properties, 7) Topological bases, 8) Topological algebras.

These lecture notes should be used as a reference book but it may also be useful for anyone who is searching for the definition of a concept or even for a beginner who, while reading it, may get a quick glance of the most important facts of the topic.

V. Totik (Szeged)

A. A. Kirillov and A. D. Gvishiani, Theorems and Problems in Functional Analysis. Problem Books in Mathematics, IX+347 pages with 6 illustrations, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

This is a translation of a Russian edition (1979). Its aim is to give a self-contained introduction to modern branches of functional analysis. It is a combination of a textbook and a problem book with detailed hints for solving the problems.

The book is divided into three parts: Theory, Problems and Hints. The chapters are subdivided into sections and the sections into subsections each containing 23 exercieses, so altogether 828 problems are posed. Many of these require only minimal skill but there are a lot of harder problems that may be nontrivial even for an expert in the field. A rough table of contents: Set theory and topology, measures and integrals, linear topological spaces and linear operaors, elements of harmonic analysis and the spectral theory of operators. In the first part — on more than 130 pages — a brief account of the most important aspects of the theory is given with complete proofs. This part may be used in a one year course, although the presentation is very concise and brief. The problem part begins with a simple exercise about equivalence relations and ends with the spectral decomposition of the selfadjoint extension of the operator $A=-(d^2/dx^2)+x^2$ with initial domain $D(A)=S(R)$. Many standard results are incorporated as exercises such as Lebesgue's density theorem, Hölder's inequality, the Stone—Weierstrass theorem etc. The problems concerning up-to-date topics such as category theory, generalized functions, characters on Abelian groups etc. provide a smooth path to these advanced matters. The hints are sufficient for working out complete solutions.

List of notation and detailed index increase the utility of the book which should be on the bookshelf of every lecturer on functional analysis and surely will enjoy great success among students, as well.

V. Totik (Szeged)

Anders Kock, Synthetic Differential Geometry (London Mathematical Society Lecture Note Series, 51), VI+311 pages, Cambridge University Press, 1981.

Synthetic differential geometry, in the sense of the book, is the theory of general differential manifolds based on the assumption of sufficiently many nilpotent elements on the "real line". The first part of the book containes a detailed exposition of the differential and integral calculus on these manifolds such as directional derivatives, Lie derivation, forms and currents, Stokes' theorem etc. In the following part categorial logic is introduced into the exposition, and in the last part several models are presented in order to compare the synthetic theory with the analytic one.

The book assumes some knowledge on abstract algebra and category theory. It is recommended to graduate students and professionals who are interested in algebraic or differential geometry or category theory.

Z. I. Szabó (Szeged)

A. I. Kostrikin, Introduction to Algebra, Universitext, XIII+575 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

This is the translation of a textbook of the present undergraduate algebra course at Moscow State University. The book reflects the Soviet approach to teaching mathematics with its emphasis on applications and problem-solving. In the first place, Kostrikin's textbook motivates many of the algebraic concepts by practical examples. For instance, the heated plate problem, coding information and the states of a molecule are used to introduce linear equations and finite fields, systems of equations over finite fields and groups and group representations, respectively. In the second place, there are a large number of exercises so that the reader can convert a vague passive understanding to active mastery of the new ideas. The harder problems have hints at the end of the book. This helps those who learn algebra outside of the framework of an organized course. In the third place, there are topics in it which are usually not part of an elementary course but which are fundamental in applications.

The book consists of two parts (Sources of algebra and Groups, Rings, Modules) and nine chapters. The first three chapters constitute an introduction to elementary linear algebra: sets, mappings, integer arithmetic, vector spaces and matrices over the field of real numbers, linear maps, systems of linear equations, determinants. The later three chapters of part one deal with groups, rings and fields, complex numbers and polynomials and roots of polynomials. In part two the reader can find more about groups (classical groups of low dimensions, group theoretical constructions, the Sylow theorems and the fundamental theorem for finite abelian groups), the elements of the group representation theory (unitary, reducible, linear and irreducible representations) as well as more about fields, rings and modules, including a section on algebras over a field.

This valuable textbook is warmly recomended to undergraduate students, as well as to anyone who wants to be familiar with basic abstract algebra and certain applications of it.

Lajos Klukovits (Szeged)

Logic Year 1979—80. The University of Connecticut, Proceedings, edited by M. Lerman, J. H. Schmerl and R. I. Soare, Lecture Notes in Mathematics 859, VIII+326 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

In 1979—80 the Mathematics Department of the University of Connecticut spensored a special year devoted to Mathematical Logic with emphasis on recursion theory and model theory. During this year a conference took place, November 11—13, 1979, with 80 participants. The papers in this volume have been based on talks presented at the conference or on seminar presentations held during the course of the year.

The majority of the 21 papers in this volume is devoted to various problems of degrees and hierarchy of recursivity which seems to be one of the blossoming branches of mathematical logic. Authors and titles from this topic: R. L. Epstein, R. Haas and R. L. Kramer: Hierarchies of sets of degrees below $0'$; P. A. Fejer and R. I. Soare: The plus-cupping theorem for the recursively enumerable degrees; S. D. Friedman: Natural α-RE degrees; C. G. Jockush: Three easy constructions of recursively enumerable sets; P. G. Kolaitis: Model theoretical characterizations in generalized recursion theory; M. Lerman: On recursive linear orderings; A. Macintyre: The complexity of types in field theory; D. P. Miller: High recursively enumerable degrees and the anti-cupping property; Y. N. Moschovakis: On the Grilliot—Harrington—MacQueen theorem; R. A. Shore: The degrees of unsolvability: global results. Without striving for completeness let us mention two further authors. M. Makkai writes about a construction that associates a certain new topos, the prime completion, with any coherent topos. T. Millar gives a necessary and sufficient condition for a universal theory to have a complete, decidable model completion and applies this result to an example concerning recursively saturated models.

