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Convergence of solutions of a nonlinear integrodifferential equation arising in compartmental systems

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In honour of Professor Béla Szőkefalvi-Nagy on his 70th birthday

1. Introduction

The compartmental models play an important role in the mathematical description of biological processes, chemical reactions, economic and human interactions [1, 2, 8]. I. GYŐRI [3, 4] used nonlinear integrodifferential equations to describe compartmental systems with pipes and propounded the question whether the bounded solutions of the model equation have limits as $t\to\infty$. If the transit times of material flow between compartments are zero, then the model equations are ordinary differential equations. In this case there are known results on the existence of the limit of solutions [9, 12]. But these methods are not applicable if the transit times are not zero. The existence of the limit of solutions is also known in the case of nonzero transit times if the model equation is linear [5] or if the so-called transport functions are continuously differentiable [11]. But there occur compartmental systems in the applications such that the transport functions do not satisfy even the local Lipschitz continuity. For example, in hydrodynamical models, where the free outflow of water through a leak at the bottom of a container has a rate proportional to square root of the amount of water in the container. If the transport functions are continuous, monotone nondecreasing and the model equation has exactly one equilibrium state then the solutions tend to this one as $t\to\infty$ [4].

In this paper we examine stationary compartmental systems with pipes, which are described by nonlinear autonomous integrodifferential equations, and the transit times of material flow through pipes are characterized by distribution functions.

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We show that the model equations have equilibrium states if and only if their solutions are bounded (the equilibrium points need not be unique). The main result of this paper guarantees the existence of the limits of the bounded solutions if the transport functions are continuous, strictly increasing functions.

2. The model equation, notations and definitions

Consider a stationary n-compartmental system with pipes. It is well-known (see e.g. [3, 4]) that the state vector $x(t)$ is the solution of the following system of integrodifferential equations:

$$(1)\qquad \dot{x}_i(t) = -\sum_{j=0}^{n} h_{ji}(x_i(t)) + \sum_{j=1}^{n} \int_0^{\tau} h_{ij}(x_j(t-s))\,dF_{ij}(s) + I_i \quad (i = 1, \dots, n),$$

where

(a) $h_{ij}: R \to R$ is a continuous, monotone nondecreasing function, $h_{ij}(0)=0$ $(i=0, 1, \dots, n;\ j=1, \dots, n)$;

(b) $\tau > 0$;

(c) $F_{ij}: [0, \tau] \to [0, 1]$ is continuous from the left, monotone nondecreasing and $F_{ij}(0)=0$, $F_{ij}(\tau)=1$ $(i, j=1, \dots, n)$;

(d) $I_i \geqq 0$ $(i=1, \dots, n)$.

Denote by $C_1, \dots, C_n$ the compartments and by C_0 the environment of the compartmental system. In equation (1) the function h_{ij} is called the transport function, which is the rate of material outflow from C_j in the direction of C_i $(i=0, 1, \dots, n;$ $j=1, \dots, n)$. The nonnegative number I_i is the inflow rate of material flow from environment C_0 into compartment C_i $(i=1, \dots, n)$.

Since in equation (1) the components of the solution vector denote material amounts, it is a reasonable claim that solutions corresponding to nonnegative initial conditions should be nonnegative, and the model equation (1) should have a unique solution for any given initial condition. In Section 3 we prove that (1) has these properties.

Let R and R^n be the set of real numbers and the n-dimensional Euclidean space, respectively, and $|\cdot|$ denotes the norm in R^n. Denote by $C([a, b], R^n)$ the Banach space of continuous functions mapping the interval $[a, b]$ into R^n with the topology of uniform convergence.

It is natural to consider the space $C([-\tau, 0], R^n)$ for the state space of (1). Let $r=2n\tau$. Obviously, without loss of generality, $C=C([-r, 0], R^n)$ may also be regarded as a state space of (1). In this paper we use C for the phase space of (1).

Denote the norm of an element φ in C by $\|\varphi\| = \max_{-r \leqq s \leqq 0} |\varphi(s)|$. If $t_0 \in R$, $A > 0$ and $x: [t_0 - r, t_0 + A) \to R^n$ is continuous, then for any $t \in [t_0, t_0 + A)$ let $x_t \in C$ be defined by $x_t(s) = x(t+s)$, $-r \leqq s \leqq 0$.

A function $x: I \to R^n$ is said to be a solution of (1) on the interval I if x is continuous on I and $x(t)$ satisfies (1) for every $t \in I$ such that $t-r \in I$. For given $\varphi \in C$ we say that $x(\varphi)$ is a solution of (1) through $(0, \varphi)$ if there is an $A>0$ such that $x(\varphi)$ is a solution of (1) on $[-r, A)$ and $x_0(\varphi)=\varphi$.

It follows from conditions (a), (b), (c), (d) that for every $\varphi \in C$ there is a solution $x(\varphi)$ of (1) through $(0, \varphi)$ and if x is a noncontinuable, bounded solution of (1) on the interval $[-r, A)$, then $A=\infty$ [7, Theorems 2.2.1, 2.3.2].

We prove in Section 3 that if $\varphi \in C$, then equation (1) has at most one solution $x(\varphi)$ through $(0, \varphi)$.

Let $x(\varphi)$ be a solution of (1) on the interval $[-r, \infty)$, $\varphi \in C$. Define the ω-limit set $\Omega(\varphi)$ of the solution $x(\varphi)$ as follows: $\Omega(\varphi)=\{\psi \in C$: there is a sequence $\{t_n\}$ such that $t_n \to \infty$ and $\|\psi - x_{t_n}(\varphi)\| \to 0$ as $n \to \infty\}$. The set $M \subset C$ is said to be invariant if for every $\psi \in M$ equation (1) has a solution y on R such that $y_0=\psi$ and $y_t \in M$ for all $t \in R$. If $x(\varphi)$ is a bounded solution of (1) on $[-r, \infty)$, then $\Omega(\varphi)$ is nonempty, compact, connected, invariant and $x_t(\varphi) \to \Omega(\varphi)$ as $t \to \infty$ [7, Corollary 4.2.1].

