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Bibliographie

Advances in Probability Theory: Limit Theorems and Related Topics, Edited by A. A. Borovkov (Translation Series in Mathematics and Engineering), XIV+377 pages, Optimization Software, Inc., Publication Division, New York, 1984. (Distributed by Springer-Verlag.)

This is the English translation of a fine collection of seventeen strong papers on limit theorems and some applications written in the best tradition of the Soviet probability school. The collection was originally published in Russian as Volume 1 of the Proceedings of the Institute of Mathematics, Siberian Branch of the USSR Academy of Sciences, Nauka, Novosibirsk, 1982.

The leading themes are the rate of convergence in the invariance principle and the theory of large deviations, applications include hypothesis testing, branching processes, stochastic differential equations, martingales in the plane, quadratic variation, queueing theory and some other fields. The authors are K. Arndt, I. S. Borisov, A. A. Borovkov, V. M. Borodihin, V. I. Chebotarev, S. G. Foss, G. P. Karev, V. I. Lotov, A. A. Mogul'skij, S. V. Nagaev, G. V. Nedogibchenko, I. F. Pinelis, A. I. Sahanenko, L. Ya. Savel'ev, V. A. Topchij, and V. V. Yurinskij.

Let us hope that the similarly strong subsequent volumes of the Novosibirsk Proceedings will be translated as well, and soon.

Sándor Csörgő (Szeged)

Asymptotic Analysis II-Surveys and New Trends, Edited by F. Verhulst (Lecture Notes in Mathematics, 985), III+497 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

The methods of asymptotic analysis are widely applied in physics, engineering sciences, biology, etc. Research papers on such applications are continually being published in various journals, but collections like the present one are necessary to enable specialists to see what goes on in a broader context. The predecessor of the present volume is *Asymptotic analysis. From theory to applications* (Lectures Notes in Mathematics, 711) published four years ago. A somewhat similar good collection is *Theory and Applications of Singular Perturbations* (Lecture Notes in Mathematics, 942) though the main feature of these proceedings is different from that of the present one and its predecessor.

This volume is divided into three parts: Survey Papers, Survey Papers with Research Aspects, and Research Papers. The list of the authors reads as follows: Z. Schuss and B. J. Matkowsky, V. F. Butuzov and A. B. Vasil'eva, J. Grasman, A. S. Bakaj, F. Verhulst, E. Sanchez-Palencia, V. Drăgan and A. Halanay; Robert E. O'Malley, Jr., P. W. Hemker, Jan A. Sanders, Richard Cushman, A. van Harten; G. G. Rafel, V. Drăgan, J. Grasman adn B. J. Matkowsky, A. H. P. van der Burgh, V. N. Bogaevsky and A. Ya. Povzner, Wiktor Eckhaus.

The main feature is the application of asymptotics to problems concerning certain differential equations, ordinary or partial, and systems motivated by practical problems and depending on parameters. Also many qualitative results are given. The papers in Part 3 "... are settling old questions in the literature and are opening up new lines of research both in the field of methods in applied mathematics and its applications".

J. Hegedűs (Szeged)

J. P. Aubin—A. Cellina, Differential Inclusions (Grundlehren der mathematischen Wissenschaften, 264), XIII+342 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

The differential equation $x'=f(t, x)$ as a model of an evolving system expresses that at every moment the evolution of the system is uniquely determined by its instantaneous state. The existence and uniqueness theorems say that the future (or even the past) of a system described by a differential equation model is uniquely determined by its state at a fixed moment. This cruelly deterministic model can be quite convenient for describing systems that arise in physics, mechanics, engineering if one ignores the influence of the environment on the system. But if one wants to describe macrosystems with mutual effects, some of which can be only estimated, one has to take into account the uncertainty in the model. This uncertainty may be caused by the absence of controls, but it can appear even in a controlled system as a consequence of the ignorance of the laws relating the controls and the states of the system. In such cases the instantaneous state of the system does not determine uniquely the velocity of its change. Mathematically, to every moment t and every state x of the system a set-valued map $F=F(t, x)$ associates to t and x the set of feasible velocities, i.e. $x' \in F(t, x)$.

This new model is perceptibly more complicated and raises new mathematical problems. Even in the simplest cases one needs a pretty wide knowledge from topology and functional analysis. Fortunately, in the two first chapters the authors give a good review on the prerequisite and an excellent introduction to the theory of set-valued maps. The existence problems are treated by the same methods (approximation, fixed-point techniques) as in the theory of differential equations but very interesting new problems arise. For example, while for ordinary differential equations th convergence of the approximate solutions implies the convergence of their derivatives, this is not the case any more for differential inclusions. In fact the convergence of the derivatives will be the main issue in each existence proof. A separate chapter is devoted to the existence and qualitative properties of solutions of differential inclusions with maximal monotone maps.

One can imagine that the set of trajectories is large in this new model, so it is very important in the theory to find a devising mechanism for selecting special trajectories. Such a mechanism is given by Optimal Contral Theory: it selects the trajectories that optimize a given criterion, a functional on the space of all trajectories. The method requires the existence of a decision maker who controls the system having a perfect knowledge of the future and makes a choice once and for all at the origin of the period of time. These requirements are not met by the interactive systems evolving, e.g., according to the laws of Darwinian evolution. They do not optimize any criterion, simply they face a minimal requirement (called viability), they must meet to remain alive. Mathematically, this means that the trajectories have to remain in a set of constraints. The viability theory guarantees the existence of such trajectories. As an application, a dynamical analog is established to the Walras equilibrium theory on the price decentralization in economy. Here the viability constraint is the requirement that the sum of the consumed commodity boundles lies in the set of available goods.

Stability theory for differential inclusions based on Lyapunov functions concludes the book.

This well-written excellent monograph summarizes the new methods and results in this very important modern field. It can be highly recommended to mathematicians and users of mathematics interested in dynamical systems and applications.

L. Hatvani (Szeged)

Michael J. Beeson, Foundations of Constructive Mathematics, Metamathematical Studies (Ergebnisse der Mathematik und ihrer Grenzgebiete, 3. Folge, Band 6) XIII+466 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1985.

The volume, devoted to constructive mathematics, consists of four parts and a historical survey. The first part, entitled "Practice and Philosophy of Constructive Mathematics", is intended to give an overview of some principal ideas on both the philosophy and the foundations of explicite existential proofs. In particular, basic methods and results important on their own right as well as for further usage are collected in this first part. The title of the second part is "Formal Systems of the Seventies". Central to the discussions in this part are three systems: theories of rules, sets and classifications proposed by Feferman, type theories due to Martin-Löf and constructive set theories of Myhill and Friedmann. The third part is devoted, to "Metamathematical Studies". In patricular, Aczel's iterative sets are investigated in detail, and, also, questions related to recursiveness, continuity and uniform continuity are considered. The final part ("Metaphilosophical Studies") is a continuation of the work started by Kreisel and Goodman in developing the "theories of constructions" which serve as the foundation to philosophical discussions on the notion of "constructive proof". The volume ends with a long survey of the historical development of the subject, emphasizing the contributions of the two outstanding forefathers Bishop and Martin-Löf.

This clearly written and attractive book is an exciting and useful reading to most mathematicians interested in the theory of algorithms, proofs and foundations of mathematics and computer sciences.

P. Ecsedi-Tóth (Szeged)

G. W. Bluman, Problem Book for First Year Calculus, (Problem Books in Mathematics), XV+ 385 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

Here is a new excellent book in the series "Problem Books in Mathematics". It contains more than 1000 problems, 300 of which are solved in great detail.

In Chapters I and II we find problems in graphing and geometry: Chapters II, IV, V and VI, respectively, contain problems belonging to physics and engineering, business and economics, biology and chemistry, and numerical methods. Finally, Chapters VII and VIII provide problems on the theory and techniques of calculus.

The organization of the first six chapters is the same. After a short discussion of background material there are several thoroughly solved problems and then proposed problems follow. (Answers to these problems can be found in Chapter IX.)

There are routine and difficult problems, almost all of which are closely connected with practical questions.