V. Totik (Szeged)

Robert Lutz and Michel Goze, Nonstandard Analysis. A Practical Guide with Applications, Lecture Notes in Mathematics 881, XIV+261, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

"This book is intended to enable the reader to use Non Standard Analysis by himself without fear, at any level of mathematical practice, from undergraduate analysis to important research areas." The necessity of an introductory work with this scope is obvious: if Nonstandard Analysis wants to be a useful tool in proving theorems or in applications then it must not assume the user to be familiar with all the model theory necessary for its rigorous foundation. As a matter of fact engineers, physicists etc. have been constantly using infinitesimals even before — remember e.g. the tricks which were applied during many university lectures on theoretical physics — nevertheless an "easy" "how to do" treatment would attract many mathematicians, since for most of them Nonstandard Analysis is rather mistery than part of mathematics. Unfortunately this book seems to have failed to accomphish its goals. While reading these notes a beginner would probably feel having got lost in the "swindles" (Lutz—Goze's terminology) of NSA. Instead of keeping a strict distinction between "real" and "extension" the authors quickly drop the stars of the transferred objects and after 10—15 pages the reader is completely ignorant of what may and may not be done in NSA. Detailed proofs and simple remarks concerning them would have helped much in understanding the material. Nevertheless Lutz and Goze's book may be a great help for those who are familiar with the elements of the nonstandard method but are unaware of the many possibilities it can grant in applications.

The lecture notes consist of four chapters. In chapter one the "elementary practice of Non-Standard Analysis" is introduced, many classical results are reviewed in the nonstandard framework. The second chapter is devoted to the logical foundations of NSA, however this introduction is far from being complete and hardly enlightens the mind of the confused inexperienced novice. The last two chapters have already the flavor of genuine applications. In Chapter III some classical topics such as integral curves of vector fields, compactness, holomorphic functions etc. are treated from a nonstandard point of view, while in the fourth chapter NSA methods are applied to perturbation problems in algebra and differential equations. Author index, glossary and the authors' good sense of humor help in reading the book.

V. Totik (Szeged)

George E. Martin, The Foundations of Geometry and the Non-Euclidean Plane, (Undergraduate Texts in Mathematics) XVI+509 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

The book is the second edition [originally published: New York: Intext Educational Publishers, 1975] of a text written for juniour, senior, and first-year graduate courses. After four introductory chapters the book starts with an axiomatic development of absolute geometry, which is a common ground between non-Euclidean and Euclidean geometries as it is independent of any assumption about parallel lines. Many models, including the Cartesian plane, are used to illustrate this system of axioms, and it is shown that this system together with one of the equivalents of Euclid's parallel postulate forms a categorical system. Part two is a very elegant development of the Bolyai-Lobachevsky geometry using many results of this theory for the study of euclidean geometry.

The text is self-contained and it is written in a very clear, enjoyable style. Beside historical materials it containes over 650 exercises, 30 of which are true-or-false questions.

Z. I. Szabó (Szeged)

J. Martinet, Singularities of Smooth Functions and Maps, (London Mathematical Society Lecture Note Series 58), XIV+256 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sidney, 1982.

The book consist of seventeen chapters which are ordered into four parts. The material of the book is based on a seminar held at the University of Michigan and a course at the Pontificia Universidade Catolica (Rio de Janeiro). The text gives a very good and significant choice from the rich subject covering the most important results and problems of the singularity theory of differentiable functions. Part one introduces the main idea by means of detailed exposition of numerous examples. Part two is devoted to the differentiable preparation theorem. In part three the preparation theorem is applied to the theory of universal deformations of function germs, by the aid of which the classification of Thom's "elementary catastrophes" is presented. Part four deals with singularities of differentiable mappings. In this part most of Mather's results are stated in their local version. For understanding the text familiarity in the basic ideas about Lie groups, modules over commutative rings and existence and uniqueness theorems for solutions of differential equations are needed.

László Gehér (Szeged)

Martingale Theory in Harmonic Analysis and Banach Spaces (Proceedings, Cleveland, Ohio, 1981), Edited by J.-A. Chao and W. A. Woyczyński (Lecture Notes in Mathematics, 939), VIII+225 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

The conference indicated in the title was held at Cleveland State University between July 13—17, 1981. Professor D. L. Burkholder was the principal speaker at the meeting and delivered a series of ten lectures. His lecture notes will appear separately in the NSF—CBMS Conference Series of the American Mathematical Society. The present volume contains papers submitted by other conference participants.

The table of contents: 1. D. Allinger: A note on strong, non-anticipating solutions for stochastic differential equations: when is path-wise uniqueness necessary? 2. K. Bichteler and D. Fonken: A simple version of the Malliavin calculus in dimension one. — Both papers are devoted to the study of stochastic differential equations. 3. H. Byczkowska and A. Hulanicki: On the support of the measures in a semigroup of probability measures on a locally compact group. 4. J.-A. Chao: Hardy spaces on regular martingales. — This paper mainly treats the generalized Walsh—Fourier series. 5. B. Davis and J. L. Lewis: The harmonic measure of porous membranes in $\mathbf{R}^3$. 6. G. A. Edgar, A. Millet and L. Sucheston: On compactness and optimality of stopping lines. — This is a survey type paper containing results both to discrete and continuous parameter processes. 7. N. A. Ghoussoub: Martingales of increasing functions. 8. J. A. Guttierez and H. E. Lacey: On the Hilbert transform for Banach space valued functions. — The authors extend some results of C. Fefferman and E. M. Stein, and G. Pisier. 9. A. T. Ławniczak: Gaussian measures on Orlicz spaces and abstract Wiener spaces. 10. C. Mueller: Exit times of diffusions. 11. C.W. Onneweer: Generalized Lipschitz spaces and Herz spaces on certain totally disconnected groups. — The absolute convergence of Fourier series of functions belonging to a generalized Lipschitz (=Besov) space and embedding theorems for Herz — and Lorentz spaces are studied. 12. C. Park: Stochastic barriers for the Wiener process and a mathematical model. 13. G. Pisier: On the duality between type and cotype. — Those X Banach spaces are studied, for which X is of type p iff X^* is of cotype p' with $1/p+1/p'=1$. 14. L. H. Riddle and J. J. Uhl: Martingales and the fine line between Asplund spaces and spaces not containing a copy of l_1. — The following theorem of Rosenthal is the starting point: A Banach space X contains no copy of l_1 iff every bounded sequence in X has a weakly Cauchy subsequence. 15. J. Rosiński: Central limit theorems for dependent random vectors in Banach spaces. — This is a relatively large survey paper. 16. J. Rosiński and J. Szulga: Product random measures and double stochastic integrals. 17. W. H. Ruckle: Absolutely divergent series and Banach operator ideals. 18. G. Schechtman: Lévy type inequality for a class of finite metric spaces. — This short note is a variation on the theme of B. Maurey, but the proof is somewhat simpler and more general. 19. W. A. Woyczyński: Asymptotic behavior of martingales in Banach spaces II. — The pesent note is a continuation of a work by the same author, and concentrates on the Marczinkiewicz—Zygmund and Brunk's type strong laws of large numbers for martingales.

