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Normal approximation for sums of non-identically distributed random variables in Hilbert spaces

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The problem of estimation of the speed of convergence in the central limit theorem in Hilbert spaces has a history of nearly twenty years. In most papers the speed is studied on the class of balls with a fixed center. This is an important and natural class of sets, but it is not very rich. However, in contrast to the finite dimensional case, one has shown that in Hilbert spaces (in general) it is impossible to construct estimates which are uniform with respect to rich classes, since not even on the class of all balls the speed of convergence in the central limit theorem in Hilbert spaces is uniform (see e.g. [1], p. 70).

The first estimates of order $n^{-1/2}$ on balls with a fixed center under the assumption of finiteness of some moments and without any other additional restrictions (such as independence of coordinates) were obtained by GÖTZE [2]. Following Götze's paper a series of papers appeared, mostly based on Götze approach, which improved and extended his results (see e.g. [3], [4], [5]).

The estimates for the case of independent not necessarily identically distributed random variables (i.non-i.d.r.v.) were obtained by BENTKUS [5] (see Theorem 3.3 in [5] or the condition (5) and the estimate (6) below). The previous results for the case of i.non-i.d.r.v. were obtained by BERNOTAS, PAULAUSKAS [6], ULYANOV [7], [8]. However, corollaries from results [6]—[8] for i.i.d.r.v. give estimates of the order $n^{-1/6}$.

In the present paper we construct estimates on some classes of Borel sets, in particularly on balls with a fixed center, for the case of i.non-i.d.r.v. Our results improve the corresponding estimates of BENTKUS [5], have the "natural" form and at the same time they are obtained under somewhat different conditions. One of the main features of our estimates is that they require minimal moment conditions and their dependence on the (truncated) moments has an explicit form. We shall use the methods of [2], [3], [5] and some ideas from [7].

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In what follows, H is a real separable Hilbert space with inner product (x, y), $x, y \in H$, and norm $|x| = (x, x)^{1/2}$, $D: H \to H$ is a bounded symmetric operator

$$|D| = \sup_{x \neq 0} |(Dx, x)|/|x|^2,$$

$h: H \to R^1$ is a linear continuous functional,

$$W(x) = (Dx, x) + h(x),$$

$$A_{a,r} = \{x \in H: W(x+a) < r\}, \quad r \geqq 0, \quad a \in H;$$

$$N_n = \{1, 2, \ldots, n\}.$$

Let $A: H \to H$ be a bounded symmetric operator with eigenvalues $\sigma_1 \geqq \sigma_2 \geqq \ldots$. We write $A \in G(\beta, k)$, $\beta > 0$, if and only if $\sigma_k \geqq \beta$. Denote $\operatorname{tr} A = \sum_{i=1}^{\infty} \sigma_i$. Let X be a random variable with values in H. Denote by $\tilde{X}$ the symmetrization of X, i.e. $\tilde{X} = X_1 - X_2$, where X_1 and X_2 are independent copies of X. For $\delta > 0$ put

$$X^\delta = X \cdot I_{\{|x| < \delta\}}, \quad X_\delta = X \cdot I_{\{|x| \geqq \delta\}},$$

where I_B is the indicator of the set $B \subset H$. By c (resp. $c(\cdot)$) with or without indices, we denote constants (resp. constants depending only on quantities in the parentheses); the same symbol may stand for different constants. Let λ_i, $i \in N_n$, be any numbers and $\Theta \subset N_n$. Put $\lambda(\Theta) = \sum_{i \in \Theta} \lambda_i$.

Our main result is the following theorem.

Theorem. *Let X_i, $i \in N_n$, be i.non-i.d.r.v. with values in H with zero means and covariance operators A_i, $i \in N_n$, respectively. Assume that there exists an operator A_0 such that*

(1) $$A_i = \lambda_i A_0, \quad \lambda_i \geqq 0, \quad i \in N_n,$$

$$\sum_{i=1}^{n} \lambda_i = 1$$

and

$$\sum_{i=1}^{n} \operatorname{tr} A_i = 1, \quad (DA_0)^2 \in G(\beta, 13), \quad \textit{for some} \quad \beta > 0.$$

Put $S_n = \sum_{i=1}^{n} X_i$, and let Z be a Gaussian $(0, A_0)$ r.v. with values in H,

Then for all $n \geqq 1$

(2) $$\Delta = \sup_{r \geqq 0} |P(S_n \in A_{a,r}) - P(Z \in A_{a,r})| \leqq c_1(1 + |a|^3)(\Lambda_2 + L_3),$$

where $\Lambda_2 = \sum_{i=1}^{n} E|X_{i1}|^2$, $L_3 = \sum_{i=1}^{n} E|X_i^1|^3$, $c_1 = c_1(|D|, |h|, \beta)$.