Let us mention a few characteristic examples. 1) A sprinter who runs the 100 metre in 10.2 seconds accelerates at a constant rate for the first 25 metres and then continues at a constant speed for the rest of the race. Find his acceleration. 2) A radar antenna is mounted 500 metres horizontally away from, and is aimed at, a rocket sitting on a launching pad. The rocker blasts off at time $t=0$ and thereafter climbs vertically with a constant acceleration of 10 m/s^2. If the antenna remains aimed directly at the rocket, how fast must it be rotating upward 10 sec after blast-off? 3) What should the speed limit be for cars on the Lions Gate Bridge in Vancouver, British Columbia, during rush hour traffic, in order to maximize the flow of traffic? (This is of course an "open-ended" problem. The answer depends on one's assumptions. There are models in the book.) 4) A manufacturer estimates that he can sell 2000 toys per month if he sets the unit price at $ 5.00. Furthermore he estimates that for each $ 0.20 decrease in price his sales will increase by 200 per month. a) Find the demand and revenue functions. b) Find the number of toys that he should sell each month in order to maximize the monthly revenue. c) What is the maximum monthly revenue? 5) A contagious disease, say smallpox, begins to spread in a community of 1000 people. This disease has the property that it spreads by contact and that each person who has it immediately and forever infects others. Initially, one person has it, and the speed of the epidemic appears to lessen after a month. Find how many people have had the disease at any time.

All in all this is a well thought out, stylish book, which one could safely put into the hands of future users of mathematics.

L. Pintér (Szeged)

K. W. Chang—F. A. Howes, Nonlinear Singular Perturbation Phenomena: Theory and Applications, (Applied Mathematical Sciences, 56), VIII+180 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

In applications one often meets differential equation models containing a "small parameter" ε. The equation belonging to $\varepsilon=0$ is called the unperturbed equation and the original one is called the perturbed equation. If the perturbed equation in normal form depends on the small parameter regularly (continuously, smoothly) then the perturbation is called regular, otherwise — singular. In the regular case the solutions of the unperturbed equation are good approximations, but in the singular case the small terms may be neglected only if certain special conditions are met. These are determined by the theory of singular perturbations.

In this book the authors are dealing with those singular perturbations when the basic (scalar or vector) equation is of second order and the second derivative of the unknown function is multiplied by a small parameter.

The best course to characterize the book is to cite some sentences from the authors' preface: "Our purpose in writing this monograph is twofold. On the one hand, we want to collect in one place many of the recent results on the existence and asymptotic behavior of solutions of certain classes of singularly perturbed nonlinear boundary value problems. On the other, we hope to raise along the way a number of questions for further study, mostly questions we ourselves are unable to answer. ... We offer our results with some trapidation, in the hope that they may stimulate further work in the challenging and important area of differential equations."

To the problems treated here the well-known methods, such as the methods of matched asymptotic expansions and two-variable expansions are not immediately applicable, differential inequality techniques coupled with geometric and asymptotic concepts are used instead. The book is concluded by very interesting examples and applications (e.g. equations from theory of nonpremixed combustion, catalytic reaction theory).

This monograph will be very useful not only for experts in singular perturbations, who get a good survey on results that were available before only from articles, but for mathematicians and science students interested in differential equations and their applications.

L. Hatvani (Szeged)

Convex analysis and optimization, Edited by J.-P. Aubin, R. B. Vinter (Research Notes in Mathematics, 57), VIII+210 pages. Pitman Advanced Publishing Program, Boston—London —Melbourne, 1982.

This book consists of 8 papers devoted to surveying new results in convex analysis and its applications in optimization theory. The papers by J. P. Aubin, J. Ekeland and A. D. Ioffe concern nonsmooth analysis. The papers by L. C. Young and R. B. Vinter illustrate the role of convexity in optimization theory and modelling problems even in cases which are not convex in a conventional sense. In their paper J. E. Jayne and C. A. Rogers investigate measurable selections of multivalued mappings. The papers by J. B. Hiriart-Urruty and J. P. Crouzeix treat a generalization of the notion of the subdifferential of a convex function.

László Gehér (Szeged)

John B. Conway, A Course in Functional Analysis (Graduate Texts in Mathematics, 96), XIV + 404 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

This is an excellent textbook on functional analysis. The author's main pedagogic purpose seems to be to help the students acquire an intuition in the subject. According to this purpose, often the book does not discuss immediately the greatest generality possible, but it starts from the particular and works its way to the more general case. The contents are arranged subject to this principle.

Chapter I introduces the geometry of Hilbert space. Chapter II begins the discussion of operators on a Hilbert space. This chapter contains the spectral theory and functional calculus for compact normal operators. In Chapter III Banach spaces are studied. Among others, the Hahn—Banach Theorem, the Open Mapping Theorem and the Principle of Uniform Boundedness are proved. Chapter IV deals with locally convex spaces and Chapter V treats the weak and weak-star topologies. In Chapter VI bounded operators on a Banach space are discussed. The subject of Chapter VII is the discussion of Banach algebras and the spectral theory for operators on a Banach space. Chapter VIII deals with C^*-algebras. Chapter IX presents the Spectral Theorem and its ramification in the framework of C^*-algebras. This chapter is concluded by the multiplicity theory of normal operators. Chapter X deals with unbounded operators in a Hilbert space and Chapter XI is devoted to the Fredholm theory.

Some of the prerequisities are presented in the appendicies. Appendix A contains, among others, a discussion of nets, Appendix B deals with the dual of $L^p(\mu)$ and Appendix C discusses the dual of $C_0(X)$. On the last pages, a rich bibliography is found, a list of symbols is given, and an index completes the book.

A great number of examples helps understand the abstract concepts, makes the presentation colorful, and enlightens the connection with some other branches of mathematics. The book also contains a great number of exercises of varying degrees of difficulty. Some of them are routine, some demand proofs left to the reader in the text, others extend the theory. By all means, the method of the discussion stimulates the reader not only to read but to do mathematics.

The prerequisities for this book are a firm foundation in measure and integration theory and some knowledge of point set topology. Analytic functions are used to furnish some examples and, in the second half of the book, analytic function theory is also applied in the proofs of results.

This book is warmly recommended first of all to graduate students. It contains a fairly large amount of material. If the reader has no time or energy to go through the whole book, he can leave out the sections marked with an asterisk. It could also serve as a handbook for a researcher in functional analysis.

E. Durszt (Szeged)

Jonh B. Conway, Functions of One Complex Variable, Second Edition (Graduate Texts in Mathematics, 11), XI + 317 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

This second edition contains some substantial revisions. These are: the inclusion of John Dixon's treatment of Cauchy's Theorem (keeping the original homotopic version), a new, elementary proof of Runge's Theorem due to Sandy Grabiner, a new appendix containing a guide for further reading and several additional exercises. These changes make this beautiful, succesful work even more exciting. It is written in a very clear style, its reading requires only the knowledge of very basic facts from calculus. This delicious, effective introductory book can be warmly recommended to every student who wants to get acquainted with the classical field of complex analysis.

L. Kérchy (Szeged)

Joseph Diestel, Sequences and Series in Banach Spaces (Graduate Texts in Mathematics, 92), XI+261 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

This volume has been written to acquaint a wide audience of mathematicians with the most important results and methods of the theory of Banach spaces. The author writes in the introduction: "I have concentrated on presenting what I believe are basic phenomena in Banach spaces that any analyst can appreciate, enjoy, and perhaps even use." The technical jargon of the area is avoided, the style is very informal, the text holds the reader. The selected material is arranged into the following chapters: Introduction. — Riesz's Lemma and Compactness in Banach Spaces.—The Weak and Weak* Topologies: an Introduction. — The Eberlien — Šmulian Theorem. — The Orlicz — Pettis Theorem. — Basic Sequences. — The Dvoretsky — Rogers Theorem. — The Claasical Banach Spaces.—Weak Convergence and Unconditionally Convergent Series in Uniformly Convex Spaces.— Extremal Tests for Weak Convegence of Sequences and Series. — Grothendieck's Inequality and the Grothendieck — Lindenstrauss — Pelczynski Cycle of Ideas. An Intermission: Ramsey's Theorem. — Rosenthal's l_1-theorem. — The Josefson — Nissenzweig Theorem. — Banach Spaces with Weak* — Sequentially Compact Dual Balls. — The Elton — Odell $(1+\varepsilon)$-Separation Theorem.