The book gives a good account of the present stage of the subject. It will certainly stimulate some of the readers to make research in this interesting field. We warmly recommend it to everybody who works either in Martingale Theory and/or in Abstract Harmonic Analysis.

F. Móricz (Szeged)

William S. Massey, Singular Homology Theory (Graduate Texts in Mathematics, Vol. 70) XII+265 pages, Springer-Verlag New York—Heidelberg—Berlin, 1980.

This book gives a systematic treatment of singular homology and cohomology theory. The author has tried to show all the standard results without unnecessary technical details and difficulties as long as it is possible. His program has been crowned with success.

Clear geometric motivation is given in and out of the first chapter devoted to the background for homology theory. Singular cubes are used rather than singular simplexes. It simplifies the proof of the invariance of the induced homomorphisms under homotopies since the product of a cube with the unit interval is a cube. Furthermore, the subdivision of a cube is simpler than the barycentric subdivision of a simplex. An appendix contains De Rham's theorem.

Massey considers his book as a sequel to his previous book in this Springer-Verlag series (Vol. 56) entitled "Algebraic Topology: An Introduction". Although the book does not really require any knowledge given in the previous one, it seems to be a textbook for a second course in algebraic topology rather than a first course since many technical details are left to the reader.

Prerequisite mathematical knowledge is as follows: minima of general topolgy and the theory of abelian groups, something about manifolds and fundamental groups, tensor product, Tor and Ext functors.

A last argument to prove that the author is right in his program is that his book is essentially shorter than other ones treating the same subject.

L. A. Székely (Szeged)

Richard S. Millman and George D. Parker, Geometry. A metric Approach with Models (Undergraduate Texts in Mathematics) VIII+355 pages, Springer-Verlag, New York—Heidelberg—Berlin 1981.

Birkhoff's metric approach to classical geometries means the use of real numbers at the building of several theories. The book develops the theory of neutral (absolute) geometry, hyperbolic geometry and of Euclidean geometry by this method. The various axioms are introduced slowly and the definitions and theorems with models, ranging from the Cartesian plane to the Poincaré upper half plane, the Taxicab plane and the Moulton plane, illustrate further these axioms. The last two chapters develop the concept of area resp. the theory of isometries in neutral geometry. Bolyai's beautiful theorem, asserting that if two poligonal regions have the same area then one can be cut up into a finite number of pieces and reassembled to form the other, is also proved here.

The book contains more than 700 problems in the exercise sets. It is an excellent introduction. It is addressed to undergraduate students and is warmly recommended to everyone who wants to make a quick acquaintance with classical geometries.

Z. I. Szabó (Szeged)

P. G. Moore, Principles of Statistical Techniques. Second Edition, VIII+288 pages, Cambridge, University Press, Cambridge—London—New York—Melbourne, 1979.

The first edition of this book was published in 1958, and it was reprinted in 1964. The second edition appeared in 1969, it was reprinted in 1974, and the nice pocket-size version under review is the first paperback edition. Such a story reflects a considerable success, and the book appears deserving it. Its longer subtitle describes it rather completely: "A first course, from the beginnings, for schools and universities, with many examples and solutions". It does not really require any mathematical prerequisites. Anybody graduating from a secondary school could, or should understand it. Nevertheless, the author provides a rather wide selection of effective tools of statistics so the the reader can tackle a whole variaty of concrete situations. The basic techniques of collection, tabulation and pictorial representation of data, of sampling and averaging; dispersion measures, tests of significance and time series are all explained through numerical examples.

The style is very nice. It sometimes represents an old world. The reviewer, for example, would find it difficult to ask his students to "catch a large number of specimens of a common species of butterfly and measure the length of the right wing of the butterflies. Do this on a number of occasions over the season...". However, not all the data is required to collect from the reader. The author has many interesting data sets of his own to work with. An earlier reviewer, cited on the cover, was right to write that "the book should prove useful to all who read it".

Sándor Csörgő (Szeged)

Jacob Palis, Jr. and Welington de Melo, Geometric Theory of Dynamical Systems. An Introduction, XII+198 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

This book is concerned with differential equations on manifolds from a global or topological point of view. Its purpose is to aquaint the reader with two central topics of the modern period of the geometric theory of dynamical systems: structural stability and genericity.

If a differential equation describes the evolution of a system, then, obviously, it cannot be supposed to be an absolutely correct model, e.g. the parameters of the system appearing in the differential equation cannot be given exactly. However, the user wishes to ensure the qualitative conclusion he draws from the equation at hand to be valid for the equation really describing his world. Probably this inspired Andronov and Pontrjagin (first of them was an engineer!) to introduce the concept of structural stability in 1937. Roughly speaking, they called a differential equation structurally stable if the differential equations near to it in a suitable metric on the space of all differential equations have the same phase portrait. 20 years later M. Peixoto proved that structurally stable differential equations form an open and dense set in the space of differential equations whose right-hand sides are defined on a compact 2-dimensional manifold, i.e. here almost all differential equations are structurally stable.

A property is said to be generic if it is satisfied by almost all differential equations. As it was defined by S. Smale, the main objective in the geometric theory of differential equations is the search for generic and stable properties.

The book gives the reader the flavour of this theory on an introductory level. The first chapter establishes the concepts and basic facts on differentiable manifolds and vector fields needed for understanding later chapters. The second chapter gives a systematic proof of the Hartman—Grobman Theorem, which says that local stability is a generic property. The same problem for periodic orbits is considered by the Kupka—Smale Theorem in Chapter 3. The last chapter is devoted to the proof of Peixoto's Theorem. There are a great number of interesting exercises of various difficulty.

We recommend this excellent book for mathematicians and students who want to get aquainted with this modern and fast developing branch of mathematics.