Corollary 1. *Assume that* $A_0 \in G(\beta, 13)$ *for some* $\beta > 0$. *Then for all* $n \geqq 1$

$$\Delta_1 = \sup_{r \geqq 0} \left| P(|S_n + a| < r) - P(|Z + a| < r) \right| \leqq c(\beta)(1 + |a|^3)(\Lambda_2 + L_3). \tag{3}$$

Corollary 2. *Assume that* $A_1 = A_2 = \dots = A_n$, $A_1 \in G(\beta, 13)$, *for some* $\beta > 0$, $E|X_i|^3 \leqq L$, $i \in N_n$. *Then for all* $n \geqq 1$

$$\Delta_1 \leqq c(\beta)(1 + |a|^3) L n^{-1/2}. \tag{4}$$

Remarks. 1. Our theorem improves estimate (3.14) of Theorem 3.3 in [5]. For completeness we recall the corresponding result proved by BENTKUS [5] (Theorem 3.3).

Let X_i, $i \in N_n$, *be i.non-i. d.r.v. with values in* H *with zero means and covariance operators* A_i, $i \in N_n$, *respectively. Assume that* $\sum_{i=1}^{n} \operatorname{tr} A_i = 1$ *and there exists a nonnegative operator* $A_0 \colon H \to H$ *such that*

$$A_i \geqq A_0 / n, \quad i \in N_n, \tag{5}$$

and $(DA_0)^2 \in G(\beta, k)$, *for some* $\beta > 0, k > 0$. *Let* q, ε, B *be any numbers*, $q > 2$, $0 < \varepsilon \leqq 1$, $B > 0$. *Then there exists a constant* $c = c(\varepsilon)$ *such that if* $k \geqq c$ *then for all* $n \geqq 1$

$$\Delta \leqq c_2(1 + |a^3|)\big(\Lambda_2 + L_3 + (n/\sigma_q^2)^{\varepsilon - 1} + \big(n \max_{1 \leqq i \leqq n} E|X_{i1}|^2\big)^B\big), \tag{6}$$

where $c_2 = c_2(\beta, \varepsilon, q, |D|, |h|, B)$, $\sigma_q = \max_{1 \leqq i \leqq n} (n^{q/2} E|X_i^1|^q)^{1/(q-2)}$.

Thus our theorem shows that the last two terms on the right hand side of (6) may be omitted. At the same time we replace condition (5) by condition (1).

2. Estimate (4) was obtained earlier by YURINSKIĬ [3].

The proof of the theorem is based on a series of lemmas.

Lemma 1. *Let* λ_i, $i \in N_n$, *be nonnegative numbers such that*

$$\sum_{i=1}^{n} \lambda_i = 1, \quad \max_{1 \leqq i \leqq n} \lambda_i \leqq 1/3.$$

Then there exist sets Θ_1, Θ_2 *such that* $\Theta_1 \cap \Theta_2 = \emptyset$, $\Theta_1 \cup \Theta_2 = N_n$, $\lambda(\Theta_i) \geqq 1/3$, $i = 1, 2$.

Proof. It is easy to see that there exists an i_0 such that $\sum_{i=1}^{i_0} \lambda_i \leqq 1/3$, $\sum_{i=1}^{i_0+1} \lambda_i > 1/3$. Put $\Theta_1 = \{1, 2, \dots, i_0 + 1\}$. The above construction implies Lemma 1.

Lemma 2. *Let* λ_i, $i \in N_n$, *be nonnegative numbers such that*

$$\sum_{i=1}^{n} \lambda_i = 1, \quad \max_{1 \leqq i \leqq n} \lambda_i \leqq R \leqq 1/3.$$

Let M_i, $i\in N_n$, *be any nonnegative numbers. Then there exists* $\Theta\subset N_n$ *such that* $R\leqq\lambda(\Theta)\leqq 3R$, $M(\Theta)/\lambda(\Theta)\leqq\sum_{i=1}^{n}M_i$.