At the end of every chapter several exercises, historical remarks and a detailed bibliography can be found.

L. Kérchy (Szeged)

Differential Geometry and Complex Analysis. A volume dedicated to the memory of Harry Ernest Rauch, Edited by I. Chavel and H. M. Farkas, XIII+222 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

This book is dedicated to the memory of the excellent mathematician Harry Ernest Rauch (1925—1979) who made significant contributions to the areas of pinching theorems of Riemannian manifolds, Teichmüller theory and theta function theory. The volume contains his biography, bibliography and the list of Ph. D. theses written under his supervision. The influence of his work on the progress of differential geometry, complex analysis and theta function theory is presented in the survey articles written by M. Berger, C. J. Earle and H. M. Farkas. The other papers published in this volume give the reader ideas about the new development of the fields of mathematics which are connected with Rauch's researches and interests. 7 papers are devoted to the study of various questions of differential geometry (by E. Calabi, J. Cheeger, M. Gromov, S. S. Chern, D. Gromoll, K. Gove, W. Klingenberg, A. Marden, K. Strebel and S. T. Yau), 3 papers to geometric group theory (L. V. Ahlfors, I. Kra, Min-Oo and E. A. Ruh), 2 papers to complex function theory (J. M. Anderson, F. W. Gehring, A. Hinkkanen and L. Bers) and 1-1 papers to Teichmüller theory (by R. D. M. Accola) and to elliptic differential operators (by L. Nirenberg).

The book is warmly recommended to everybody working in differential geometry and complex function theory.

Péter T. Nagy (Szeged)

Harold M. Edwards, Galois Theory (Graduate Texts in in Mathematics Vol. 101) XII+152 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

This book is a constructive and concrete introduction to Galois theory presenting an exposition of the theory in tems near enough to Galois' original work, the "Mémoire sur les conditions de résolubilité des equations par radicaux" which is contained as Appendix 1 in this book in an English translation due to the author.

It is devided into eight parts, each of which is followed by a set of exercises.

After a brief historical outline of the solutions of quadratic, cubic and quartic equations from 1700 B. C. to 1600 A. D., the Lagrange (Vandermonde) resolvent is introduced and its application in solving equations of degree 2, 3 and 4 is discussed briefly.

A subsequent part is devoted to the cyclotomic equations and to some results of Gauss on constructing p-gons by ruler and compass.

The remaining five parts can be considered, with one major exception, as a detailed version of Galois' original work, which contains full and rigorous proofs (that are missing at Galois). The exception mentioned is the material on factorization of polynomials due to Kronecker. This gives clear meaning to the computations with roots of algebraic equations that Galois and Lagrange performed without inhibition and comment.

Appendices 2 and 3 help the reader understand the original text of Galois making connections with the full (and modern) treatment of the problem contained in the book.

This interesting volume ends with the complete solutions of the exercises.

Lajos Klukovits (Szeged)

Leonard Euler, Elements of Algebra, LX+593 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

The eighteenth century was, par excellence, a textbook age in mathematics, never before had so many books appeared in so many editions. Euler also composed a popular algebra textbook, the famous "Volstandige Anleitung zur Algebra", which appeared at St. Petersburg in 1770. The exceptional didactic quality of this book is attributed to the fact that it was dictated by the blind author through a relatively untutored domestic. In 1774 appeared the French version of this book, translated by M. Bernoulli, with Additions composed by Lagrange.

The present volume is a reprint of the fifth English edition which appeared in 1840. This version is a translation, due to J. Hewlett, of the French edition, which was prefixed by "A Memoire of the Life and Character of Euler" by Francis Horner. It also contains a paper of C. Truesdell under the title "Leonard Euler, Supreme Geometer" which was originally published by the University of Wisconsin Press in 1972.

This book, which summarises the eighteenth century knowledge on determinate and indeterminate quantities and equations, illustrates — as other algebra textbooks of the century — a tendency toward increasingly algorithmic emphasis, while at the same time there remained considerable uncertanity about the logical bases of the subject.

We warmly recommend this book to everybody who wants to read "The Masters".

Lajos Klukovits (Szeged)

L. R. Foulds, Combinatorial Optimization for Undergraduates, XII+228 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

This is a book on combinatorial optimization covering the main topics in this field and written especially for undergraduates. It is a good introductory book in which clarity and plausability have priority over rigorous arguments. Each topic begins with solving a concrete numerical example, the general problem and several algorithms come only afterwards. Some exercises are also provided. Sometimes, when it serves a better understanding of an algorithm, rigorous proofs also occur.

The first two chapters deal with linear programming (simplex method, transportation problem, assignment problem), integer programming and dynamic programming. Chapter 3 deals with the optimization on graphs and networks; minimal spanning trees, shortest paths, the maximum-flow problem, the minimum-cost-flow problem and activity networks are discussed. Chapter 4 handles various problems in operation research, engineering and biology; this chapter includes the travelling salesman problem, the vehicle scheduling problem, evolutionary trees, car pooling and facilities layout.

"The book is intended for undergraduates in mathematics, engineering, business, or the phisical or social sciences. It may also be useful as a reference text for practising engineers and scientists."

G. Czédli (Szeged)

Daniel S. Freed—Karen K. Uhlenbeck, Instantons and Four-Manifolds, (Mathematical Sciences Research Institute Publications, 1), 232 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

This book is devoted to the systematic study of topological, differential geometric and nonlinear analytic methods used for the proof of Donaldson's and Freedman's theorems on the existence of exotic differentiable structures on four-manifolds. As originally Donaldson pointed out, these topological theorems are closely related to the properties of solution spaces of Yang-Mills and self-dual equations. The solution space of self-dual equations on a four-manifold M is divided out by a natural equivalence giving the "moduli space" $\mathfrak{M}$. This moduli space can be regarded as an oriented five-manifold with point singularities, the neighbourhoods of singular points are cones on the complex projective plane and M is the boundary of $\mathfrak{M}$. The geometric description of the moduli space can be applied to the investigation of exotic topologies of the basic manifold M.

The reader is assumed to be familiar with the theory of elliptic differential operators and the smoothing theory of manifolds.

Péter T. Nagy (Szeged)

J. K. Hale—L. T. Magalhães—W. M. Oliva, An Introduction to Infinite Dimensional Dynamical Systems-Geometric Theory (Applied Mathematical Sciences, Vol. 47), 195 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

"The purpose of these notes is to outline an approach to the development of a theory of dynamical systems in infinite dimensions which is analogous to the theory of finite dimensions... the discussion centers around retarded functional differential equations although the techniques and several of the results apply to more general situations; in particular, to neutral functional differential equations, parabolic partial differential equations and some other types of partial differential equations" as the authors write in the introduction.

The book consists of eleven sections and an appendix. The first section comprises an abstract formulation of a class of dynamical systems and propounds some of the basic questions that should be discussed.

Sections 3 and 4 are concerned with retarded functional differential equations on manifolds. An existence and uniqueness theorem is stated and some important properties of the solution map are given.

For ordinary differential equations the Kupka—Smale theorem asserts that the property that all critical points and periodic orbits are hyperbolic and the stable and unstable manifolds intersect transversally is generic in the class of all ordinary differential equations in R^n. Although presently there is no complete proof of the Kupka—Smale theorem for retarded functional differential equations, some interesting results in this direction are proved in Section 4.

The attractivity properties of the invariant sets and limit sets are studied in Section 5.

The set of all initial data of global bounded solutions of a given retarded functional differential equation is called the attractor. The structure of the attractor (dimension, smoothness) is investigated in Sections 6 and 7.

Section 8 discusses the difficulties of the generalization (if it exists) of the Hartman—Grobman theorem for retarded functional differential equations.

Poincaré's compactification method is extended in Section 9 to get the behaviour at infinity of solutions of a linear delay equation.