L. Hatvani (Szeged)

Steve Smale, The Mathematics of Time (Essyas on Dynamical Systems, Economic Processes, and Related Topics), VI+151 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1980.

In the preceding review on Palis and Melo's book we tried to sketch what is geometrical theory and global analysis of dynamical systems. Undoubtedly, one of the most important people of this theory is Stephen Smale. For his outstanding research in differential topology and in global analysis he was awarded the Fields Medal of the International Mathematical Union in 1966.

The first piece in this collection of his earlier papers and addresses is his celebrated paper on "Differentiable dynamical systems" published originally in *Bulletin of the American Mathematical Society* in 1967. The Notes following the body of its new edition are of the greatest interest, where the author completes his "classical" work with up-to-date results and gives a riport on the history of the problems and conjectures he proposed in his original paper. In the second half of the book one can find some expository essays and addresses: What is global analysis?; Stability and genericity in dynamical systems; Personal perspectives on mathematics and mechanics; Dynamics in general equilibrium; Some dynamical questions in mathematical economics; On the problem of reviving the ergodic hypotheses of Boltzmann and Birkhoff. The book is concluded by personal confessions: "On how I got started in dynamical systems". The reader can get acquainted with such "intimacies" from the author's life as how he, as a topologist, entered into the mathematical world of ordinary differential equations; how "extraordinarily" he was impressed to meet the "powerful group of four young mathematicians: Anosov, Arnold, Novikov and Sinai in Moscow".

This nice book is of interest not only to topologist and global analysists, but also to those whose primary fields are applied mathematics, differential equations, physics, or mathematical economics.

L. Hatvani (Szeged)

Sudhakar G. Pandit—Sadashiv G. Deo, Differential Systems Involving Impulses (Lecture Notes in Mathematics, 954), VII+102 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

In the classical analysis of differential systems the right-hand side is assumed to be continuous or integrable, so the solutions are continuous functions. However, in many physical problems the right-hand side of the modelling differential equation involves some perturbations of discontinuous behaviour. For example, the bang-bang principle in the optimal control theory shows that the parameters often have to change in an impulsive manner. Biological systems (heart beats, models for biological neural nets) exhibit an impulsive behaviour, too. These systems are described by so called "measure differential equations". The derivative involved in these equations is the distributional derivative, the solutions are functions of bounded variations. Consequently, the methods of classical analysis are not sufficient to describe the impulsive behaviour of systems.

These notes give a good unified survey on the results from several research papers published during the last fifteen years dealing with the basic problems such as existence, uniqueness, stability, boundedness and asymptotic equivalence associated with measure differential equations.

L. Hatvani (Szeged)

Probability Measures on Groups. Proceedings, Oberwolfach, Germany, 1981. Edited by H. Heyer (Lecture Notes in Mathematics, 928), X+477 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

This collection contains the text of 22 lectures presented at the Sixth Conference in the series "Probability Measures on Groups" held at the Mathematisches Forschunginstitut, Oberwolfach, Germany, June 28—July 4, 1981. The subjects of this meeting cover various areas of stochastics and analysis including probability theory and potential theory on algebraic-topological structures as well as their interrelations with the structure theory of locally compact groups, Banach spaces and Banach lattices. The Editor of this volume classified the papers into four groups: (i) Probability measures on groups, semigroups and hypergroups, (ii) Stochastic processes with values in groups, (iii) Connections between probability theory on groups and abstract harmonic analysis, (iv) Applications of probability theory on algebraic-topological structures to quantum physics.

Lajos Horváth (Szeged)

Processes Aléatoires à Deux Indices (Colloque E. N. S. T. et C. N. E. T., Paris 1980), Edité par H. Korezlioglu, G. Mazziotto et J. Szpirglas (Lecture Notes in Mathematics, 863), IV+274 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

This volume contains the texts of the talks made at the Conference "Two-parameter Stochastic Processes" held in Paris on June 30 and July 1, 1980, under the support of "l'École Nationale Supérieure des Télécommunications" (E.N.S.T.) and "Centre National d'Études des Télécommunications" (C.N.E.T.).

The table of contents: 1. P. A. Meyer; Théorie élémentaire des processus à deux indices. — This is a nice introduction into the subject, embracing the main notions and results up to the decomposition theorems of martingales with indices in $\mathbf{R}_+ \times \mathbf{R}_+$ (continuous case) and $\mathbf{N} \times \mathbf{N}$ (discrete case) and the stochastic integrals. 2. D. Bakry: Limites "quadrantales" des martingales. — This talk closely attaches to the previous one by Meyer. 3. A. Millet: Convergence and regularity of stong submartingales. 4. G. Mazziotto, E. Merzbach et J. Szpirglas: Discontinuités des processus croissants et martingales à variation intégrable. 5. G. Mazziotto et J. Szpirglas: Sur les discontinuités d'un processus cad-lag à deux indices. 6. J. Brossard: Régularité des martingales à deux indices et inégalités de normes. — This is a good summarization of the methods how to obtain moment inequalities foı the maximum partial sum and the martingale square function. 7. M. Ledoux: Inégalités de Burkholder pour martingales indexées par $\mathbf{N} \times \mathbf{N}$. 8. D. Nualart: Martingales à variation indépendante du chemin. 9. M. Zakai: Some remarks on integration with respect to weak martingales. — This gives interesting contributions to stochastic integration in the plane. 10. M. Dozzi: On the decomposition and integration of two-parameter stochastic processes. — While the previous talk treats weak martingales, this one does strong martingales. 11. J. B. Walsh: Optimal increasing paths. — Among other things, the author proves some Fatou type theorems concerning fine and nontangential limits of biharmonic functions at the distinguished boundary of a bicylinder. 12. D. Nualart and M. Sanz: The conditional independence property in filtrations associated to stopping lines. 13. X. Guyon et B. Prum: Identification et estimation de semi-martingales représentables par rapport à un Brownien à un indice double. 14. A. Al-Hussaini and R. J. Elliott: Stochastic calculus for a two parameter jump process. — The authors obtain some new formulae, which cannot be written as special cases of those for the two parameter Wiener or Poisson processes. 15. H. Korezlioglu, P. Lefort et G. Mazziotto: Une propriété markovienne et diffusions associées.