Let $N \subset \{1, 2, ..., n\}$ and define the directed graph $D_N=(V(D_N), A(D_N))$ to equation (1) as follows: $V(D_N)=\{v_i\colon i \in N\}$ is the set of vertices, $A(D_N)= =\{a_{ij}\colon h_{ij}(\cdot) \not\equiv 0,\ (i,j) \in N \times N\}$ is the set of arcs, where the arc a_{ij} is said to join v_j to v_i, v_j is the tail of a_{ij} and v_i is its head. A directed (v_j, v_i)-walk W from v_i to v_j is a finite non-null sequence $W=(a_{i_1 i_0}, a_{i_2 i_1}, ..., a_{i_k i_{k-1}})$, where $a_{i_1 i_0}, a_{i_2 i_1}, ..., a_{i_k i_{k-1}} \in A(D_N)$ and $i_0=i$, $i_k=j$. If $i_0, i_1, ..., i_k$ are distinct, then the walk $W=(a_{i_1 i_0}, a_{i_2 i_1}, ..., a_{i_k i_{k-1}})$ is called a directed (v_{i_k}, v_{i_0})-path. Two vertices v_i, v_j are diconnected in D_N if there are a directed (v_i, v_j)-path, and a directed (v_j, v_i)-path in D_N. The diconnection is an equivalence relation on set N. The directed subgraphs $D_{N_1}, D_{N_2}, ..., D_{N_k}$ induced by the resulting partition $(N_1, N_2, ..., N_k)$ of N are called the dicomponents of D_N. It is easy to see that there exists a dicomponent $D_{N_{i_0}}$ of D_N such that if $i \in N_{i_0}$ and $j \in N \setminus N_{i_0}$, then $a_{ij} \notin V(D_N)$.

3. Uniqueness, boundedness and some technical lemmas

In this section we prove some easy lemmas, which are necessary to the proof of the main result.

Define the functional $U\colon C \times C \to [0, \infty)$ as follows:

$$U(\varphi, \psi) = \sum_{i=1}^{n} \Big[|\varphi_i(0) - \psi_i(0)| + \sum_{j=1}^{n} \int_0^{\tau} \int_0^{s} \big| h_{ij}(\varphi_j(-u)) - h_{ij}(\psi_j(-u)) \big| \, du \, dF_{ij}(s) \Big],$$

$$\varphi = (\varphi_1, ..., \varphi_n), \quad \psi = (\psi_1, ..., \psi_n) \in C.$$

Lemma 1 claims the monotonicity of functional U along the solutions of (1).

Lemma 1. *If x and y are solutions of* (1) *on the interval* $[-r, A)$ *then* $U(x_t, y_t)$ *as a function of* t *is monotone nonincreasing on* $[0, A)$.

Proof. Let $u(t)=U(x_t, y_t)$, $t\in[0, A)$. Since x and y are solutions of (1) on $[-r, A)$, we have

$$\frac{d}{dt}[x_i(t)-y_i(t)] =$$

$$= -\sum_{j=0}^{n}[h_{ji}(x_i(t))-h_{ji}(y_i(t))]+\sum_{j=1}^{n}\int_0^{\tau}[h_{ij}(x_j(t-s))-h_{ij}(y_j(t-s))]\,dF_{ij}(s)$$

$$(t\in[0, A), \quad i = 1, \ldots, n).$$

Thus, from the monotonicity of functions h_{ij} it follows that

$$D^+|x_i(t)-y_i(t)| \leqq$$

$$\leqq -\sum_{j=0}^{n}|h_{ji}(x_i(t))-h_{ji}(y_i(t))|+\sum_{j=0}^{n}\int_0^{\tau}|h_{ij}(x_j(t-s))-h_{ij}(y_j(t-s))|\,dF_{ij}(s)$$

$$(t\in[0, A), \quad i = 1, \ldots, n).$$

Hence it is easy to see that

$$D^+u(t) \leqq$$

$$\leqq \sum_{i=1}^{n}\Big[-\sum_{j=0}^{n}|h_{ji}(x_i(t))-h_{ji}(y_i(t))|+\sum_{j=1}^{n}\int_0^{\tau}|h_{ij}(x_j(t-s))-h_{ij}(y_j(t-s))|\,dF_{ij}(s)+$$

$$+\sum_{j=1}^{n}|h_{ij}(x_j(t))-h_{ij}(y_j(t))|-\sum_{j=1}^{n}\int_0^{\tau}|h_{ij}(x_j(t-s))-h_{ij}(y_j(t-s))|\,dF_{ij}(s)\Big] =$$

$$= -\sum_{i=1}^{n}|h_{0i}(x_i(t))-h_{0i}(y_i(t))| \leqq 0 \quad (t\in[0, A)),$$

which, by using differential inequality [10, p. 15], completes the proof.

R. M. Lewis and B. D. O. Anderson [11] proved similar result provided that functions h_{ij} are continuously differentiable.

The uniqueness for the initial-value problem of (1) follows easily from Lemma 1.

Corollary 1. *For every* $\varphi\in C$ *equation* (1) *has a unique solution* $x(\varphi)$ *through* $(0, \varphi)$.

By using Lemma 1 and the properties of the ω-limit set one can readily verify that:

Corollary 2. *If* $\varphi \in C$ *and* $\psi \in \Omega(\varphi)$ *then there exists a sequence* $\{t_n\}$ *such that* $t_n \to \infty$ *and*

$$\sup_{u \geqq 0} |x(\varphi)(t_n+u) - x(\psi)(u)| \to 0 \quad as \quad n \to \infty.$$

Define the functions $H_i\colon R^n \to R$ by

$$H_i(z_1, \ldots, z_n) = -\sum_{j=0}^{n} h_{ij}(z_i) + \sum_{j=1}^{n} h_{ij}(z_j) + I_i \quad (i = 1, \ldots, n),$$

where $(z_1, \ldots, z_n) \in R^n$. If $z^* \in R^n$ and $H_i(z^*) = 0$, $i = 1, \ldots, n$, then the constant function z^* is a solution of (1) on R, i.e. z^* is an equilibrium point of (1).

From Lemma 1 it is clear that the existence of an equilibrium point of (1) guarantees the solutions of (1) to be bounded.

Corollary 3. *If there exists* $z \in R^n$ *such that*

$$H_i(z) = 0 \quad (i = 1, \ldots, n), \tag{2}$$

then every solution of (1) *is bounded on* $[-r, \infty)$.

