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Homomorphically complete classes of automata with respect to the α_2-product

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Dedicated to Professor K. Tandori on his 60th birthday

Homomorphically complete classes of automata with respect to the general product were characterized by A. A. LETIČEVSKIĬ in [8]. In order to decrease the complexity of the general product F. Gécseg introduced the concept of α_i-products in [5]. The notion of α_0-product coincides with that of the loop-free product used by J. HARTMANIS (cf. [7]). It is known that there exists no homomorphically complete finite class of automata for the α_0- or α_1-product (cf. [4]). Using a result in [3], P. DÖMÖSI (cf. [1]) succeeded in proving that there is a single automaton homomorphically complete with respect to the α_2-product. In the present paper we show that a class of automata is homomorphically complete with respect to the α_2-product if and only if it is homomorphically complete with respect to the general product. Thus, Letičevskiĭ's criterion can be used to describe those classes which are homomorphically complete with respect to the α_2-product. Our result can also be used to show that for every $i \geqq 2$, the α_i-product is homomorphically as general as the general product (cf. [2]).

By an automaton we shall always mean a finite automaton. Given a finite system $\mathbf{A}_t=(A_t, X_t, \delta_t)$ $(t=1, \ldots, n,\ n\geqq 1)$ of automata together with a finite set of input signs X and a family of feedback functions $\varphi_t\colon A_1\times\ldots\times A_n\times X\to X_t$ $(t=1, \ldots, n)$ we can form the general product (cf. [6]) $\prod(\mathbf{A}_1, \ldots, \mathbf{A}_n|\varphi)=(A_1\times\ldots\times A_n, X, \delta)$ where $\delta((a_1, \ldots, a_n), x)=(\delta_1(a_1, x_1), \ldots, \delta_n(a_n, x_n))$, provided that $a_t\in A_t$, $x\in X$, $x_t=\varphi_t(a_1, \ldots, a_n, x)$ $(t=1, \ldots, n)$. If $i\geqq 0$ is a given integer and none of the feedback functions φ_t depends on the states a_s with $t+i\leqq s\leqq n$, then we come to the notion of α_i-products introduced in [5]. Further, if for each t, φ_t only depends on its last variable (the input sign) then we get the concept of the quasi-direct product. If all the $\mathbf{A}_t$-s coincide then we speak about a general, α_i- or quasi direct power

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according to the cases described above. We say that an automaton $\mathbf{A}=(A, X, \delta)$ homomorphically realizes an automaton $\mathbf{B}=(B, X, \delta')$ if $\mathbf{B}$ is a homomorphic image of a subautomaton of $\mathbf{A}$. *A* class $\mathcal{K}$ of automata is called homomorphically complete with respect to the general product (homorphically complete, for short) if every automaton can be homomorphically realized by a general product of automata belonging to $\mathcal{K}$. Homorphically α_i-complete classes are similarly defined. By Letičevskiĭ's result in [8], a class of automata is homomorphically complete if and only if it contains an automaton $\mathbf{A}=(A, X, \delta)$ having states $a_0, a_1, a_1' \in A$ such that $a_1 \neq a_1'$, further, $\delta(a_0, x)=a_1$, $\delta(a_0, x')=a_1'$, $\delta(a_1, p)=a_0$ and $\delta(a_1', p')=a_0$ hold for some input signs $x, x' \in X$ and strings $p, p' \in X^*$.

We are going to show that homomorphically complete classes with respect to the α_2-product are exactly the homomorphically complete classes. For this reason we have to prove that if an automaton satisfies Letičevskiĭ's criterion then it is homomorphically complete with respect to the α_2-product. Let us denote by $\mathbf{U}=(U, \{x_1, x_2\}, \delta)$ an automaton with the following properties:

(i) $U=\{u_0, \dots, u_{k_1-1}\} \cup \{u_0', \dots, u_{k_2-1}'\}$ where $k_1, k_2 \geqq 1$, $k_1>1$ or $k_2>1$, further, $u_0=u_0'$, $u_i \neq u_j$ if $i \neq j$ $(0 \leqq i, j<k_1)$ and $u_i' \neq u_j'$ if $i \neq j$ $(0 \leqq i, j<k_2)$,

(ii) $\delta(u_0, x_1)=u_1$, $\delta(u_0', x_2)=u_1'$, $\delta(u_i, x_j)=u_{i+1}$ $(i=1, \dots, k_1-1,\ j=1, 2)$, $\delta(u_i', x_j)=u_{i+1}'$ $(i=1, \dots, k_2-1, j=1, 2)$ where we have used the notations $u_{k_1}=u_0$ and $u_{k_2}'=u_0$,

(iii) $u_1 \neq u_1'$.