The book collects together materials that have been widely scattered in the literature, and is likely to be of special interest to those who work on the field of Stochastic Processes endowed with a partially ordered index set.

F. Móricz (Szeged)

Elmer G. Rees, Notes on Geometry (Universitext), VIII+109 pages with 99 figures, Springer-Verlgag, New York—Heidelberg—Berlin 1983.

There are several ways to introduce the classical geometries into university syllabuses. The most general method is the so-called axiomatic method which is in some cases rather cumbersome and not very informative. The present book shows how to give an introduction to geometries that is short and nevertheless is of rich content, taking a concrete viewpoint rather than an axiomatic one.

In the first part the Euclidean geometry is considered with a detailed examination of isometries and crystals. Projective geometry (Part II) and hyperbolic geometry (Part III) are treated from the point of view of Felix Klein's Erlanger Programme, supplemented with some topological aspects.

There is a large number of exercises throughout the notes, many of these are staightforward and are meant to test the reader's understanding.

The book is recommended to undergraduate studens and to teachers of elementary geometry.

Z. I. Szabó (Szeged)

G. F. Roach, Green's Functions, XIV+325 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sidney, 1982.

The first edition of this book was published in 1970. The author's aim was to give a self-contained and systematic introduction to the theory of Green's functions. The success of the book is shown by this new edition.

In my opinion the advantage of this work is that it gives for scientists a mathematically thoroughly developed tool to the investigation of boundary value problems associated with either ordinary or partial differential equations, and, at the same time, for postgraduate students a clear and well-motivated exposition of the problem showing also the necessity of the generalization of some notions (e.g. Riemann integral-Lebesgue integral).

Two new chapters (Calculations of particular Green's functions and approximate Green's functions) and four appendices (Summary of the Green's function method, Operators and expressions, The Lebesgue integral, Distributions) have been added in this edition. Especially the new appendices are very useful for those readers who lack the necessary mathematical background to understand more advanced accounts. (The other chapters are: The concept of a Green's function, Vector spaces and linear transformations, Systems of finite dimension, Continuous functions, Integral operators, Generalized Fourier series and complete vector spaces, Differential operators, Integral equations, Green's functions in higher-dimensional spaces.)

Summarizing, this is a well-written text giving the reader a picture how notions, proofs, and applications arise in this field.

L. Pintér (Szeged)

Derek J. S. Robinson, A Course in the Theory of Groups, Graduate Texts in Mathematics 80, XVIII+481 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

This book is an excellent up-to-date introduction to the theory of groups. It is general yet comprehensive, covering various branches of group theory. The fifteen chapters contain the following main topics: free groups and presentations, free products, decompositions, Abelian groups, finite permutation groups (including the Mathieu groups), representations of groups, finite and infinite soluble groups, group extensions, generalizations of nilpotent and soluble groups, finiteness properties.

The reader is expected to have at least the knowledge and maturity of a graduate student who has completed the first year of study at a North American university or of a first year research student in the U.K. He or she should be familiar with the more elementary facts about rings, fields and modules, possess a sound knowledge of linear algebra and be able to use Zorn's Lemma and transfinite induction. However, no knowledge of homological algebra is assumed. There are some 650 exercises, found at the end of each section. They must be regarded as an integral part of the text.

This book is highly recommended everybody who wants to read research texts in more specialized areas of groups theory.

Lajos Klukovits (Szeged)

Klaus Schittkowski, Nonlinear Programming Codes, (Lecture Notes in Economics and Mathematical Systems, 183) VIII+242 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

In recent years a lot of effort has been made to implement efficient and reliable optimization programs for the solution of complex nonlinear systems. The author undertook collect many programs developed by various researchers, and so gives a designer the possibility to decide which optimization program could solve his problem in the most desirable way.

A reader who is interested in selecting a program for the numerical solution of his problem should start with Chapter I where the problem is formulated. Chapter II gives the mathematical background of the different methods. Chapter III is divided into two sections. The first one contains a table with different technical details (program length, the original precision of the program etc.). More detailed information is contained in the second section where all the programs are described individually. Chapter IV shows how test problems with predetermined solutions are generated. In Chapter V the reader finds different performance criteria (efficiency, global convergence, and the performance examinations). Final conclusions and some technical remarks are gathered in Chapter VI.

The two appendices contain the numerical data for constructing test problems and a sensitivity analysis.

G. Galambos (Szeged)

Laurent Schwartz, Geometry and Probability in Banach Spaces, Notes by Paul R. Chernoff (Lecture Notes in Mathematics, 852,) X+101 pages, Spinger Verlag, Berlin—Heidelberg—New York, 1981.

These Notes correspond to a course of lectures which was given by Prof. Laurent Schwartz at the University of California, Berkeley, in April-May 1978. It is Prof. Paul Chernoff who gives here a good account of these lectures.

The book summarizes a great number of new results, many of them found by mathematicians of the French school, in particular by Laurent Schwartz, Bernard Maurey, and Gilles Pisier. These results cover relationships between geometrical properties, properties of functional analysis, and probabilistic properties in Banach spaces. The present subject turns around the L^r spaces, $1 \leqq r \leqq +\infty$.

The book contains 19 lectures arranged into four chapters and a new result of Pisier.

Ch. 1 gives a rapid account of the main ideas of the book, presents the Pietsch factorization theorem with applications, etc.

Ch. 2 is devoted to the study of cylindrical probabilities and radonifying maps, in particular, to P. Levy's p-stable laws, p-Pietsch spaces, the continuity and Hölder properties of the Brownian motion.

Ch. 3 is entitled by "Types and Cotypes". Let $\{\varepsilon_n : n \in \mathbf{N}\}$ be independent random variables, with values ± 1 with probability 1/2. A Banach space E is said to be of type p, $1 \leqq p \leqq 2$, it $\sum_n |x_n|^p < \infty$ implies that $\sum_n \varepsilon_n x_n$ is almost surely convergent; of cotype q, $2 \leqq q \leqq +\infty$, if the almost sure convergence of $\sum_n \varepsilon_n x_n$ implies $\sum_n |x_n| < +\infty$; where $x_n \in E$, $n \in \mathbf{N}$. It is remarkable that, for $1 \leqq r \leqq 2$, L^r is of type r and cotype 2, and nothing better; for $2 \leqq r < +\infty$, type 2 and cotype r, and nothing better; while L^∞ is very bad, it is of only type 1 and cotype $+\infty$.