Corollary 3 is reversible in the following sense: if equation (1) has a bounded solution on $[-r, \infty)$ then equation (2) has a solution.

Lemma 2. *If* x *is a bounded solution of* (1) *on* $[-r, \infty)$ *and* $M_i = \overline{\lim}_{t \to \infty} x_i(t)$, $m_i = \underline{\lim}_{t \to \infty} x_i(t)$, $i = 1, \ldots, n$, *then*

(i) $H_i(M_1, \ldots, M_n) = 0 \quad (i = 1, \ldots, n)$;
(ii) $H_i(m_1, \ldots, m_n) = 0 \quad (i = 1, \ldots, n)$;
(iii) $h_{0i}(m_i) = h_{0i}(M_i) \quad (i = 1, \ldots, n)$.

Proof. We first prove that $H_i(M_1, \ldots, M_n) \geqq 0$, $i = 1, \ldots, n$. Suppose this is not true. Then there is an $i_0 \in \{1, \ldots, n\}$ such that $H_{i_0}(M_1, \ldots, M_n) < 0$. Let $a = H_{i_0}(M_1, \ldots, M_n)$. Since functions h_{ij} are continuous, there exists $\varepsilon > 0$ such that

$$-\sum_{j=0}^{n} h_{ji_0}(M_{i_0} - \varepsilon) + \sum_{j=1}^{n} h_{i_0 j}(M_j + \varepsilon) + I_{i_0} < \frac{a}{2}. \tag{3}$$

Let T be chosen so that if $t \geqq T$, then

$$\sup_{t \geqq T - \tau} x_j(t) \leqq M_j + \varepsilon \quad (j = 1, \ldots, n). \tag{4}$$

By using relations (3), (4) and the monotonicity of functions h_{ij} we have

$$\dot{x}_{i_0}(t) \leqq -\sum_{j=0}^{n} h_{ji_0}(M_{i_0} - \varepsilon) + \sum_{j=1}^{n} h_{i_0 j}(M_j + \varepsilon) + I_{i_0} < \frac{a}{2} < 0$$

on the set $\{t \geqq T: x_{i_0}(t) \in [M_{i_0}-\varepsilon, M_{i_0}+\varepsilon]\}$. This contradicts the definition of M_{i_0}, proving the statement. By similar arguments we obtain $H_i(m_1, \ldots, m_n) \leqq 0$, $i=1, \ldots, n$. From the above and the equality $\sum_{i=1}^{n} H_i(z_1, \ldots, z_n) = \sum_{i=1}^{n} [I_i - h_{0i}(z_i)]$ it follows that

$$0 \leqq \sum_{i=1}^{n} H_i(M_1, \ldots, M_n) - \sum_{i=1}^{n} H_i(m_1, \ldots, m_n) = -\sum_{i=1}^{n} [h_{0i}(M_i) - h_{0i}(m_i)] \leqq 0,$$

which proves the lemma.

The proof of Lemma 2 is based on the idea of [4, Th. 3.2.1].

Corollary 4. *If equation* (2) *has exactly one solution, then for every solution* x *of* (1) *the limit* $\lim_{t\to\infty} x(t)$ *exists.*

Corollary 5. *If there exists* $i_0 \in \{1, \ldots, n\}$ *such that function* h_{0i_0} *is strictly monotone increasing, then for every bounded solution of* (1) *the limit* $\lim_{t\to\infty} x_{i_0}(t)$ *exists.*

Lemma 3. *If* $M_i, m_i, i=1, \ldots, n$, *are real numbers and*

(i) $M_i > m_i \quad (i = 1, \ldots, n)$,

(ii) $H_i(M_1, \ldots, M_n) = 0 \quad (i = 1, \ldots, n)$,

(iii) $H_i(m_1, \ldots, m_n) = 0 \quad (i = 1, \ldots, n)$,

then for every $\varepsilon \in (0, \min_{i=1,\ldots,n} (M_i - m_i))$ *there exists* $z^*(\varepsilon) = (z_1^*, \ldots, z_n^*) \in R^n$ *such that*

(iv) $M_i - \varepsilon \leqq z_i^* \leqq M_i \quad (i=1, \ldots, n)$,

(v) *there is an* $i_0 \in \{1, \ldots, n\}$ *such that* $z_{i_0}^* = M_{i_0} - \varepsilon$,

(vi) $H_i(z_1^*, \ldots, z_n^*) = 0 \quad (i=1, \ldots, n)$.

Proof. From the equality $\sum_{i=1}^{n} H_i(z_1, \ldots, z_n) = \sum_{i=1}^{n} [I_i - h_{0i}(z_i)]$, the monotonicity of functions h_{0i} and (ii), (iii) it follows that $\sum_{i=1}^{n} H_i(z_1, \ldots, z_n) = 0$ for $z_i \in [m_i, M_i]$, $i=1, \ldots, n$. Let $\varepsilon \in (0, \min_{i=1,\ldots,n} (M_i - m_i))$ be given. Define the sequence $\{z_1^{(k)}, \ldots, z_n^{(k)}\}_{k=0}^{\infty}$ as follows:

(a) $z_i^{(0)} = M_i - \varepsilon \quad (i=1, \ldots, n)$,

(b) assume that $(z_1^{(k+1)}, \ldots, z_{j-1}^{(k+1)}, z_j^{(k)}, \ldots, z_n^{(k)})$ is defined such that $M_i - \varepsilon \leqq z_i^{(k+1)} \leqq M_i$, $i=1, \ldots, j-1$, $M_i - \varepsilon \leqq z_i^{(k)} \leqq M_i$, $i=j, \ldots, n$. Let $z_j^{(k+1)}$ be chosen according as $H_j(z_1^{(k+1)}, \ldots, z_{j-1}^{(k+1)}, z_j^{(k)}, \ldots, z_n^{(k)}) \leqq 0$ or >0. If $H_j(z_1^{(k+1)}, \ldots, z_{j-1}^{(k+1)}, z_j^{(k)}, \ldots, z_n^{(k)}) \leqq 0$ then let $z_j^{(k+1)} = z_j^{(k)}$. If $H_j(z_1^{(k+1)}, \ldots, z_{j-1}^{(k+1)}, z_j^{(k)}, \ldots, z_n^{(k)}) > 0$ then choose $z_j^{(k+1)}$ such that $z_j^{(k)} < z_j^{(k+1)} \leqq M_j$ and $H_j(z_1^{(k+1)}, \ldots, z_{j-1}^{(k+1)}, z_j^{(k+1)}, z_{j+1}^{(k)}, \ldots, z_n^{(k)}) = 0$. Since $H_j(z_1^{(k+1)}, \ldots, z_{j-1}^{(k+1)}, M_j, z_{j+1}^{(k)}, \ldots, z_n^{(k)}) \leqq H_j(M_1, \ldots, M_n) = 0$, the number $z_j^{(k+1)}$ exists.