Section 10 proves stability theorems for Morse—Smale maps.

The book is finished by bibliographical notes (Section 11) and an appendix written by K. P. Rybakowski entitled "An Introduction to the Homotopy Index Theory in Noncompact Spaces".

These notes give not only a unified exposition of the fundamental results in this field but also contain speculations on some of the possible directions for future research.

T. Krisztin (Szeged)

Infinite-Dimensional Systems, Proceedings, Retzhof, 1983. Edited by F. Kappel and W. Schappacher (Lecture Notes in Mathematics, 1076), VII+278 pages, Springer-Verlag, Berlin—Heidelberg —New York—Tokyo, 1984.

This volume contains lectures at the Conference on Operator Semigroups and Applications held in Retzhof, Austria, June 5—11, 1983. "The aim of this conference was to stimulate the exchange of ideas and methods and to provide information on recent advances in various directions of research" as the editors write in the introduction. This aim is completely fulfilled.

Most of the papers have the common feature that they use the so-called semigroup approach. This method can quite effectively be applied for the investigation of partial differential equations or partial functional differential equations when the original equation is rewritten as an ordinary or a functional differential equation in a suitable infinite dimensional Banach space. In addition, there are lectures taken from various branches of the theory of differential equations.

Since there is no room to list the titles of all of the 22 papers, here are only the main key words and phrases to show that this volume is important and interesting for everyone who is interested in the theory and applications of operator semigroups: generators of positive semigroups, Wiener's theorem and semigroups, nonlinear diffusion problems, abstract Volterra equations, stability by Lyapunov functionals, extrapolation spaces, wave propagation, Burgers systems, age-dependent population dynamics, semilinear periodic-parabolic problems, approximation of semigroups, differentiability of semigroups, semigroups generated by convolution equations, the Sharpe—Lotka theorem, integrable resolvent operators, functional evolution equations, rate of convergence in singular perturbations.

T. Krisztin (Szeged)

Dennis Kletzing, Structure and Representations of Q-Groups (Lecture Notes in Mathematics 1084), VI+290 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

A Q-group is a finite group all of whose ordinary complex representations have rationally valued characters. The representation of a group by means of permutations is always a basic question in the theory considered and the work of Frobenius and Young shows that the rationally represented characters of the symmetric and hyperoctahedral groups are generalized permutation characters. One can also mention Artin's theorem in this connection, asserting that a rationally represented character of a group may be written as a linear combination of permutation characters with rational numbers as coefficiens.

The book is a clear exposition and a further extension of this theory concentrating on the following two questions: I. What can be said about the structure of a Q-group? II. Under what circumstances can we conclude that the rationally represented characters of a Q-group are generalized permutation characters? Chapters 1 and 2 are devoted to question I, while Chapters 3 and 4 concentrate on II.

The most important invariant of this theory is the smallest positive integer $\gamma(G)$ with the property that $\gamma(G)\chi$ is a generalized permutation character of the group G whenever χ is a rationally represented character of G. Furthermore, the concepts of local classes, local characters and local splitting are basic in this consideration. The main result of the book is the following theorem proved in Chapter 4: If G is a Q-group which is locally split on every local class, the $\gamma(G)=1$. Several applications of these results to the symmetric groups and Weyl groups are considered.

The volume assumes that the reader is familar in group theory, commutative algebra and the ordinary representation theory of finite groups. It will be of interest to researches working in finite groups and their representation theory and also to algebraic geometers.

Z. I. Szabó (Szeged)

M. A. Krasnosel'skiĭ,—P. P. Zabreĭko, Geometrical Methods of Nonlinear Analysis (Grundlehren der mathematischen Wissenschaften, 263) XIX+409 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

The appearance of the computers has had an influence also on the development of the theory of nonlinear operator equations. The numerical methods have come by a very important role. However, there are many problems in the theory which are essential also from the point of view of practice, and the numerical approach to which is difficult or not possible at all. Such are the stability of solutions, bifurcation and branching of solutions, criteria for the existence of periodic solutions, the analysis of the structure of the solutions, etc. Even the numerical analysis raises such problems for the iteration processes and other procedures. Undoubtedly, the geometrical or topological methods are the most important and widely used tools for the study of these problems. The book, which is a translation of the Russian original edition from 1975, gives a systematic and complete treatment of these methods.

The central concept is an elemantary integer-valued invariant for vector fields called rotation. This tool makes it possible to investigate not only single operator equations, but general classes of operators which permit certain modifications of the equations, e.g., continuous deformation or perturbation of constituents of the equation. The first four chapters are devoted to computing and estimating the rotations of different vector fields and applications to periodic problems. Chapters 5 and 6 deal with the existence of the solutions and nontrivial solutions and with the estimation of the number of solutions for equations with nonlinear operators. Chapter 7 illuminates well how useful this approach can be in many topics of nonlinear analysis and qualitative theory (approximation methods for the solution of nonlinear operator equations and stability theory for instance). Among others, it is proved here that if a system has an isolated equilibrium state which is Lyapunov-stable, then, in general, it cannot be continuously deformed so that it reaches an equilibrium state which is asympotically stable. The last chapter deals with the influence of small perturbations.

The results are illustrated by applications to nonlinear vibrations, nonlinear mechanical problems, nonlinear integral equations and boundary value problems.

This book will be indispensable for mathematicians and other scientists interested in qualitative problems for operator equations.

L. Hatvani (Szeged)

Lie Group Representations III, Proceedings of the Special Year held at the University of Maryland, College Park 1982—1983. Edited by R. Herb, R. Johnson, R. Lipsman and J. Rosenberg (Lecture Notes in Mathematics, 1077), X+454 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

During the academic year 1982—1983, the Department of Mathematics of the University of Maryland conducted its traditional Special Year in the modern theory of Lie group representations. The articles by the invited speakers, distinguished researchers, have been published in a 3-volume set, the last of which is the present book.

The table of contents: 1. L. Corwin: Matrix coefficients of nilpotent Lie groups. 2. L. Corwin: Primary projections on nilmanifolds. 3. L. Corwin: Solvability of left invariant differential operators on nilpotent Lie groups. 4. M. Cowling and A. Korányi: Harmonic analysis on Heisenberg type groups from a geometric viewpoint. 5. M. Duflo: On the Plancherel formula for almost algebraic real Lie groups. 6. M. Felnsted-Jensen: Harmonic analysis on semisimple symmetric spaces. A method of duality. 7. B. Helffer: Partial differential equations on nilpotent groups. 8. S. Helgason: Wave equations on homogeneous spaces. 9. R. Howe, G. Ratcliff and N. Wildberger: Symbol mappings for certain nilpotent groups. 10. H. Moscovici: Lefschetz formulae for Hecke operators. 11. R. Penney: Harmonic analysis on unbounded homogeneous domains in C^n. 12. W. Rossmann: Characters as contour integrals. 13. L. Preiss Rothschild: Analyticity of solutions of partial differential equations on nilpotent Lie groups. 14. V. S. Varadarajan: Asymptotic properties of eigenvalues and eigenfunctions of invariant differential operators on symmetric and locally symmetric spaces. 15. G. J. Zuckerman: Quantum physics and semisimple symmetric spaces.

The collection of the research papers published in this and the other two volumes (Lectures Notes in Mathematics, 1024 and 1041) gives an up-to-date account of the areas of the theory of Lie group representations which are of current interest. So this excellent book is highly recommended to everybody who works in this field or on related subjects of mathematics and mathematical physics.