It is obvious that if an automaton $\mathbf{A}$ satisfies Letičevskiĭ's criterion then for some k_1 and k_2 an automaton $\mathbf{U}$ having properties (i), (ii) and (iii) above can be isomorphically embedded into an α_1-power of $\mathbf{A}$ with a single factor. Therefore, if each automaton $\mathbf{U}$ is homomorphically complete with respect to the α_2-product then so is $\mathbf{A}$. In this way it is enough to show that any automaton $\mathbf{U}$ is homomorphically complete with respect to the α_2-product.

In the next two lemmas we fix an automaton $\mathbf{U}$ and denote by k the l.c.m. of k_1 and k_2. For every integer i we shall denote by u_i and u_i' the states u_r resp. u_s' with $r \in \{0, \dots, k_1-1\}$, $s \in \{0, \dots, k_2-1\}$ and such that $i \equiv r \pmod{k_1}$ and $i \equiv s \pmod{k_2}$. First we prove that all the automata $\mathbf{S}_m=(\{1, \dots, mk\}, \{x_1, x_2\}, \delta)$ can be homomorphically realized by α_2-powers of $\mathbf{U}$, where the transitions in $\mathbf{S}_m$ are defined by $\delta(i, x_1)=j$ if and only if $j \equiv i+1 \pmod{mk}$ and

$$\delta(i, x_2)=\begin{cases} j & \text{where } j \equiv i+1 \pmod{mk} \quad \text{if } i \not\equiv 0 \pmod{k}, \\ 1 & \text{if } i \equiv 0 \pmod{k}. \end{cases}$$

Lemma 1. *Each automaton* $\mathbf{S}_m$ $(m \geqq 1)$ *can be homomorphically realized by an* α_2*-power of* $\mathbf{U}$.

Proof. Let $\mathbf{C}=(\{1, \dots, k\}, \{x\}, \delta_{\mathbf{C}})$ be a counter, i.e. $\delta_{\mathbf{C}}(i, x) \equiv i+1 \pmod{k}$. It is quite obvious that **C** is isomorphic to a quasi-direct power of **U** with two factors. We define an α_2-product $\mathbf{A}=(A, \{x_1, x_2\}, \delta)=\Pi(\mathbf{C}, \underbrace{\mathbf{U}, \ldots, \mathbf{U}}_{mk \text{ times}}|\varphi)$ with

$$\varphi_1(i, v_1, x_j) = x,$$

$$\varphi_{1+t}(i, v_1, \ldots, v_t, v_{t+1}, x_j) = \begin{cases} x_2 & \text{if} \quad v_{t+1} = u_1', \\ x_1 & \text{otherwise,} \end{cases}$$

$$\varphi_{1+mk}(i, v_1, \ldots, v_{mk}, x_1) = \begin{cases} x_2 & \text{if} \quad i\in\{1, \ldots, k-1\} \quad \text{and} \quad v_{mk-i+1} = u_1', \\ & \text{or} \quad i = k, \quad (v_1, \ldots, v_{mk}) = (u_1', \ldots, u_k', u_1, \ldots, u_{(m-1)k}), \\ x_1 & \text{otherwise,} \end{cases}$$

$$\varphi_{1+mk}(i, v_1, \ldots, v_{mk}, x_2) = \begin{cases} x_2 & \text{if} \quad i\in\{1, \ldots, k-1\} \quad \text{and} \quad v_{mk-i+1} = u_1', \\ & \text{or} \quad i = k, \\ x_1 & \text{otherwise,} \end{cases}$$

where $1\leqq t<mk$, $i\in\{1, \ldots, k\}$, $v_1, \ldots, v_{mk}\in U$ and $j=1$ or $j=2$.