Since the sequence $\{z_i^{(k)}\}_{k=0}^{\infty}$ is monotone nondecreasing and bounded, $z^*(\varepsilon)$

can be defined by $z_i^*=\lim_{k\to\infty} z_i^{(k)}$, $i=1, \ldots, n$, $z^*(\varepsilon)=(z_1^*, \ldots, z_n^*)$. We now prove that $z^*(\varepsilon)$ has properties (iv, (v), (vi). By the definition of $z^*(\varepsilon)$ (iv) is obviously satisfied. If (vi) is not true, then from $\sum_{i=1}^n H_i(z_1^*, \ldots, z_n^*)=0$ it follows that there is an $i_0\in\{1, \ldots, n\}$ such that $H_{i_0}(z_1^*, \ldots, z_n^*)>0$. Since H_{i_0} is continuous, one can find a number N such that $H_{i_0}(z_1^{(k+1)}, \ldots, z_{i_0}^{(k+1)}, z_{i_0+1}^{(k)}, \ldots, z_n^{(k)})>(1/2)H_{i_0}(z_1^*, \ldots, z_n^*)>0$ for $k\geqq N$. But this contradicts the definition of $z_{i_0}^{(k+1)}$. If (v) is not true, then we can choose a number k_i for every $i\in\{1, \ldots, n\}$ such that $z_i^{(k_i+1)}>z_i^{(k_i)}=M_i-\varepsilon$. Let $k_0=\max\{k_1, \ldots, k_n\}$ and $j=\max\{i\in\{1, \ldots, n\}: k_i=k_0\}$. The definition of the sequence $\{z_i^{(k)}\}$ implies $H_i(z_1^{(k_i+1)}, \ldots, z_{i-1}^{(k_i+1)}, z_i^{(k_i)}, \ldots, z_n^{(k_i)})>0$, $i=1, \ldots, n$. From the structure of H_i, the monotonicity of functions h_{ij} and the construction of $\{z_i^{(k)}\}$ it follows that $H_i(z_1^{(k_0+1)}, \ldots, z_{j-1}^{(k_0+1)}, z_j^{(k_0)}, \ldots, z_n^{(k_0)})\geqq 0$ for $i\neq j$. Thus $\sum_{i=1}^n H_i(z_1^{(k_0+1)}, \ldots, z_{j-1}^{(k_0+1)}, z_j^{(k_0)}, \ldots, z_n^{(k_0)})>0$, which is a contradiction.

The following lemma includes the nonnegativity of trajectories and a comparison result.

Lemma 4 [13, Theorems 1,3]. *If* $\varphi, \psi\in C$, $\psi_i(s)\geqq\varphi_i(s)\geqq 0$ *for* $s\in[-r, 0]$, $i=1, \ldots, n$, *and* $x(\varphi)(\cdot)$, $x(\psi)(\cdot)$ *are solutions of* (1) *on* $[-r, \infty)$ *through* $(0, \varphi)$, $(0, \psi)$, *then* $x_i(\psi)(t)\geqq x_i(\varphi)(t)\geqq 0$ *for* $t\in[0, \infty)$, $i=1, \ldots, n$.

Lemma 5 [6, Theorem 3.1]. *Assume that* $\varphi\in C$ *and* $x(\varphi)$ *is a bounded solution of* (1) *on* $[-r, \infty)$. *If there exists a nonempty set* $H\subset(0, r]$ *such that*

(i) $\dot{x}_1(\psi)(0)\leqq 0$ *for all* $\psi\in\Omega(\varphi)$ *such that* $\psi_1(0)=\max_{-r\leqq s\leqq 0}\psi_1(s)$;

(ii) $\{\psi_1(-u): u\in H\}=\{\psi_1(0)\}$ *for all* $\psi\in\Omega(\varphi)$ *such that* $\dot{x}_1(\psi)(0)=0$ *and* $\psi_1(0)=\max_{-r\leqq s\leqq 0}\psi_1(s)$;

(iii) *either there exist* $r_1, r_2\in H$ *such that* r_1/r_2 *is irrational or these set* H *is infinite;*

then for any $\psi\in\Omega(\varphi)$ *the limit* $\lim_{t\to\infty} x_1(\psi)(t)$ *exists.*

Lemma 6. *Assume that* $\varphi\in C$, $x(\varphi)$ *is a bounded solution of* (1) *on the interval* $[-r, \infty)$ *and* $\psi\in\Omega(\varphi)$. *If the limit* $\lim_{t\to\infty} x(\psi)(t)$ *does not exist, then there are subsets* N_1, N_2 *of* $\{1, \ldots, n\}$ *and real numbers* c_i, $i\in\{1, \ldots, n\}\setminus N_1$, *such that*

(i) $N_2\subseteqq N_1$;

(ii) $x_i(\psi)(\cdot)\equiv c_i$ *for* $i\in\{1, \ldots, n\}\setminus N_1$;

(iii) *the limit* $\lim_{t\to\infty} x_i(\psi)(t)$ *does not exist for all* $i\in N_1$;

(iv) D_{N_2} *is a dicomponent of* D_{N_1};

(v) *for every* $i\in N_2$

$$\dot{x}_i(\psi)(t)=-\sum_{j\in N_2\cup\{0\}}\tilde{h}_{ji}(x_i(\psi)(t))+\sum_{j\in N_2}\int_0^\tau \tilde{h}_{ij}(x_j(\psi)(t-s))\,dF_{ij}(s)+\tilde{I}_i \tag{5}$$

$$(t\in R),$$

where $\tilde{h}_{ij}(\cdot)=h_{ij}(\cdot)$, $i, j\in N_2$, $\tilde{h}_{0i}(\cdot)=h_{0i}(\cdot)+\sum_{j\in N\setminus N_2} h_{ji}(\cdot)$, $\tilde{I}_i=I_i+\sum_{j\in N\setminus N_1} h_{ij}(c_j)$, $i\in N_2$.