Ch. 4 is the longest part of the book, mainly dealing with the questions in connection with ultrapowers and superproperties. Given a property P of Banach spaces, P is said to be a superproperty if two Banach spaces E and F are such that E has P and F is finitely representable in E, then F also has P. A number of interesting results are treated in this chapter: the Maurey and the Grothendieck factorization theorems, the nonexistence of $(2+\varepsilon)$-Pietsch spaces, the results of Pisier on martingale type and cotype, etc.

The presentation is rather tight. Some proofs are omitted, some are merely outlined. Practically there is no bibliography in the text. The "Séminaires de l'École Polytechnique" are indicated as general references.

This thin book is the first attempt to collect the main ideas of the new branch of mathematics which deals with the functional-analytic, geometric and probabilistic properties of Banach spaces. We warmly recommend it firstly to those who have some acquaintance with this heavy but fascinating subject.

F. Móricz (Szeged)

Zbigniew Semadeni, Schauder Bases in Banach Spaces of Continuous Functions, Lecture Notes in Mathematics 918, IV+136 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

Zbigniew Semadeni is the author of a successful monograph "Banach Spaces of Continuous Functions". This fact and the booming interest in bases of Banach spaces guarantee the success of these lecture notes. To tell the truth, the author might have written a more complete, more attractive monograph by including the proofs of the most important "hard theorems" on bases in $C(X)$ (e.g. Karlin's theorem about the non-existence of unconditional bases in $C(0, 1]$ or Pelczynski's related theorem; Olveskii's result that a uniformly bounded orthonormal system cannot be a basis in $C[0, 1]$ etc.), but the book, as it stands, is a good introduction into the topic (concerning bases in more general Banach spaces we mention the books "Bases in Banach spaces I—II" by I. Singer (Springer) and "An Introduction to Nonharmonic Fourier Sieres" by R. M. Young (Academic Press)). Many references and a detailed bibliography help the interested reader to proceed on to finer topics. The style is that of a text-book and the book is more or less self-contained (it is a bit embrassing that the definition of "uniform cross-norm" and "tensor product" is not given). It contains several exercises, although these require mostly standard manipulations. The proofs are clearly presented but the nature of the material does not allow quick reading: it is not at all an easy task to catch up with a consideration about or construction of a pyramidal basis in higher dimension.

The lecture notes are divided into four chapters. The first one is a general introduction into the properties of bases in Banach spaces. Only the most important topics are treated: duality, stability and some properties of unconditional bases.

Chapter 2 contains the "classical part" of the book: the broken-line construction in one variable. The most frequently investigated systems of Haar, Faber—Schauder, Walsh and Franklin are introduced here. The definition of the Haar—system is somewhat misleading since the values at jump points are indifferent only if we consider the Haar functions as the elements of L^∞ and the author mentiones also the uniform convergence of the Haar expansion to a continuous function.

Chapter 3 is devoted to the multidimensional case. Everybody reading the previous chapter will feel that the same might be done in higher dimension but to write the construction down is another thing. Although this chapter is not very attractive, the author has good reasons for dealing with the higher dimensional case so lengthy: "For two (or perhaps even four) decades it has been known how to construct... bases, consisting of certain spline functions... In spite of the regularity of the construction of these bases and their nice properties, they have not yet attracted people working in numerical methods. A reason... may be that in the existing literature the descriptions... are geometrical, ..., without explicit formulas...". In Chapter 3 pictures help to follow the construction, and formulas are given for the coefficients etc. In the last paragraph the celebrated results of Ciesielski—Shoenfield and Bockariev concerning bases in $C^k[0, 1]^d$ and A are sketched.

The material in Chapter 4 is quite new. A detailed proof is given for the existence of monotone bases in separable spaces $C(X)$ (the solution of the basis problem in these spaces) and several interesting extensions of the mentioned Olevskii result are listed. It is a pity that the proofs of these last theorems are left out.

V. Totik (Szeged)

Set Theory and Model Theory, Proceedings, Bonn 1979, edited by R. B. Jensen and A. Prestel, Lecture Notes in Mathematics 872, IV+174 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

On the occasion of Gisbert Hasenjaeger's 60-th birthday a symposium on set theory and model theory was held at Bonn, 1979 June 1—3. All of the contributors to these proceedings are former students and co-workers of Professor Hasenjaeger, and the papers are all dedicated to him. K. J. Devlin presents a new morass construction which leads to Souslin and Kurepa K_2-trees as limits of directed systems of countable trees. H. D. Donder shows how coarse morasses in L can be used to answer combinatorial questions in L, e.g. how Kurepa trees with additional properties can be obtained using the "natural" global coarse morass in L. S. Koppelberg reveals several properties of the partially ordered set of isomorphism type structures of complete Boolean algebras, such as their being distributive lattices with Stone and Heyting algebras as duals. A. Prestel introduces a suitable definition of pseudo real closed (prc)-fields and shows, among others, that with this definition every algebraic extension of a prc-field is again a prc-field. Finally, T. von der Twer simplifies Paris and Harrington' famous proof concerning the incompleteness of Peano's arithmetic by avoiding probabilities in PA.

V. Totik (Szeged)

Statistique non Parametrique Asymptotique, Proceedings, Rouen, France 1979. Edité par J. P. Raoult (Lecture Notes in Mathematics, 821), VIII+175 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

This volume contains seven papers presented at the meeting "Journées Statistiques" Rouen, June 13—14, 1979. Three papers by Balascheff and Dupont, Harel, and Rüschendorf deal with the asymptotic behaviour of multivariate empirical processes. Using the weak convergence of the multivariate emprical process, Deheuvels presents nonparametric tests of independence. Adaptive rank tests and midrank statistics are considered in the papers of Albers and Ruymgaart. Collomb gives results on convergence in probability, with probability one, and in L_q, $1<q<\infty$, of the $k-NN$ estimator of a multivariate regression function.

Lajos Horváth (Szeged)

Ottó Steinfeld, Quasi-ideals in Rings and Semigroups, Disquisitiones Mathematicae Hungaricae 10., Akadémiai Kiadó, Budapest, 1978.