Let B consist of those elements $(i, v_1, \ldots, v_{mk})\in A$ for which there exist an integer $j\in\{1, \ldots, mk\}$ and $v_1', \ldots, v_{(m+1)k}'\in U$ satisfying the following three conditions:

(i) $i \equiv j \pmod k$, $v_{mk-j+1} = u_1'$,

(ii) $(v_{tk+1}', \ldots, v_{(t+1)k}') = (u_1, \ldots, u_k)$ or $(v_{tk+1}', \ldots, v_{(t+1)k}') = (u_1', \ldots, u_k')$,

(iii) $(v_1, \ldots, v_{mk}) = (v_{i+1}', \ldots, v_{i+mk}')$.

It is not difficult to check that $\mathbf{B}=(B, \{x_1, x_2\}, \delta)$ is a subautomaton of **A** and $\mathbf{S}_m$ is a homomorphic image of **B** under the mapping $\psi\colon B\to\{1, \ldots, mk\}$ defined by $\psi((i, v_1, \ldots, v_{mk}))=\min\{j|1\leqq j\leqq mk,\ i\equiv j \pmod k,\ v_{mk-j+1}=u_1'\}$.

Note that φ_1 was independent of its variables, therefore our contruction gives rise to a homomorphic realization of $\mathbf{S}_m$ by an α_2-power of **U**. As none of the functions φ_{1+t} $(1\leqq t<mk)$ depended on the input sign $\mathbf{S}_m$ can also be homomorphically realized by an α_2-power of **U** in such a way that the feedback functions, except the last one, are independent of the input sign.

Next we show that all shift-registers can be homomorphically realized by an α_2-power of **U**. As usual, by a shift-register on a fixed alphabet $X=\{x_1, \ldots, x_n\}$ we shall mean any automaton isomorphic to one of the automata $\mathbf{R}_m=(X^m, X, \delta)$ $(m\geqq 1)$, where X^m denotes the set of all strings $y_1\ldots y_m$ of length m on X, and $\delta(y_1\ldots y_m, y)=y_2\ldots y_m y$ $(y_1, \ldots, y_m, y\in X)$.

Lemma 2. *Every shift-register can be homomorphically realized by an α_2-power of* **U**.

Proof. As $\mathbf{R}_{m_1}$ is a homomorphic image of $\mathbf{R}_{m_2}$ whenever $m_1\leqq m_2$, it is enough to show that shift-registers $\mathbf{R}_{mk}$ with $m\geqq n$ can be homomorphically realized by an α_2-power of **U**.

Let $\mathbf{C}=(\{1, \ldots, mk\}, \{x\}, \delta_{\mathbf{C}})$ be a counter having mk states. We shall define an α_2-product $\mathbf{A}=(A, X, \delta)=\prod(\mathbf{C}, \underbrace{\mathbf{U}, \ldots, \mathbf{U}}_{(m+n)mk^2 \text{ times}}|\varphi)$ where the last $(m+n)mk^2$ components will be treated as mk buffers b_i of length $(m+n)k$. The counter will point to the buffer used last. That is, if $i\in\{1, \ldots, mk\}$ is the first component of a state of $\mathbf{A}$ then b_i contains the input sign arrived for the last time. Buffers are used in a circular way: if $i<mk$ then b_{i+1}, otherwise b_1 is the buffer available next. Consequently, the mk signs arrived last will be contained by buffers $b_{i+1}, \ldots, b_{mk}, b_1, \ldots, b_i$ in this order. We shall use the states $u_1\neq u_1'$ to encode a sign by the fixed mapping $\tau\colon X\to\{u_1, u_1'\}^n$, $\tau(x_j)=u_1^{j-1}u_1'u_1^{n-j}$ $(j=1, \ldots, n)$. Therefore, in order to store a sign x_j into the next available buffer we shall set the $(m+j)k$-th component of this buffer to u_1', and set all the $(m+j')k$-th components for $j'=1, \ldots, j-1, j+1, \ldots, n$ to u_1. During this transition all already stored input signs will be shifted with one place to the left, the values of the first components of the buffers underflow.