Proof. Let $N_0=\{i\in\{1, \dots, n\}$: the limit $\lim_{t\to\infty} x_i(\psi)(t)$ exists$\}$ and $c_i=\lim_{t\to\infty} x_i(\psi)(t)$, $i\in N_0$. From the definition of $\Omega(\varphi)$ and Corollary 2 it follows that $\lim_{t\to\infty} x_i(\varphi)(t)=c_i$ and $x_i(\psi)(\cdot)\equiv c_i$, $i\in N_0$. Let $N_1=\{1, \dots, n\}\setminus N_0$ and define the set N_2 as follows: D_{N_2} is a dicomponent of D_{N_1} such that if $i\in N_2$ and $j\in N_1\setminus N_2$ then $a_{ij}\notin V(D_{N_1})$. Clearly (iii), (iv), (v) are satisfied.

4. Convergence of the bounded solutions

In this section we give a sufficient condition for the existence of the limit of bounded solutions of (1).

Theorem. *If for every* $i, j\in\{1, \dots, n\}$ *either function* $h_{ij}(\cdot)$ *is strictly monotone increasing or* $h_{ij}(\cdot)\equiv 0$, *then, for each* $\varphi\in C$ *such that* $x(\varphi)$ *is a bounded solution of* (1) *on* $[-r, \infty)$, *the limit* $\lim_{t\to\infty} x(\varphi)(t)$ *exists.*

Proof. Assume that $\varphi\in C$, $x(\varphi)$ is a bounded solution of (1) on $[-r, \infty)$ and $\lim_{t\to\infty} x(\varphi)(t)$ does not exist. By Corollary 2 if $\psi\in\Omega(\varphi)$ then $\lim_{t\to\infty} x(\psi)(t)$ does not exist, either. Using Lemma 6 one can construct the equation (5), which has a bounded solution on $[-r, \infty)$ such that its components do not tend to constant as $t\to\infty$. Our aim is to show that equation (5) has not such a solution. This contradiction will prove Theorem.

Since (5) is a special case of (1), without loss of generality we can assume that $N_1=N_2=\{1, \dots, n\}$ in Lemma 6, i.e. $x(\varphi)$ is a solution of (1) on $[-r, \infty)$ such that for every $i\in\{1, \dots, n\}$ the limit $\lim_{t\to\infty} x_i(\varphi)(t)$ does not exist.

Let $M_i=\overline{\lim}_{t\to\infty} x_i(\varphi)(t)$ and $m_i=\underline{\lim}_{t\to\infty} x_i(\varphi)(t)$, $i=1, \dots, n$. By Corollary 2 and the definition of $\Omega(\varphi)$, for every $\psi\in\Omega(\varphi)$

$$M_i=\overline{\lim_{t\to\infty}}\, x_i(\psi)(t),\quad m_i=\underline{\lim_{t\to\infty}}\, x_i(\psi)(t)\quad (i=1, \dots, n) \tag{6}$$

and

$$m_i\leqq x_i(\psi)(t)\leqq M_i\quad (t\in R;\ i=1, \dots, n). \tag{7}$$

We now show that for every $\psi\in\Omega(\varphi)$

$$\max_{-r\leqq s\leqq 0}\psi_i(s)=M_i\quad (i=1, \dots, n) \tag{8}$$

and

$$\min_{-r\leqq s\leqq 0}\psi_i(s)=m_i\quad (i=1, \dots, n). \tag{9}$$

If (8) is not true for $\psi\in\Omega(\varphi)$, then without loss of generality one can assume that there exists $\varepsilon_0>0$ such that

(10) $$\max_{-r\leqq s\leqq 0}\psi_1(s)\leqq M_1-\varepsilon_0.$$

Let $i\in\{2,\ldots,n\}$ in the case $n>1$. Since v_1, v_i are diconnected in $D_{\{1,\ldots,n\}}$, there exists a directed (v_i, v_1)-path $W=(a_{i_1i_0}, a_{i_2i_1},\ldots,a_{i_mi_{m-1}})$ in $D_{\{1,\ldots,n\}}$, where $i_0=1$, $i_m=i$. Suppose that for some $k\in\{0,1,\ldots,m-1\}$ there is an $\varepsilon_k>0$ such that

(11) $$\max_{-r+k\tau\leqq s\leqq 0}\psi_{i_k}(s)\leqq M_{i_k}-\varepsilon_k.$$

From the strict monotonicity of function $h_{i_{k+1}i_k}(\cdot)$, (7), (11) and Lemma 2 it follows that on the set

$$S=\{t\in[-r+(k+1)\tau,0]\colon \psi_{i_{k+1}}(t)=M_{i_{k+1}}\}$$

we have

$$\dot{x}_{i_{k+1}}(\psi)(t)\leqq-\sum_{j=0}^{n}h_{ji_{k+1}}(M_{i_{k+1}})+\sum_{\substack{j=1\\ j\neq i_k}}^{n}h_{i_{k+1}j}(M_j)+h_{i_{k+1}i_k}(M_{i_k}-\varepsilon_k)+I_{i_{k+1}}<$$

$$<H_{i_{k+1}}(M_1,\ldots,M_n)=0.$$

On the other hand, (7) and $x_{i_{k+1}}(t)=M_{i_{k+1}}$ imply $\dot{x}_{i_{k+1}}(\psi)(t)=0$, i.e. S is an empty set. Thus, there exists $\varepsilon_{k+1}>0$ such that

(12) $$\max_{-r+(k+1)\tau\leqq s\leqq 0}\psi_{i_{k+1}}(s)\leqq M_{i_{k+1}}-\varepsilon_{k+1}.$$