Quasi-ideals were introduced by the author of the book in 1956. This is the first monograph in the field, and it gives a fairly complete discussion of the results attained in these two decades. The book is completely self-contained — perhaps even too much so, as it gives definitions of literally all notions and rather meticulous proofs. It is very clearly written and well readable.

The first four chapters contain basic notions, examples and bits of general ring and semigroup theory used throughout the book. In §§ 5—7 the basic facts concerning minimal quasi-ideals of rings and minimal and 0-minimal ideals of semigroups are given, with a particular stress (in § 7) for semiprime rings and semigroups. § 8 deals with decomposition theorems for some classes of semiprime rings. An interesting result here is Theorem 8.1 which provides different decompositions for semiprime rings satisfying the minimum condition for principal left ideals. The next paragraph throws light on the behaviour of quasi-ideals in regular rings and semigroups, and § 10 contains analoga of the results of § 8 for regular semigroups (as for semiprime semigroups they don't hold in general). The last two chapters of the main part deal with the characterization of regular duo elements, rings and semigroups, and with different ways of generalization.

There is an Appendix on quasi-absorbents in so-called groupoid-lattices. This notion, too, was introduced by the author in 1970. It seems to be a good tool for finding the common roots of some properties of rings, groups and semigroups, and is far from being completely exhausted. Its possibilities are shown in this Appendix by an abstract version of Theorem 8.1 and its analoga.

There are over 20 problems in the text, collected also in a list at the end of the book. An important role in the book is played by examples, in particular counter-examples showing the limits of parallelism between rings and semigroups.

G. Pollák (Szeged)

Stochastic Integrals, Proceedings, LMS Durham Symposium, 1980. Edited by D. Williams (Lecture Notes in Mathematics, 851), IX+540 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

The volume is devided into three parts. The first part contains three introductory articles to help make some of the later material accessible to a wider audience. Williams gives a self-contained introduction to some important concepts such as continuous martingales and the associated martingale representation, the Stroock—Varadhan theorem and its consequences for martingale representation, the Girsanov theorem. The main theme of this survey is the modern theory of the Kolmogorov forward (or Fokker—Planck) equation. Roger's paper provides a brief summary of the construction and the fundamental properties of stochastic integrals. Various kinds of integration are described by Elliott.

Longer research and survey papers are in the second part of the book. These 13 surveys, written by excellent probabilists, cover a wide part of stochastics. We mention only the following topics: Markov processes in quantum theory, Malliavin calculus, set-parametered martingales, Bessel processes and infinitely divisible laws, probability functionals of diffusion processes. The book ends with five shorter papers presented at the London Mathematical Society Durham Symposium.

Lajos Horváth (Szeged)

Árpád Szabó, The Beginings of Greek Mathematics, 358 pages, Akadémiai Kiadó Budapest, 1978.

This is the English edition of the German original, published by the same publishing house in 1969. The carefully written book is not intended to be an introduction to Greek mathematics (for this purpose the reader can consult the book of van der Waerden, Science Awakening). Its aim is to bring the problems associated with the early history of deductive science to the attention of classical scholars, and historians and philosophers of science.

The method used undoubtedly distinguishes this book from most of its prodecessors. It is based on a very careful investigation of original texts (the author is a classical philologist). Using this the author reconstructs the history of the early Greek mathematics, the origin of the axiomatic method.

The axiomatic method must have been existed before Euclid, but previous historians credited it to Aristotle or Plato. The author's idea is that the founders of this method were the Pythagoreans under the influence of the Eleatic philosophy.

In the first two parts of the book we can read the early history of the theory of irrationals and the pre-Euklidean theory of proportions. Part 3, the main part of the book, deals with the construction of mathematics within a deductive framework. The appendix "How the Pythagoreans discovered Proposition II. 5 of the Elements" serves to illustrate the kind of research which needs to be undertaken if we are to acquire a new understanding of the historical development of Greek mathematics.

Lajos Klukovits (Szeged)

Allen Tannenbaum, Invariance and System Theory: Algebric and Geometric Aspects (Lecture Notes in Mathematics, 845), X+161 pages, Springer-Verlag, Berlin-Heidelberg—New York, 1981.

These lecture notes are based on a series of lectures given by the author at the Mathematical System Theory Institute of the ETH, Zürich in 1980. The autor's purpose is to draw the attention of theoretical mathematicians and convince them that there are, in system theory, some interesting and deep problems from pure mathematics to be solved, and to introduce people working in system theory to the ideas of algebraic geometry, differential geometry, algebraic topology and invariant theory.

In the first three parts the author gives a good survey on some topics of algebraic geometry, system theory and invariant theory. Parts IV and V are devoted to the global and local moduli of linear time-invariant dynamical systems, respectively.

In Part VI the "system realization problem" is discussed which concerns the construction of a state space model of a system from its input/output behaviour. Part VII is concerned with the geometry of rational transfer functions. Finally, in Part VIII the stabilization through feedback is treated.

We can recommend these lecture notes to both theoretical mathematicians and system theory people interested in theoretical approaches in system theory.

L. Hatvani (Szeged)

The Correspondence Between A. A. Markov and A. A. Chuprov on the Theory of Probability and Mathematical Statistics. Edited by Kh. O. Ondar, Translated from the Russian by Charles and Margaret Stein, XVII+181 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1981.

"I note with astonishment that in the book of A. A. Chuprov, Essays on the Theory of Statistics, on page 195, P. A. Nekrasov, whose work in recent years represents an abouse of mathematics, is mentioned next to Chebyshev. A. Markov". The story begins with this postcard, of 2 November 1910, from Markov (1856—1922) to Chuprov (1874—1926), and this of course settles the tone of the lively correspondence that followed in the next seven years between the two of them. We all know that Markov is an outstanding figure in the history of mathematics in general, and of the theory of probability and mathematical statistics in particular. Chuprov, not to be measured to Markov, was a fairly good statistician of his time under the influence of Lexis, Bortkiewicz, Quetelet and the emerging school of Pearson from one side, and of the inheritance of the Russian school of probability from the other. The latter, owing especially to Chebishev, Liapunov and Markov, was way ahead of the West at the time. The topics of the exchanges range wide: Lexis's coefficient of dispersion, the notorious "law of small numbers" of Bortkiewicz, the law of large numbers (including, at the time, what is nowadays the central limit theorem), Slutsky's work, Pearson's curves, the expectation of a ratio of dependent variables, Markov's linguistic statistics, etc.