L. Gy. Fehér (Szeged)

Linear and Complex Analysis Problem Book, 199 Research Problems, Edited by V. P. Havin, S. V. Hruščëv and N. K. Nikol'skii (Lecture Notes in Mathematics, 1043), XVIII+719 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

The editors write in the Preface: "The most exciting challenge to a mathematician is usually not what he understands, but what still eludes him. This book reports what eluded a rather large group of analysts in 1983 whose interests have a large overlap with those of our Seminar." The 199 problems of the volume derive from more than 200 mathematicians dealing with spectral theory or complex analysis all over the world. They show the main trends of research and the likely directions of further development in these areas. This volume is an extensively expanded version of a former Russian edition which contained 99 problems and was published in 1978. Almost half of these previous problems have been partly or completely solved in the meantime. The solutions are included either as commentaries or in the last "Solutions" chapter. The editors organized the material into the following chapters: 1. Analysis in Function Spaces, 2. Banach Algebras, 3. Probabilistic Problems, 4. Operator Theory, 5. Hankel and Toeplitz Operators, 6. Singular Integrals, BMO, H^p, 7. Spectral Analysis and Synthesis, 8. Approximation and Capacities, 9. Uniqueness, Moments, Normality, 10. Interpolation, Bases, Multipliers, 11. Entire and Subharmonic Functions, 12. C^n, 13. Miscellaneous Problems. Each chapter begins with an introduction, in which, as the editors say, "we try to help the reader to grasp quickly the main point of the chapter, to record additional bibliography, and sometimes also the explain our point of view on the subject or to make historical comments".

This book is a representative collection of problems exciting a wide circle of mathematicians. Every specialist of these fields will certainly read it with great pleasure.

L. Kérchy (Szeged)

D. H. Luecking—L. A. Rubel, Complex Analysis, A Functional Analysis Approach (Universitext), VI+176 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

Let G be an open set on the complex plane and let $H(G)$ denote the set of analytic functions defined in G. $H(G)$ is a topological vector space with the topology of uniform convergence on compact sets. The main point of this book is a representation theorem for the dual space of $H(G)$. The central theorems of complex analysis, like Runge's theorem, Cauchy's integral theorem, Mittag-Leffler's theorem, are derived in an easy way from this result. This "approach via duality is entirely consistent with Cauchy's approach to complex variables, since curvilinear integrals are typical examples of linear functionals". At some places the authors digress even to functions of several variables. At the end of each chapter numerous exercises help the understanding.

This nice book can be a stimulating, delicious reading for every student who is familar with the elements of complex analysis.

L. Kérchy (Szeged)

B. Malgrange, Lectures on the Theory of Functions of Several Complex Variables, 132 pages, Tata Institute Lectures on Mathematics, Bombay, and Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

The book is a reissued form of the author's lecture notes published originally in 1958. It provides a very short and well-selected introduction to finite dimensional holomorphic convexity, sheaves, cohomology and Stein manifolds. The treatment is heuristical and it can be considered as almost self-contained: only some quite basic knowledge of holomorphy in one dimension, manifolds with differential forms and locally convex topoligical vector spaces is required for an intelligent reading. The book is divided into three parts: the first focusing on analytic continuation, domains of holomorphy and convexity; the second dealing with d''-cohomology of the cube and the first Cousin problem; the third introducing to the theory of coherent analytic sheaves on Stein manifolds. Each part ends with exercises which give complementary material and with a list of references. There is also a list for supplementary reading but up to date only to 1958.

L. Stachó (Szeged)

J. Marsden—A. Weinstein, Calculus I—II—III (Undergraduate Texts in Mathematics), 1224 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

It is a recurrent care of university lecturers teaching calculus that other sciences (e.g. physics) taught simultaneously to their students require the notions of the derivative and integral very early. In the case of the classical arrangement of topics in a calculus course (real number, limit, continuity, derivation, integration), it is typical that students have to apply notions and methods before they thoroughly get acquainted with them.

Probably it was this challenge that motivated the authors to write this textbook, which is the outgrowth of their experinences while teaching calculus at Berkeley. It is intended for a three-semester sequence with six chapters covered per semesters (four or five student contact hours per week are calculated). At the end of the first semester the students master differentiation and integration and can apply them to solve problems in other sciences. Meanwhile the notion of limit is used intuitively: "If the value of $f(x)$ approximates the number l for x close to x_0, then we say that f approaches the limit l as x approaches x_0." The details about limits involving epsilonics to the different kinds of limits are presented in the second semester, in the context of l'Hospital's rule and infinite series. The differential equations are also found in this semester. The third semester is devoted to functions of vector variables, curves and surfaces, and vector analysis.

The chapters are devided into sections. The first sentence of every section summarizes the main point in italics. Then the notion or method to be introduced is prepared by examples with solutions. The definitions and theorems are placed in emphasizing boxes and are illuminated by further examples and exercises. Every section is concluded by many-many exercises.

The examples and exercises of the book deserve particular attention. Calculus — as the other branches of mathematics — can be picked up only actively, i.e. by solving problems, guessing and proving theorems on one's own way. To do this the students need such well-selected examples and exercises as those in this book. The exercises closing the sections are graded into three consecutive groups: the first exercises are routine, next come those that are still based directly on the examples and the text, the last ones are difficult (these are marked with a star). Answers to odd-numbered exercises are available in the back of the book, and every other odd exercise (i.e. Ex. 1, 5, 9, 13, ...) has a complete solution in the student guide. (The book is supplemented by a Student Guide and an Instructor's Guide.) Anwers to even-numbered exercises are not available to the student. The freshmen who want to know whether or not they posses the prerequisites for reading the book can find several orientation quizzes with answers and a review section to bridge the gap between previous training and the book. The applications show how close calculus is to problems of real world.

As often experienced, it is very difficult to interpret the basic graphs of functions with vector variable and to imagine the complicated surfaces. In this book this is made easier by 1256 figures including plenty of carefully chosen artwork and computer-generated graphics.

This excellent book is highly recommended to every student who wants to learn calculus actively and/or needs it for applications, and to every teacher of calculus who wants to make his classes enjoyable and useful.

L. Hatvani (Szeged)

Tamás Matolcsi, A Concept of Mathematical Physics: Models for Space-Time, 236 pages, Akadémiai Kiadó, Budapest, 1984.

The difficulties physicists encountered in quantum field theory resulted in a crisis of theoretical physics that began in the fifties and lasts up till now. This critical stage of physics bears a strong resemblance to the crisis mathematics faced at the end of the 19th century when a number of paradoxes came to light. The way out from the present troubles of theoretical physics is quite likely to be found in an analogous manner too. What we need is a theoretical physics operating only with notions and only in ways of complete mathematical exactness. As P. A. M. Dirac said: "Any physical or phylosophical ideas that one has must be adjusted to fit the mathematics. Not the way around."

In a critical reaction of mathematical physics, first of all one has to conscientiously analyse even the most common notations. The book under review deals with this task for the space-time notations of physics. It is of primary importance because the entities space-time, matter and field constitute the notational basis present day's physics is built upon.

The book is divided into two parts. Part Ones, Space-Time Models, consists of three chapters. These treat the norrelativistic space-time model, the special relativistic one and the general relativistic space-time models, respectively. Special care is taken for notations like world lines of particles, observers, reference systems and so on. The Galilean, the Lorentz and the Poincaré groups are described in detail. The reader will find useful the exercises given at the end of each chapter of Part One.

The second part of the book is devoted to a concise explanation of the mathematical tools used in the first part. A good account of the material covered here can be given by enumerating the headlines of the chapters in this part: *Tensorial Operations, Pseudo-Euclidean Spaces, Affine Spaces,* Smooth Manifolds, Lie Groups.

Although the exact mathematical formulation of space-time notations of physics constitutes the essence of this work, the physical motivations and interpretations are also sketched by the author. Everyone who is interested in a mathematically clear setting of the basic space-time ideas will find this book useful and enjoyable to read.

L. Gy. Fehér (Szeged)

V. A. Morozov, Methods for Solving Incorrectly Posed Problems, XVIII+257 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

Although the original Russian edition of this book was published ten years ago, it is just as well interesting for mathematicians and engineers today as then.

Several problems of analysis require the solution of the equation (E) $Au=f$, $f\in F$, where $A:D_A\to Q_A$, $D_A\subset U$, $Q_A\subset F$, U and F are metric spaces. The following definition goes back to J. Hadamard: We say that the problem (E) is well-posed if the following three requirements are satisfied: a) Solvability: $Q_A=F$; b) Uniqueness: $Au_1=Au_2$ for any $u_1, u_2\in D_A$ implies $u_1=u_2$; c) Stability: the inverse operator A^{-1} is continuous on F. (This definition was formulated in 1902.) Any mathematical model of a physical problem requires the properties a), b), c). If E does not satisfy all the conditions a), b), c), then this problem is called "ill-posed". Hadamard has constructed an ill-posed problem which became a classical example of the theory. Later many branches of mathematics and natural sciences produced examples involving ill-posed problems, e.g. the continuation for analytic and harmonic functions, automatic control, thermophysics, nuclear physics, the supersonic body problem, biophysical problems etc.