Since (10) is satisfied and (12) follows from (11), by using mathematical induction it can be seen that for some $\varepsilon_i>0$

$$\max_{-r+m\tau\leqq s\leqq 0}\psi_i(s)\leqq M_i-\varepsilon_i.$$

Since $i\in\{2,\ldots,n\}$ was arbitrary and W was a path, we have $m\leqq n-1$ and for some $\varepsilon>0$

(13) $$\max_{-\tau\leqq s\leqq 0}\psi_i(s)\leqq M_i-\varepsilon\quad(i=1,\ldots,n).$$

Apply Lemma 3: there exists $z^*=(z_1^*,\ldots,z_n^*)\in R^n$ such that $H_i(z_1^*,\ldots,z_n^*)=0$ and $M_i-\varepsilon\leqq z_i^*\leqq M_i$ for every $i\in\{1,\ldots,n\}$, $z_{i_0}^*=M_{i_0}-\varepsilon$ for some $i_0\in\{1,\ldots,n\}$. By (13) and Lemma 4 it follows that

$$x_{i_0}(\psi)(t)\leqq M_{i_0}-\varepsilon\quad(t\geqq 0),$$

which contradicts (6). Thus (8) is proved. By similar arguments one can show (9).

Let $T_{ij}\subset[0,\tau]$ denote the support of the Lebesgue—Stieltjes type measure induced by the distribution function F_{ij}, $i,j=1,\ldots,n$.

Define the set

$$H=(0,r]\cap\Big\{\bigoplus_{k=1}^{m} T_{i_k i_{k-1}}\colon (a_{i_1 i_0}, a_{i_2 i_1}, \dots, a_{i_m i_{m-1}}) \text{ is a directed } (v_1,v_1)\text{-walk in } D_{\{1,\dots,n\}}\Big\}.$$

See a special case at the end of this section. Since every two vertices are diconnected in $D_{\{1,\dots,n\}}$, for every $a_{ij}\in A(D_{\{1,\dots,n\}})$ there exists a directed (v_1,v_1)-walk W in $D_{\{1,\dots,n\}}$ such that $a_{ij}\in W$ and the length of W is at most $2n-1$. Thus, if H is empty, then $T_{ij}=\{0\}$ for every (i,j) such that $h_{ij}(\cdot)\not\equiv 0$, i.e. equation (1) is an ordinary differential equation. In this case τ can be an arbitrary small positive number. From this and (8), (9) it follows that $M_i=m_i$, $i=1,\dots,n$, which is a contradiction. Further on let us suppose that the set H is nonempty.

As regards the structure of set H we distinguish two cases:

Case. 1. Either there exist $r_1, r_2\in H$ such that r_1/r_2 is irrational or the set H is infinite.

Case 2. $H=\{p_1r^*, p_2r^*, \dots, p_Kr^*\}$, where $r^*>0$, $0<p_1<\dots<p_K\leqq r/r^*$, p_i is an integer for each $i=1,\dots,K$ and $(p_1,\dots,p_K)=1$ (( ) denotes the greatest common divisor).

Case 1. Set H just satisfies condition (iii) of Lemma 5. (8) implies condition (i) of Lemma 5. To verify condition (ii) of Lemma 5 it will be sufficient to show that for each $\psi\in\Omega(\varphi)$, from $\dot x_1(\psi)(0)=0$, $\psi_1(0)=M_1$ it follows that $\psi_1(0)=\psi_1(-u)$ for all $u\in H$. Let $u=\sum_{k=1}^{m} t_{i_k i_{k-1}}\in H$, where $t_{i_k i_{k-1}}\in T_{i_k i_{k-1}}$, $k=1,\dots,m$. If $\dot x_1(\psi)(0)=0$ and $\psi_1(0)=M_1$ then by equation (1)

$$0=-\sum_{j=0}^{n} h_{j1}(M_1)+\sum_{j=1}^{n}\int_0^{\tau} h_{1j}(x_j(\psi)(-s))\,dF_{1j}(s)+I_1. \tag{14}$$

From Lemma 2

$$0=-\sum_{j=0}^{n} h_{j1}(M_1)+\sum_{j=1}^{n}\int_0^{\tau} h_{1j}(M_j)\,dF_{1j}(s)+I_1. \tag{15}$$

From (8), (14), (15) and the monotonicity of functions $h_{1j}(\cdot)$ with the notation $i_m=1$

$$0=\int_0^{\tau}\big[h_{i_m i_{m-1}}(M_{i_{m-1}})-h_{i_m i_{m-1}}(x_{i_{m-1}}(\psi)(-s))\big]\,dF_{i_m i_{m-1}}(s). \tag{16}$$

Since function $h_{i_m i_{m-1}}(\cdot)$ is strictly increasing and $t_{i_m i_{m-1}}\in T_{i_m i_{m-1}}$, (16) implies

$$\psi_{i_{m-1}}(-t_{i_m i_{m-1}})=M_{i_{m-1}}. \tag{17}$$

Using (8), (17) it is easy to see that $\dot x_{i_{m-1}}(\psi)(-t_{i_m i_{m-1}})=0$. Continuing this procedure we get

$$\psi_{i_{k-1}}\left(-\sum_{j=k}^{m} t_{i_j i_{j-1}}\right) = M_{i_{k-1}} \quad (k = 1, \ldots, m). \tag{18}$$

In the case $k=1$ relation (18) gives just $\psi_1(-u)=M_1$, which was to be proved. From (6) and Lemma 5 it follows $M_1=m_1$, which is a contradiction.

Case 2. Define the nonempty sets $A_0, A_1, \ldots, A_m$ as follows:

(i) $\bigcup_{p=0}^{m} A_p=\{1, \ldots, n\}$;

(ii) $A_0=\{1\}$;

(iii) $A_p=\{i\colon i\in\{1, \ldots, n\}\setminus\bigcup_{k=0}^{p-1} A_k$ and there exists $j\in A_{p-1}$ such that $a_{ji}\in \in A(D_{\{1,\ldots,n\}})\}$, $p=1, \ldots, m$.