Now we put this into a precise form by defining the feedback functions of the product. For every $i\in\{1, \ldots, mk\}$, $v_1, \ldots, v_{mk}\in U$, $j\in\{1, \ldots, n\}$ and $t(1\leq t<(m+n)mk^2)$ we put $\varphi_1(i, v_1, x_j)=x$,

$$\varphi_{1+t}(i, v_1, \ldots, v_t, v_{t+1}, x_j)=\begin{cases} x_2 & \text{if } \ v_{t+1}=u_1', \\ x_1 & \text{otherwise,}\end{cases}$$

provided that $t\not\equiv 0 \pmod{k}$, and if $t\equiv 0 \pmod{k}$ then

$$\varphi_{1+t}(i, v_1, \ldots, v_t, v_{t+1}, x_j)=$$

$$=\begin{cases} x_2 \text{ if } t=i'(m+n)k-(n-j)k \text{ where } i'\in\{1, \ldots, mk\} \text{ is determined by } i+1\equiv i' \\ \qquad \pmod{mk}, \\ \quad \text{or } v_{t+1}=u_1' \text{ and there exists an integer } i'\in\{1, \ldots, mk\} \text{ with } i+1\equiv i' \\ \qquad \pmod{k},\ (i'-1)(m+n)k<t<i'(m+n)k, \\ \quad \text{or there exist } i'\in\{1, \ldots, mk\},\ r\in\{2, \ldots, k\} \text{ such that } i+r\equiv i' \pmod{k}, \\ \qquad v_{t-k+r}=u_1' \text{ and } (i'-1)(m+n)k<t\leq i'(m+n)k, \\ x_1 \text{ otherwise,}\end{cases}$$

and similarly,

$$\varphi_{1+m(m+n)k^2}(i, v_1, \ldots, v_{m(m+n)k^2}, x_j)=$$

$$=\begin{cases} x_2 \text{ if } i=mk-1;\ j=n, \\ \quad \text{or there exists } r\in\{2, \ldots, k\} \text{ with } i+r\equiv mk \pmod{k} \\ \quad \text{and } v_{m(m+n)k^2-k+r}=u_1', \\ x_1 \text{ otherwise.}\end{cases}$$

Next we give a subautomaton $\mathbf{B}=(B, X, \delta)$ of $\mathbf{A}$ and a homomorphism of $\mathbf{B}$ onto $\mathbf{R}_m$. This will be accomplished by the help of the auxiliary functions $\varrho_j\colon A\to U^n$ $(j=1, \ldots, mk)$ and $\varrho\colon A\to U^{mnk}$. Suppose that $a=(i, v_1^1, \ldots, v_{(m+n)k}^1, \ldots, v_1^{mk}, \ldots,$

$\ldots, v^{mk}_{(m+n)k})$, $j\in\{1, \ldots, mk\}$. If $j>i$ then we put

$$\varrho_j(a) = v^j_{j-i+k} \ldots v^j_{j-i+nk} \in U^n$$

else

$$\varrho_j(a) = v^j_{mk-(i-j)+k} \ldots v^j_{mk-(i-j)+nk} \in U^n.$$

By $\varrho(a)$ we shall denote the string

$$\varrho(a) = \varrho_{i+1}(a) \ldots \varrho_{mk}(a)\varrho_1(a) \ldots \varrho_i(a) \in U^{mnk}.$$

Now let B consist of all those elements $a=(i, v^1_1, \ldots, v^1_{(m+n)k}, \ldots, v^{mk}_1, \ldots, v^{mk}_{(m+n)k})$ which satisfy the following conditions:

(i) There exists a string $y_1 \ldots y_{mk} \in X^{mk}$ with $\varrho(a)=\tau(y_1)\ldots\tau(y_{mk})$,