In solving ill-posed problems, the major part of the theory and methods comes from famous Soviet mathematicians. If one has to mention only one name, it is A. N. Tikhonov's. He introduced useful new concepts and discovered several fundamental methods and results. The author of this book also obtained important new results in this field and this fact increases significantly the interest of this book, which in fact discusses mainly the author's investigations. Several theorems can be found in English here for the first time.

L. Pintér (Szeged)

J. D. Murray, Asymptotic Analysis (Applied Mathematical Science, 48), VII+164 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

An earlier version of this book was published by Clarendon Press, Oxford, 1974. Here the very practical Chapter 7 is added on matched asymptotic methods in singular perturbation theory and on multi-scale perturbation methods and suppression of secular terms. The questions discussed in Chapters 1—6 also did not lose their actuality in the past decade. In fact, the domain of possible applications of the methods that are considered in this book becomes wider and wider.

The book is based on lectures given in the mathematics departments at Oxford University and New York University. Chapter 1 contains some necessary definitions, e.g. the definitions of order relations o, O, asymtotic sequences, expansions and series, with many illustrative examples and useful exercises. In the following four chapters the reader can be acquainted with the methods for obtaining analytical approximations, asymptotic expansions to integrals most frequently met in practice, depending on a large or small parameter. In Chapters 6 and 7 the most important asymptotic methods for obtaining asymptotics to solutions of ordinary differential equations with a large or small parameter are given and discussed, for example, the WKB method and the matched expansions. These methods are useful even if the problem posed (strictly speaking) does not have a solution, or if the existence of the solution is not clear.

The table of contents reads as follows. 1. Asymptotic Expansions 2. Laplace's Method for Integrals 3. Method of Steepest Descents 4. Method of Stationary Phase 5. Transform Integrals 6. Differential Equations 7. Singular Perturbation Methods.

The material can easily be covered by undergraduates or graduates with some knowledge of functions of a complex variable and of ordinary differential equations. Many useful examples and exercises are given, there are 25 illustrations and the history of asymptotic methods is also discussed. The book is written in a clear style and without unnecessary details: "Heuristic reasoning, rather than mathematical rigor, is often used to justify a procedure, or some extension of it." It is warmly recommended to everyone interested in differential equations with a large or small parameter and also to other mathematicians or physicists interested in asymptotic expansions, and, finally to every scientists and students interested in mathematical analysis.

J. Hegedűs (Szeged)

Nonlinear Analysis and Optimization, Proceedings of the International Conference held in Bologna, Italy, May 3—7, 1982. Edited by C. Cinti (Lecture Notes in Mathematics, 1107), VI+214 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

The conference was organized in honour of Lamberto Cesari, who has recently completed his 70-th year.

In the first lecture D. Graffi talked "On the contributions of Lamberto Cesari to applied mathematics" analysing his works on nonlinear oscillation and nonlinear optics, or more generally, wave propagation in nonlinear media. The second lecture was an interesting essay on the role of applied mathematics and its connection to the physical world, pure mathematics and other sciences. This question has always given rise to much controversy but everybody will probably agree with the following statements of the author, J. Serrin: "... let me add one overarching principle: 'applied' mathematics should be 'good' mathamatics, and whould be marked by the same clarity which all mathematicians necessarily strive for. ... In summary, mathematics can be both necessary and sufficient, bringing order, elegance and beauty to parts of science which otherwise can seem complex, disjoint and confining."

The papers dedicated to L. Cesari suit the subjects of his scientific activity: A. Bensoussan, J. Frehse, Nash point equilibria for variational integrals; H. W. Engl, Behaviour of solutions of nonlinear alternative problems under perturbations of the linear part with rank change; R. P. Gossez, On a property of Orlicz—Sobolev spaces; P. Hess, S. Senn, Another approach to elliptic eigenvalue problems with respect to indefinite weight functions; S. Hildebrandt, Some results on minimal surfaces with free boundaries; R. Kannan, Relaxation methods in nonlinear problems; K. Kirchgässner, Waves in weakly-coupled parabolic systems; J. Mawhin, M. Willem, Variational methods and boundary value problems for vector second order differential equations and applications to the pendulum equation; M. Roseau, Stabilité de régime des machines tournantes et problèmes associés.

In his paper "Nonlinear optimization" L. Cesari gives existence theorems for multidimensional problems of optimal control and for problems of the calculus of variations concerning integrals of an extended Lagrangian on a multidimensional domain.

L. Hatvani (Szeged)

Yasuo Okuyama, Absolute Summability of Fourier Series and Orthogonal Series (Lecture Notes in Mathematics, 1067), VII+18 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

With the exception of some classical results such as Bernstein's theorems and some of the authors results from the years 1975—81, the results and problems of the theory of absolute summability of Fourier and orthogonal series considered in this monograph are from the years 1935—1973. Its object is to show that a lot of the classical criteria for absolute convergence can be systematically proved from the point of wiew of best approximation. As absolute summability is a natural extension of absolute convergence, this monograph contains several criteria for the absolute summability of non-absolute convergent Fourier series and orthogonal series. First of all the author collected theorems for absolute Nörlund and Riesz summabilities because absolute Cesaro summability is better known to analysts. This fact is also clear from the contents: Absolute Convergence of Orthogonal Series; Absolute Nörlund Summability Almost Everywhere of Fourier Series; Absolute Nörlund Summability Almost Everywhere of Orthogonal Series; Absolute Riesz Summability Almost Everywhere of Orthogonal Series; Absolute Nörlund Summability Factors of Fourier Series; Absolute Nörlund Summability Factors of Conjugate Series of Fourier Series; Local Property of Absolute Riesz Summability of Fourier Series; Local Property of Absolute Nörlund Summability of Fourier Series.

The reference list contains 96 papers. The book is useful for analysts and graduate students of the field.

I. Szalay (Szeged)

D. P. Parent, Exercises in Number Theory (Problem Books in Mathematics), X+541 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

This book is a translation of the French original published in 1978. D. P. Parent is a pseudonym for 12 French authors, who have collected and chosen these problems in number theory. Many exercises in this book were used for the first time in university examinations. The book is not just a simple exercises-book. There is an introduction to each chapter which contains a summary of the fundamental theorems and notations necessary for the solution of the problems. This theoretical preparation makes the book very useful and gives aid for attempting the solutions. Each chapter is devoted to exactly one area of number theory. Of course it could not have been the aim of the authors that the whole theory be covered in their ten chapters. Solutions are provided for all the problems. The level of these problems, over 150 in number, varies. But these problems have a common feature: every one of them represents an important fact in number theory. This in itself is sufficient to praise the careful work of the authors.

László Megyesi (Szeged)

Probability Theory on Vector Spaces III, Proceedings of a Conference held in Lublin, Poland, August, 24—31, 1983. Edited by D. Szynal and A. Weron (Lecture Notes in Mathematics, 1080), V+373 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

Not too long ago, two main characteristics distinguished probability theory as a separate branch of mathematics. One was that this theory used results from almost every other branches of mathematics, the other that this theory rarely contributed to developments in other fields. While the first feature is even more true nowadays, one can assess several streams of a trend opposite to the second.

One particular appearance of this trend is an activity to understand the structure of some abstract vector spaces by means of probabilistic methods. These proceedings fit into this stream, more or less. One of the editors separates three main features of these proceedings. One is the study of stable distributions in abstract settings, represented by four papers. The second one is the investigation of vector-valued processes and Hilbert space methods in stochastic processes, to which topic eight papers are devoted. Weak and strong limit theorems in Hilbert, Orlicz, Banach, or "even" Polish spaces are studied in six papers, three papers deal with ergodic theorems in von Neumann algebras. The remaining four papers are devoted to the study of random functional spaces, elliptically contoured measures, almost sure limits of continuous linear functionals, and to summability questions in Banach lattices.