Markov, known as "Neistovyi Andrei" to his contemporaries, throws himself vehemently into all the issues raised. In a firm consciousness of his authority he is not afaid to go into rather technical computations feverishly (on the three days 18—20 November 1910 he mailed not less then ten letters and postcards to Chuprov; they both lived in Petersburg), and to comment quite coarsely on what he feels a poor work. This is in accord with the contemporary discription above, which, in Jerzy Neyman's translation, is "Andrew the irrepressible, who does not pull any punches". Chuprov's role is seen, by the reviewer, as that of a prudent stimulator. We must be grateful to him that he was able to bring this out of Markov. Many of Chuprov's letters have not been found. The book contains 25 letters from him and 80 letters or postcards from Markov.

The idea of the translation has been put forward by the late Jerzy Neyman who wrote a charming introduction to the book. This is followed by the editor's preface outlining the life and work of Markov and Chuprov. Following the letters we find the editor's explanatory review of the correspondence. The first two of the four appendices are Markov's (negative) review of Chuprov's book and Chuprov's (positive) review of a posthumus edition of Markov's book on probability. The book is ended by Markov's and Chuprov's addresses at the bicentenary celebration of Bernoulli's law of large numbers in 1913, held by the Russian Academy upon the initiative of Markov. (This celebration must have been unique in the whole world.) The correspondence is really very exciting, one can hardly put the book down before finishing. The translation is excellent. I recommend reading it to every probabilist and statistician. It is a pity that I can probably never learn Neistovyi Andrei's limerick, mentioned in Professor Neyman's introduction, "not suited for the ears of ladies".

Sándor Csörgő (Szeged)

The Geometric Vein. The Coxeter Festschrift, edited by Chandler Davis, Branko Grünbaum and F. A. Sherk, VIII+598 pages with 5 color plates, 6 halftones and 211 line illustrations, Springer-Verlag, New York—Heidelberg—Berlin 1981.

H. S. M. Coxeter is one of the most inspiring geometers in the present century. Close to a hundred mathematicians from eight countries gathered on the Coxeter Symposium (held at the University of Toronto, 21—25 May 1979) testifying the deep influence of Coxeter's works in several fields of geometry such as the theory of polytopes and honeycombs, geometric transformations, groups and presentations of groups, extremal problems and combinatorial geometry.

The Geometric Vein is the collection of the lectures given at this Symposium, containing altogether 41 papers. Thus it would be impossible to give a detailed survey in this review. The reader can read papers among others by J. H. Conway, E. Ellers, G. Ewald, L. Fejes Tóth, B. Grünbaum, W. Kantor, P. McMullen, C. A. Rogers, B. A. Rosenfeld, J. J. Seidel, G. C. Shephard, J. Tits, W. T. Tutte, I. M. Yaglom.

The book is arranged very carefully and the papers are written in a brilliant style. Most of the papers are understandable also for the undergraduate studens. So they are warmly recommended to everyone who wants an insight into a very geometric geometry.

Z. I. Szabó (Szeged)

Arthur T. Winfree, The Geometry of Biological Time (Biomathematics, Volume 8), XIV+530 pages, 290 illustrations, Springer-Verlag, New York—Heidelberg—Berlin, 1980.

From opening and closing of flowers to heartbeat, from cell division to pupal eclosion of insects, from pattern formation of mashrooms to the migration of fishes, from bird navigation to the female cycle, from spatial wave organisation of catalytic oxidation to sleeping, from pacemaker neurons in the optic nerve of crayfish and in the brain of various flies to mating habits, and in fact in the whole life outside and inside us, we encounter all kinds of periodic patterns with circadian, seasonal and various other rythmicities, exhibited collectively or individually, and with various organising phase singularities. The encyclopaedic masterpiece under review visualizes Nature for us as mutually synchronized communities of chemical, physical and, as a main topic, biological clocks. The first ten chapters [Circular logic — Phase singularities (Screwy results of circular logic) — The rules of the ring— Ring popolations — Getting off the ring — Attracting cycles and isochrons — Measuring trajectories of a circadian clock — Populations of attractor cycle oscillators — Excitable kinetics and excitable media — The varieties of phaseless experience; in which the geometrical orderliness of rhythmic organization breaks down in diverse ways] constitute the more theoretical first part of the book, with elementary topological facts and little catastrophe theory, and a few differential equations here and there. Fortunately, Life is not raped by mathematics at all. Only the flavour is mathematical and the reviewer, being very far from what he thinks to be a biologist, has the feeling that this is modern biology of the highest quality. The second half of the book, the author calls it the Bestiary, with 13 chapters [The firefly machine — Energy metabolism in cells — The malonic acid reagent ("Sodium Geometrate") — Electrical rhythmicity and excitability in cell membranes — The aggregation of slime mould amoebae — Growth and regeneration – Arthropod cuticle — Pattern formation in the fungi — Circadian rythms in general — The circadian clooks of insect eclosion — The flower of Kalanchoe — The cell mitotic cycle — The female cycle], is a collection of extraordinary wealth of particular experimental systems of living organisms about which the first part theorises.

It is almost unbelievable that such a book could have been written. No doubt, its intrinsic theoretical value, its wealth, the fantastically easy-flowing style, and its flexible, broad and open view will make it a classic. The bibliography contains more than 1300 items, only the author index fills 14 pages. Some 290 illustrations, sometimes really beautiful, invite the attention of the reader, but the book is also very cheap (US $ 33). Biologists, physiologists, physicists, chemists and perhaps also applied mathematicians will find it a good and rewarding reading. Why then advertising it in these *Acta*, for readers almost exclusively in pure mathematics? The point is the recreational. Many of us, exhausted by daily abstraction or sophisticated calculation, would still like to have something slightly mathemtical but "real" around in the evening. This book is an ideal choice for such a purpose. It is partly dedicated "to those readers who, expecting wonders to follow so grand a title as it flaunts, may feel cheated by its actual content". I expected nothing to follow so grand a title as it flaunts, and I found wonders.

Sándor Csörgő (Szeged)