(ii) If $j\in\{1, \ldots, mk\}$, $r\in\{1, \ldots, k\}$ and $j\equiv i+r \pmod k$ then $\{v^j_1 \ldots v^j_k, \ldots, \ldots, v^j_{(m+n-1)k+1} \ldots v^j_{(m+n)k}\} \subseteq U_r$ where U_r denotes a set of four strings:

$$U_r = \{\underbrace{u_{2-r} \ldots u_{-1}u_0}_{r-1} \underbrace{u_1 \ldots u_{k-r+1}}_{k-r+1}, \underbrace{u_{2-r} \ldots u_{-1}u_0}_{r-1} \underbrace{u'_1 \ldots u'_{k-r+1}}_{k-r+1},$$

$$\underbrace{u'_{2-r} \ldots u'_{-1}u'_0}_{r-1} \underbrace{u_1 \ldots u_{k-r+1}}_{k-r+1}, \underbrace{u'_{2-r} \ldots u'_{-1}u'_0}_{r-1} \underbrace{u'_1 \ldots u'_{k-r+1}}_{k-r+1}\},$$

(iii) If $j\in\{1, \ldots, mk\}$ then $v^j_t \ldots v^j_{(m+n)k} = u_1 \ldots u_{(m+n)k-t+1}$ where $t=j-i+nk+k$ if $j>i$ and $t=mk-(i-j)+nk+k$ if $j\leqq i$.

It can be seen that with the definition above **B** becomes a subautomaton of **A** and the mapping ψ: $B\to X^m$ determined by $\psi(a)=y_1\ldots y_{mk}$ if and only if $\varrho(a)= =\tau(y_1)\ldots\tau(y_{mk})$ is a homomorphism of **B** onto $\mathbf{R}_{mk}$. As the counter **C** is an X-subautomaton of $\mathbf{S}_m$, by Lemma 1 and the fact that φ_1 is a constant mapping, we obtain a homomorphic realization of $\mathbf{R}_{mk}$ by an α_2-power of U.

Now we are ready to state our

Theorem. *Every homomorphically complete class of automata is homomorphically complete with respect to the α_2-product.*

Proof. Given a homomorphically complete class of automata, by the result of A. Letičevskiĭ in [8], there is an automaton $\mathbf{U}_0$ in this class such that for some k_1, k_2 $(k_1, k_2\geqq 1, k_1\neq 1$ or $k_2\neq 1)$ the automaton **U** can be isomorphically embedded into an α_1-power of $\mathbf{U}_0$ with a single factor. Therefore it is enough to show that every automaton $\mathbf{A}=(A=\{a_1, \ldots, a_m\}, X=\{x_1, \ldots, x_n\}, \delta)$ can be homomorphically realized by an α_2-power of this automaton **U**. In order to prove this statement we form an α_2-product $\mathbf{B}=(B, X, \delta')=\prod(\mathbf{R}_{mk}, \mathbf{S}_m, \underbrace{\mathbf{U}, \ldots, \mathbf{U}}_{mk \text{ times}}|\varphi)$ where $\mathbf{R}_{mk}$ and $\mathbf{S}_m$ are the automata described previously, and for any $y_1\ldots y_{mk}\in X^{mk}$, $i\in\{1, \ldots, mk\}$, $v_1, \ldots, v_{mk}\in U$,

$j\in\{1, \ldots, n\}$ and $t(1\leqq t<mk)$

$$\varphi_1(y_1\cdots y_{mk}, i, x_j) = x_j,$$

$$\varphi_2(y_1\cdots y_{mk}, i, v_1, x_j) = \begin{cases} x_2 & \text{if } v_1 = u_1' \text{ and } i\equiv 0 \pmod k, \\ x_1 & \text{otherwise,}\end{cases}$$

$$\varphi_{2+t}(y_1\cdots y_{mk}, i, v_1, \ldots, v_t, v_{t+1}, x_j) =$$

$$= \begin{cases} x_2 & \text{if } v_{t+1} = u_1', \\ & \text{or } t\equiv 0 \pmod k \text{ and } v_{t-r+1} = u_1' \text{ for an integer} \\ & \quad r\in\{1, \ldots, k-1\} \text{ with } i\equiv r \pmod k, \\ & \text{or } v_1 = u_1',\ i\equiv 0,\ t\equiv 0 \pmod k \text{ and } \delta(a_{i/k}, y_{mk-i+2}\cdots y_{mk}x_j) = a_{t/k}, \\ x_1 & \text{otherwise,}\end{cases}$$

and similarly,

$$\varphi_{2+mk}(y_1\cdots y_{mk}, i, v_1, \ldots, v_{mk}, x_j) =$$

$$= \begin{cases} x_2 & \text{if } v_{mk-r+1} = u_1' \text{ for an } r\in\{1, \ldots, k-1\} \text{ satisfying } i\equiv r \pmod k, \\ & \text{or } v_1 = u_1', i\equiv 0 \pmod k \text{ and } \delta(a_{i/k}, y_{mk-i+2}\cdots y_{mk}x_j) = a_m, \\ x_1 & \text{otherwise.}\end{cases}$$