Let the function $S\colon \{2, 3, \ldots, n\}\to\{1, 2, \ldots, n\}$ eb defined in the following way: $S(i)\in A_{p-1}$ and $a_{S(i)i}\in A(D_{\{1,\ldots,n\}})$ whenever $i\in A_p$, $p=1,\ldots, m$. Let $\psi\in\Omega(\varphi)$, $y=x(\psi)$ and define the function

$$V(t) = \sum_{i=1}^{n} z_i(t),$$

where $z_1(t)=y_1(t)$ and

$$z_i(t) = \int_0^{\tau} \cdots \int_0^{\tau} y_i\left(t-\sum_{m=1}^{k} s_m\right) dF_{i_k i_{k-1}}(s_k) \ldots dF_{i_1 i_0}(s_1)$$

for $i=2, 3, \ldots, n$, where $i_0, i_1, \ldots, i_k$ are defined by $i_0=i$, $i_k=1$ and $S(i_m)=i_{m+1}$ for $m=0, 1, \ldots, k-1$. Obviously $i_0, i_1, \ldots, i_k$ may depend on i. Let $M_0=\sum_{i=1}^{n} M_i$ and $m_0=\sum_{i=1}^{n} m_i$. It is clear from (7) and the definition of V that

$$m_0 \leqq V(t) \leqq M_0 \quad (t\in R).$$

From the invariance of set $\Omega(\varphi)$ we have $y_t\in\Omega(\varphi)$ for all $t\in R$. By similar arguments as in Case 1, for every $t\in R$ from $y_1(t)=M_1$ it follows that $y_i(t-\sum_{m=1}^{k} s_m)=M_i$ whenever $s_m\in T_{i_m i_{m-1}}$, $m=1, \ldots, k$; moreover $y_1(t-u)=M_1$ for each $u\in H$. Clearly, $V(t)=M_0$ implies $y_1(t)=M_1$. Thus, from $V(t)=M_0$ it follows that $V(t-u)=M_0$ for $u\in H$. Similarly, from $V(t)=m_0$ we obtain $V(t-u)=m_0$, $u\in H$. Hence and from (8), (9), (19) we have

$$\max_{-r\leqq s\leqq 0} V(t+s) = M_0, \quad \min_{-r\leqq s\leqq 0} V(t+s) = m_0 \quad (t\in R). \tag{20}$$

Since $(p_1, \ldots, p_K)=1$, from elementary number theory, there exist integers $n_1, \ldots, n_K$ such that $\sum_{k=1}^{K} n_k p_k=1$. Let

$$N = \sum_{k=1}^{K} n_k^+ p_k-1 \left(= \sum_{k=1}^{K} n_k^- p_k\right),$$

where n_k^+ and n_k^- are the positive and negative parts of n_k.

If $h=\sum_{k=1}^{K} a_k p_k$, where a_k is nonnegative integer, $k=1, \dots, K$, then hr^* is the sum of the elements of set H. Thus, from $V(t)=M_0$ and $V(t)=m_0$ it follows that $V(t-hr^*)=M_0$ and $V(t-hr^*)=m_0$, respectively. For every integer l, which is not less than N^2, the number lr^* is the sum of the elements of H. This is evident from the following:

$$l = N^2+k = N^2+aN+b = (N+a)N+b =$$

$$= (N+a)\sum_{k=1}^{K} n_k^- p_k + b\sum_{k=1}^{K} n_k p_k = \sum_{k=1}^{K} [(N+a)n_k^- + bn_k]p_k,$$

where k, a, b are nonnegative integers, $k=aN+b$, $b<N$.

Thus, from (20) it follows that there exist numbers $t_1, t_2 \in R$ such that

$$V(t_1-ir^*) = M_0, \quad V(t_2-ir^*) = m_0 \quad (i = 0, 1, 2, \dots). \tag{21}$$

From Lemma 2, (7) and the monotonicity of functions h_{0i} we have

$$\sum_{i=1}^{n} [-h_{0i}(y_i(t))+I_i] = 0 \quad (t \in R).$$

Thus, by using that y is a solution of (1), one gets

$$\dot{V}(t) = \sum_{i=1}^{n} \dot{z}_i(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(t),$$

where

$$w_{11}(t) = \int_0^{\tau} h_{11}(y_1(t-s))\,dF_{11}(s) - h_{11}(y_1(t)),$$

$$w_{1i}(t) = \int_0^{\tau} h_{1i}(y_i(t-s))\,dF_{1i}(s) -$$

$$-\int_0^{\tau}\dots\int_0^{\tau} h_{1i}\Big(y_i\Big(t-\sum_{m=1}^{k} s_m\Big)\Big)\,dF_{i_k i_{k-1}}(s_k)\dots dF_{i_1 i_0}(s_1) \quad (i \geqq 2),$$

$$w_{j1}(t) = \int_0^{\tau}\dots\int_0^{\tau} h_{j1}\Big(y_1\Big(t-s-\sum_{m=1}^{l} s_m\Big)\Big)\,dF_{j1}(s)\,dF_{j_l j_{l-1}}(s_l)\dots dF_{j_1 j_0}(s_1) -$$

$$-h_{j1}(y_1(t)) \quad (j \geqq 2),$$

$$w_{ij}(t) = \int_0^{\tau}\dots\int_0^{\tau} h_{ij}\Big(y_j\Big(t-s-\sum_{m=1}^{k} s_m\Big)\Big)\,dF_{ij}(s)\,dF_{i_k i_{k-1}}(s_k)\dots dF_{i_1 i_0}(s_1) -$$

$$-\int_0^{\tau}\dots\int_0^{\tau} h_{ij}\Big(y_j\Big(t-\sum_{m=1}^{l} s_m\Big)\Big)\,dF_{j_l j_{l-1}}(s_l)\dots dF_{j_1 j_0}(s_1) \quad (i, j \geqq 2),$$

where $j_0, j_1, \dots, j_l$ are defined by $j_0=j$, $j_l=1$ and $S(j_m)=j_{m+1}$ for $m=0, 1, \dots, l-1$. Let W be a (v_j, v_1)-path in $D_{\{1,\dots,n\}}$. Then $(W, a_{j_1 j_0}, a_{j_2 j_1}, \dots, a_{j_l j_{l-1}})$ and $(W, a_{ij}, a_{i_1 i_0}, a_{i_2 i_1}, \dots, a_{i_k i_{k-1}})$ are (a_1, a_1)-walks in $D_{\{1,\dots,n\}}$ such that their lengths are at most $2n-1$. Thus, from the definition of H it follows that there exists a nonnegative u such that