Those who liked the predecessor Lecture Notes No. 655 and No. 828 will certainly enjoy reading the present continuation.

Sándor Csörgő (Szeged)

R. Remmert, Funktionentheorie I (Grundwissen Mathematik 5), XIII+324 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

Who needs yet another book on complex analysis? Nowadays this question seems to be quite reasonable. If the new book has a characteristic feature together with a careful and clear exposition, then we need it. This book possesses these properties. The presentation, though rigorous, is never fussy. In every chapter the reader finds historical remarks. Perhaps this is the most characteristic of this work. For example, reading the interesting history of the Cauchy Integral Theorem one can better understand the difficulties surrounding this result. The Eisenstein's theory of trigonometric functions is interesting for almost everyone. Sometimes we overlook excellent ideas. This book is rich in such ideas.

I found the first volume a very good book and I am looking forward to the second one.

L. Pintér (Szeged)

R. A. Rosenbaum—G. Philip Johnson, Calculus: Basic Concepts and Applications, XVI+422 pages, Cambridge University Press, Cambridge—London—New York—Rochelle—Sidney—Melbourne, 1984.

Various kinds of books exist on calculus. Nevertheless, when we must choose one for our students we can hardly find any suitable.

The cruicial point is that, taking into account the students'age and thorough grounding in mathematics, the spirit of the development must be intuitive with several examples to provide motivation and to clarify concepts only with just a few proofs, yet, the statements must be careful. Such a development may be a good basis of further courses in analysis. In general, in calculus one emphasizes problem solving rather than theory. This is sometimes misleading. If one cannot resist the temptation, the statements will be careless so that the students will have to unlearn many of them later.

This book is a well organized text. The examples, problems are multifarious and instructive. Let us mention one of them proposed at the end of the first chapter (entitled Functional relationships): "The harmonic series is $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\dots$, a) Calculate $H_n=1+\frac{1}{2}+\dots+\frac{1}{n}$ for $n=1, 2, \dots, 12$. b) Do you think that the H_n will ultimately behave like geometric series with $-1<r<1$, that is, that H_n will get closer and closer to some limiting value as n increases? c) With a programmable calculator or computer, find H_{50}, H_{100} and H_{200}. Also find how large n must be to make $H_n>4$, $H_n>6$ and $H_n>8$. Do these results reinforce or change your guess about the answer to b)? d) You should have found in a) that H_{10} is a little more than 2.9. Note that each of the terms $\frac{1}{11}, \frac{1}{12}, \dots, \frac{1}{99}$

exceeds $\frac{1}{100}$, so $\frac{1}{11}+\frac{1}{12}+\ldots+\frac{1}{100} > \left(\frac{1}{100}\right)90=0.9$. Hence $H_{100}>2.9.+0.9=3.8$. (Actually $H_{100}\approx 5.4$). Similarly the sum of the next 900 terms is greater than 0.9, so $H_{1000}>3.8+0.9=4.7$; and the sum of the next 9000 terms is greater than 0.9, so $H_{1000}>4.7+0.9=5.6$ Use this argument to make a definite statement about H_n as n increases without bound." The authors encourage the students to ask questions constantly: Why is it done this way? Could it not have been accomplished more easily as follows? Is this hypothesis really needed? Does not the following example contradict the statement in the text? How do this problem and its solution compare with other problems I have solved and with situations I know apart from my math course? This as well as the teaching experience striking on every page of the book reminds the reviewer of the outstanding books of Pólya. The authors find the ideal balance between problem solving and theory. The format is pleasing and the printing is excellent. The book is wholeheartedly recommended to students and to teachers who teach calculus.

L. Pintér (Szeged)

Yu. A. Rozanov, Markov Random Fields, IX+201 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

This is the English translation, by Constance M. Elson, of the original Russion edition, Nauka, Moscow, 1980. The present research monograph appears to be the first detailed overview of a relatively new field of research in probability.

When departing from the real line and considering a random field $\{X_t,\ t\in T\subseteq \mathbf{R}^d\}$, $d\geqq 2$, the notions of "past", "present" and "future", crucial to the Markov property of univariate stochastic processes, loose their unique meaning. There are, therefore, several possible ways to define a "Markov property". The one adopted in this book, so far being the most successful one and dating back to P. Lévy and H. P. McKean, is the following: for each open $S\subseteq T$ and for each sufficiently small neighbourhood Γ^ε of the boundary Γ of S, the σ-algebras $\sigma\{X_t,\ t\in \bar{S}\}$ and $\sigma\{X_t,\ t\in T\setminus(S\cup\Gamma)\}$ are independent given the σ-algebra $\sigma\{X_t,\ t\in\Gamma^\varepsilon\}$.

Chapter 1 presents necessary technical prerequisites including consistent conditional distributions and Gaussian processes on infinite-dimensional spaces. Chapter 2 starts with the above definition and studies this Markov property in a systematic manner. Chapter 3 is devoted to the study of the Markov property for generalized random functions, i.e., continuous linear mappings of the Schwartz space of infinitely many times differentiable functions on T into the L^2 space on the underlying propbability space, mainly under the assumption of the existence of a dual process. Finally, Chapter 4 gives conditions, in terms of the spectral density, for a vector-valued stationary generalized random field to be Markov.

Sándor Csörgő (Szeged)

N. Z. Shor, Minimization Methods for Non-Differentiable Functions (Springer Series in Computational Mathematics 3) VIII+162 pages, Springer-Verlag Berlin—Heidelberg—New York—Tokyo, 1985.

The main purpose of this book is to give methods for solving nonsmooth optimization problems; it is an English translation ot the original Russian edition. The text starts with an introductory part (Chapter 1) which introduces and investigates special classes of non-differentiable functions, and defines generalized concepts of gradients — namely the concepts of subgradients — either by making use of separability theorems or via the process of taking limits. In Chapter 2 the generalized gradient methods can be found in detailed various versions. Stepsize selection in most of these methods plays a significant role. The relations of these methods to the methods of Fejér-type approximations are shown and the fundamentals of ε-subgradient methods are briefly presented. Chapter 3 is devoted to the description of gradient-type algorithms with space dilation and to the study of the convergence and speed of convergence of these algorithms. Chapter 4 deals with the use of the subgradient methods in iterative algorithms for solving linear and convex programming problems with the aid of some decomposition schemes.

László Gehér (Szeged)

E. Solomon, Games Programming, XI+257 pages, Cambridge University Press, Cambridge—New York—Melbourne, 1984.

"The time is ripe for owners of home computers to raise their sights" — says the author in the Preface.

This is not a book of program listings. The reader is introduced to fundamental concepts of a full utilisation of the machine. References to particular machines are kept to a minimum. The first part of the book deals with those general aspects of computers which are involved in game programming methodology, e.g. program design methods, language processing, program testing, reading play commands, structured data, text handling, random walks, the mathematics of motion, etc. The second part is concerned with simulation games, e.g. war games and management games, where the computer plays the role of the moderator. The third part discusses the implementation of abstract games in which the machine acts as the opponent. Amongs the topics touched are: algorithms and heuristics, game trees, minimax search, the α—β algorithm, self-improving programs.

I. Gyémánt (Szeged)

D. W. Stroock, An Introduction to the Theory of Large Deviations (Universitext), VIII+196 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

These notes are based on a course which the author gave at the University of Colorado. In the Preface he writes: "My intention was to provide both my audience as well as myself with an introduction to the theory of large deviations." He more than achieved this goal and wrote an excellent textbook as well. The first part is devoted to extensions of Cramér's theorem and the second part deals with the theory of large deviations from ergodic phenomena.

Contents: 0. Introduction — 1. Brownian Motion in Small Time, Strassen's Iterated Logarithm — 2. Large Deviations, Some Generalities — 3. Cramér's Theorem — 4. Large Deviation Principle for Diffusion — 5. Introduction to Large Deviations from Ergodic Phenomena — 6. Existence of a Rate Function — 7. Identification of the Rate Function — 8. Some non-Uniform Large Deviation Results — 9. Logarothmic Sobolev Inequalities.