Let $C\subseteq B$ contain all states $b=(y_1\ldots y_{mk}, i, v_1, \ldots, v_{mk})\in B$ with the following property: there are $r\in\{1, \ldots, k\}$ and $t\in\{1, \ldots, m\}$ such that $i\equiv r \pmod k$, $tk+i-r\leqq \leqq mk$ and

(i) $v_{tk-r+1}\ldots v_{mk}v_1\ldots v_{tk-r}=u_1'\ldots u_k'u_{k+1}\ldots u_{mk}$ if $i\geqq k$, and

(ii) $v_{tk-r+1}\ldots v_{tk}v_1\ldots v_{tk-r}v_{(t+1)k-r+1}\ldots v_{mk}v_{tk+1}\ldots v_{(t+1)k-r}=u_1'\ldots u_k'u_{k+1}\ldots u_{mk}$ if $i<k$. It is easy to show that $\mathbf{C}=(C, X, \delta')$ is a subautomaton of $\mathbf{B}$. Indeed, assume that $b\in C$ and the integers t and r are determined as previously, and let $y\in X$. Then $\delta'(b, y)=(y_2\ldots y_{mk}y, i', v_1', \ldots, v_{mk}')$ where i' and $v_1', \ldots, v_{mk}'$ are determined according to the three cases below:

Case 1. If $r\neq k$ and $i>k$; or $r=k$ and $t\neq 1$ then $i'=i+1$, $v_1'=v_2, \ldots, \ldots, v_{mk-1}'=v_{mk}$, $v_{mk}'=v_1$. (Observe that now $k\leqq i<mk$.)

Case 2. If $r\neq k$ and $i<k$ then $i'=i+1$, $v_1'=v_2, \ldots, v_{tk-1}'=v_{tk}$, $v_{tk}'=v_1$, $v_{tk+1}'=v_{tk+2}, \ldots, v_{mk-1}'=v_{mk}$, $v_{mk}'=v_{tk+1}$.

Case 3. If $r=k$ and $t=1$ then $i'=1$, $v_1'=v_2, \ldots, v_{sk-1}'=v_{sk}$, $v_{sk}'=v_1$, $v_{sk+1}'= =v_{sk+2}, \ldots, v_{mk-1}'=v_{mk}$, $v_{mk}'=v_{sk+1}$ where $s\in\{1, \ldots, m\}$ is determined by $\delta(a_{i/k}, y_{mk-i+2}\ldots y_{mk}y)=a_s$. It can be checked that $b'\in C$ in all the three cases above.

In order to complete the proof we have to give a homomorphism ψ of $\mathbf{C}$ onto $\mathbf{A}$. Let $b=(y_1\ldots y_{mk}, i, v_1, \ldots, v_{mk})\in C$ be artibtrary. Then there are uniquely determined integers $r\in\{1, \ldots, k\}$ and $t\in\{1, \ldots, m\}$ fulfilling $i\equiv r \pmod k$, $tk+i-r\leqq mk$ and such that either condition (i) or (ii) holds according to $i\geqq k$ or $i<k$. Put

$\psi(b)=\delta(a_{(tk+i-r)/k}, y_{mk-i+1}\dots y_{mk})$. Then, corresponding with the previously listed three cases, one can easily verify that ψ is a homomorphism. On the other hand ψ is obviously surjective.

We have seen that **A** is homomorphically realized by **B**. From this the result follows by the lemmas, the fact that $\mathbf{S}_m$ was homomorphically realized by an α_2-power of **U** in such a way that with the exception of the last feedback function none of the feedback functions depended on the input sign, further, by observing that in our construction of **B**, φ_1 only depends on the input sign. This ends the proof of the Theorem.

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