$$u+s+\sum_{m=1}^{k} s_m = p_i r^* \quad\text{and}\quad u+\sum_{m=1}^{l} \tau_m = p_j r^*$$

for all $s\in T_{ij}$, $s_m\in T_{i_m i_{m-1}}$, $m=1, \dots, k$, $\tau_m\in T_{j_m j_{m-1}}$, $m=1, \dots, l$, for some nonnegative integers p_i, p_j, where p_i and p_j may depend on s_m, s, τ_m. That is, for $i, j\in \{1, \dots, n\}$ functions $w_{ij}(t)$ have the following structure

$$w_{ij}(t) = \sum_{k=1}^{K_1} a_k v(t-b_k r^*+u)-\sum_{m=1}^{K_2} c_m v(t-d_m r^*+u),$$

where $\sum_{k=1}^{K_1} a_k = \sum_{m=1}^{K_2} c_m = 1$, $u\in R$, b_k and d_m are nonnegative integers for $k=1, \dots, K_1$, $m=1, \dots, K_2$, and the function $v\colon R\to R$ is bounded on R. See a special case at the end of this section.

If $|v(t)|\leqq a$ for $t\in R$ and $b=\max_{k=1,\dots,K_1, m=1,\dots,K_2}\{b_k, d_m\}$, then

$$\frac{1}{L+1}\Big|\sum_{l=0}^{L}\Big(\sum_{k=1}^{K_1} a_k v(t-b_k r^*+u-lr^*)-\sum_{m=1}^{K_2} c_m v(t-d_m r^*+u-lr^*)\Big)\Big| =$$

$$= \frac{1}{L+1}\Big|\sum_{k=1}^{K_1} a_k \sum_{l=0}^{L} v(t+u-(b_k+l)r^*)-\sum_{m=1}^{K_2} c_m \sum_{l=0}^{L} v(t+u-(d_m+l)r^*)\Big| \leqq$$

$$\leqq \frac{1}{L+1}\Big|\sum_{k=1}^{K_1} a_k \sum_{s=b}^{L} v(t+u-sr^*)-\sum_{m=1}^{K_2} c_m \sum_{s=b}^{L} v(t+u-sr^*)\Big|+\frac{2ab}{L+1} =$$

$$= \frac{1}{L+1}\Big|\sum_{s=b}^{L} v(t+u-sr^*)\Big(\sum_{k=1}^{K_1} a_k-\sum_{m=1}^{K_2} c_m\Big)\Big|+\frac{2ab}{L+1} = \frac{2ab}{L+1}\to 0$$

as $L\to\infty$ uniformly in t on R. Hence we have

$$\text{(22)}\qquad \frac{1}{L+1}\sum_{l=0}^{L} \dot V(t-lr^*)\to 0 \quad\text{as}\quad L\to\infty$$

uniformly in t on R.

On the other hand from (21) it follows that

$$\int_{t_2}^{t_1} \frac{1}{L+1}\sum_{l=0}^{L} \dot V(t-lr^*)\,dt = \frac{1}{L+1}\sum_{l=0}^{L}\int_{t_2-lr^*}^{t_1-lr^*} \dot V(t)\,dt = M_0-m_0>0$$

for all $L=0, 1, 2, \dots,$ which contradicts (22).

This completes the proof.

Remarks. The proof of Theorem for Case 2 is based on the idea of [6, Theorem 3.2].

We remark that the monotonicity conditions for functions h_{ij} cannot be omitted: if the functions are not monotone nondecreasing, then the equation (1) may have periodic solution [10].

We do not know whether the strict monotonicity conditions for h_{ij} is a necessary condition for the convergence of solutions of (1).

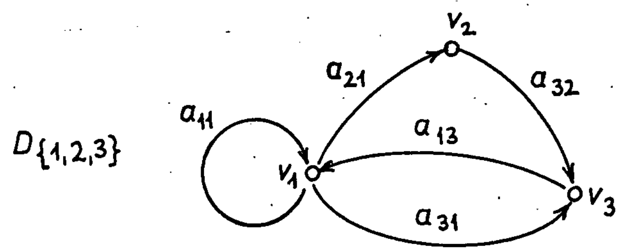
To illustrate the above proof we give a special case. Let us consider the system

$$\dot{x}_1(t) = -h_{11}(x_1(t))-h_{21}(x_1(t))-h_{31}(x_1(t))+h_{11}(x_1(t-1))+h_{13}(x_3(t-2))$$

$$\dot{x}_2(t) = -h_{32}(x_2(t))+h_{21}(x_1(t-1))$$

$$\dot{x}_3(t) = -h_{13}(x_3(t))+h_{31}(x_1(t-2))+\frac{1}{2}h_{32}(x_2(t))+\frac{1}{4}h_{32}(x_2(t-1))+\frac{1}{4}h_{32}(x_2(t-2)),$$

where functions $h_{11}, h_{21}, h_{31}, h_{13}, h_{32}$ are strictly increasing. Here directed graph $D_{\{1,2,3\}}, \tau, r, T_{ij}, H, A_p, V(t)$ and $\dot{V}(t)$ are the following:



$$\tau = 3; \quad r = 18;$$

$$T_{11} = \{1\}, \quad T_{13} = \{2\}, \quad T_{21} = \{1\}, \quad T_{31} = \{2\}, \quad T_{32} = \{0, 1, 2\};$$

$$H = \{1, 2, 3, \ldots, 18\};$$

$$A_0 = \{1\}, \quad A_1 = \{3\}, \quad A_2 = \{2\},$$

$$V(t) = y_1(t)+y_3(t-2)+y_2(t-2)/2+y_2(t-3)/4+y_2(t-4)/4;$$

$$\dot{V}(t) = [-h_{11}(y_1(t))+h_{11}(y_1(t-1))]+[-h_{31}(y_1(t))+h_{31}(y_1(t-4))]+$$
$$+[-h_{21}(y_1(t))+h_{21}(y_1(t-3))]/2+[-h_{21}(y_1(t))+h_{21}(y_1(t-4))]/4+$$
$$+[-h_{21}(y_1(t))+h_{21}(y_1(t-5))]/4.$$

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