Lajos Horváth (Ottawa and Szeged)

N. M. Swerdlow—O. Neugebauer, Mathematical Astronomy in Copernicus's De Revolutionibus (Studies in the History of Mathematics and Physical Sciences 10), 204 figures, Part 1: XVI+537 pages, Part 2: 538—711 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

Copernicus's "De Revolutionibus Orbium Coelestium" is considered as one of the last, and historically the most important, astronomical work in the Ptolemaic tradition, i.e., using circular orbits to describe the motions of the planets. On the basis of his own and his Greek, Arabic and European predecessors' observations Copernicus's aim was to develop a new heliocentric planetary theory with a moving earth. He attributed three fundamental and a number of secondary motions to the earth.

Except the first eleven chapters of Book I of the "De Revolutionibus" which was devoted to the general description of the universe and of the location and motion of the earth, the whole work consisting of six books is considered in these two volumes. The omission can be explained by the fact that these topics have been treared by several authors.

Part 1 of this book consists of six chapters. The first one is a general introduction containing a detailed biography of Copernicus and an outline of his astronomy. In the remaining five chapters — the headings are: Trigonometry and Spherical Astronomy, The Motion of the Earth, Lunar Theory and Related Subjects, Planetary Theory of Longitude, Planetary Theory of Latitude — we can read a detailed review on Copernicus's book, on his mathematical methods and on his methods of calculation. Part 2 (in a separate volume) contains tables and figures that illustrate the material of Part 1.

This book is unmatched in depth and detail in the literature on Copernicus, and it can be considered as one of the most detailed exposition of a major scientific work ever written.

Piroska Fekete and Lajos Klukovits (Szeged)

A. N. Tikhonov—A. B. Vasil'eva—A. G. Sveshnikov, Differential Equations, VIII+238 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

The theory of differential equations has a special position among the fields of mathematics. It is directly connected to the practice: the models of many evolving, moving systems and processes in the real word are differential equations. So this theory can be considered as a branch of applied mathematics. On the other hand, it has its own life which is independent of the origin of the problems. In this regard it is one of the fields of pure mathematics that raises and solves very deep problems and is in connection with other fields. According to this twofold feature of the theory, the books on differential equations can be either practice-oriented or of a theoretical character. It is not an easy task to write a good book of the first type, especially because it should be readable and understandable with respect to both the contents and the mode of the presentation for users.

Now here is such a book. It is based on a course which has been taught for several years at the Physics Department and the Department of Computational and Cybernetics of Moscow State University. The reader can find interesting physical problems leading to differential equations. Besides the standard topics (existence, uniqueness, dependence of solutions on initial values and parameters, linear equations, boundary value problems, stability theory, first order partial differential equations), two special chapters are included. In one of them we become acquainted with various methods for numerical solution of initial values as well as boundary value problems, and such fundamental notions as the convergence of difference schemes, approximation and stability. The other one gives a brilliant introduction to the asymptotics of solutions of differential equations with respect to a small parameter (in other words, to the theory of regular and singular perturbations).

The style of the book suits its practice-orientad character. If a new idea to be introduced requires complicated techniques, it is presented at first in the possibly simplest case and only after in the general case.

This English translantion can be recommended even to those who have access to the original Russian edition because it includes important improvements.

L. Hatvani (Szeged)

V. S. Varadarajan, Lie Groups, Lie Algebras and their Representations (Graduate Texts in Mathematics, Vol. 102), XIV+430 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

This is a reprint of an introduction to Lie groups, Lie algebras and their representations of wide-ranging popularity which was originally published in the Prentice-Hall Series in Modern Analysis, 1974.

The algebraic as well as the analytic aspects of the theory of Lie groups and their finite-dimensional representations are discussed in detail in the book. The first chapter gives an introductory exposition of the main results of manifold theory that are used throughout the book. In the second chapter, all the basic concepts and results of the general theory of Lie groups and Lie algebras are introduced. To mention some examples: the Lie and the enveloping algebra of a Lie group, the properties of the exponential map, the adjoint representation and the Baker-Campbell-Hausdorff formula are treated here. The third chapter is devoted to the structure theory of Lie algebras. The most important results discussed in this chapter are: the theorems of Lie and Engel on nilpotent and solvable Lie algebras, Cartan's criterion for semisimplicity, Weyl's theorem asserting the semisimplicity of all finite-dimensional representations of a semisimple Lie algebra, and the decomposition theorems of Levi and Mal'cev. The final, most substantial, chapter contains an exchaustive development of the structure and representation theory of semisimple Lie algebras and Lie groups. The classical Lie algebras and the classification of simple Lie algebras over the field of complex numbers are treated here. The representation theory is examined from both the infinitesimal and the global points of view which is an extraordinary merit of this book.

The book under review excels with the clarity of the exposition of its very extensive subject matter which is, otherwise, even widened further by the large number of exercises placed at the end of each chapter. It is a must for everybody interested in the theory of Lie groups, Lie algebras and their representations, from graduate students to specialists and users of the theory.

L. Gy. Fehér (Szeged)

S. Watanabe, Lectures on Stochastic Differential Equations and Malliavin Calculus, 110 pages, Tata Institute of Fundamental Research, Bombay, and Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

Consider the partial differential equation (∗) $\partial u/\partial t = Au$, $u(0, x) = f(x)$, where A is a second-order differential operator of the form

$$A = \frac{1}{2}\sum_{i,j=1}^{d} a_{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{d} b_i(x)\frac{\partial}{\partial x_i} + c(x), \quad x = (x_1, \dots, x_d) \in \mathbf{R}^d.$$

In typical cases a stochastic differential equation with respect to a vector-valued Wiener process can be associated with (∗) such that the formal expectation

$$p(t, x, y) = E\Big(\exp\Big\{\int_0^t c(X(s,x))\, ds\Big\}\, \delta_y(X(t, x))\Big)$$

where X is the solution of the stochastic equation and δ_y is the Dirac function at $y \in \mathbf{R}^d$, is the fundamental solution of (∗). However, $\delta_y(X(t, x))$ has no meaning as a Wiener functional, a measurable function from a Banach space of continuous vector-valued functions endowed with the supremum norm to $\mathbf{R}^d$. "The purpose of these lectures is to give a correct mathematical meaning to the formal expression $\delta_y(X(t, x))$."

This is a way of presenting Paul Malliavin's infinite-dimensional calculus, introduced in 1976, a stochastic calculus of variations for Wiener functionals. The volume is based on lectures given by the author in 1983 at the Tata Institute, Bangalore, India.

Sándor Csörgő (Szeged)

André Weil, Number Theory: An Approach through History; From Hammurapi to Legendre, XXI + 375 pages, Birkhäuser, Boston—Basel—Stuttgart, 1983.

This book is an historical exposition of number theory. The author examines texts that span roughly thirty-six centuries of arithmetical work from an Old Babylonian tablet, detable to the time of Hammurapi to Legendre's Essai sur la Théorie des Nombres (1798). In this very interesting volume A. Weil accompanies the reader into the workshop of four major authors of number theory: Fermat, Euler, Lagrange and Legendre.

Chapter I., the Protohistory, deals with ancient results, e.g., perfect numbers, Pythagorean triangles (partly after the Old Babylonian tablet PLIMPTON 322), indeterminate equations. In Chapter II we can read about Fermat's and his correspondents' works on number theory, e.g., infinite descent, quadratic residues, the prime divisors of sums of two squares. Chapter III deals with Euler's contributions to the subject: large primes, sums of four squares, square roots and continued fractions, Diophantine equations, zeta-function, etc. The last chapter, An Age of Transition: Lagrange and Legendre, deals with indeterminate equations and binary quadratic forms. All the chapters end with appendices in which the reader can find the modern treatments of some problems mentioned in the text, and some detailed original proofs.

While "... enriched by a broad knowledge of intellectual history, Number Theory represents a major contribution to the understanding of our cultural heritage", we recommend this book to the broad mathematical community.

Lajos Klukovits (Szeged)