Proof. As in Lemma 1 it is easy to see that there exist $\Theta_1, \Theta_2, \ldots, \Theta_m$, such that $\Theta_i\cap\Theta_j=\emptyset$, $i\neq j$, $\bigcup_{i=1}^{m}\Theta_i=N_n$ and $R\leqq\lambda(\Theta_i)\leqq 3R$, $i=1,2,\ldots,m$. Assume that the ratios $M(\Theta_i)/\lambda(\Theta_i)$ are arranged in the following way

$$\frac{M(\Theta_{i_1})}{\lambda(\Theta_{i_1})}\leqq\frac{M(\Theta_{i_2})}{\lambda(\Theta_{i_2})}\leqq\ldots\leqq\frac{M(\Theta_{i_m})}{\lambda(\Theta_{i_m})}.$$

This implies

$$M(\Theta_{i_1})/\lambda(\Theta_{i_1})\leqq\sum_{i=1}^{n}M_i.$$

Put $\Theta=\Theta_{i_1}$. Lemma 2 is proved.

Lemma 3. *Let X be a r.v. with values in H and* $EX=0$, $E|X|^2<\infty$. *Then for any* $z\in H$, $\delta>0$

$$E((\tilde{X}^1)^\delta, z)^2\geqq 2E(X,z)^2-4|z|^2(E|X_1|^2+2E|X^1|^3/\delta), \tag{7}$$

where $\tilde{X}^1=(X^1)^\sim$, $X_1=XI_{\{|x|\leqq 1\}}$.

Proof. We have

$$E((\tilde{X}^1)^\delta, z)^2=E(\tilde{X}^1,z)^2-E((\tilde{X}^1)_\delta,z)^2\geqq E(\tilde{X}^1,z)^2-|z|^2E|(\tilde{X}^1)_\delta|^2. \tag{8}$$

$$E(\tilde{X}^1,z)^2=2E(X^1,z)^2-2(E(X^1,z))^2. \tag{9}$$

As in (8) we get

$$E(X^1,z)^2\geqq E(X,z)^2-|z|^2E|X_1|^2. \tag{10}$$

Since $EX=0$,

$$E(X^1,z)=-E(X_1,z). \tag{11}$$

Moreover

$$|E(X_1,z)|\leqq|z|(E|X_1|^2)^{1/2}. \tag{12}$$

From (9)—(12) it follows that

$$E(\tilde{X}^1,z)^2\geqq 2E(X,z)^2-4|z|^2E|X_1|^2. \tag{13}$$

Furthermore

$$E|(\tilde{X}^1)_\delta|^2\leqq E|\tilde{X}^1|^3/\delta\leqq 8E|X^1|^3/\delta. \tag{14}$$

Using (8), (13) and (14) we get (7). Lemma 3 is proved.

Lemma 4. *Let A, B be any bounded linear operators in H,* $A\geqq 0$. *Then the sets of non-zero eigenvalues of the operators AB, BA and* $A^{1/2}BA^{1/2}$ *are the same (taking into account the multiplicity of the eigenvalues).*

Proof. See VAKHANIA [9], p. 84, or Lemma 2.3 in [5].

Lemma 5. *Let* $X_i(Y_i)$, $i\in N_n$, *be i.non-i.d.r.v. (independent Gaussian r.v.) with values in H with zero means and covariance operators* A_i, $i\in N_n$, *respectively. Let R be any positive number,* $R\leqq 1/6$. *Assume that*

$$A_i=\lambda_i A_0,\quad \lambda_i\geqq 0,\quad i\in N_n,$$

(15) $$\sum_{i=1}^n \operatorname{tr} A_i=\sum_{i=1}^n \lambda_i=1,\quad (DA_0)^2\in G(\beta,13)\ \textit{ for some }\ \beta>0,$$

$$\max_{1\leqq i\leqq n}\lambda_i\leqq R,\quad \Lambda_2\leqq R_\beta/(200|D|^2).$$

Let $\Theta_1\cap\Theta_2=\emptyset$, $\Theta_1\cup\Theta_2=N_n$. *Put* $A=\sum_{i\in\Theta_1}\operatorname{cov}V_i$, $B=\sum_{i\in\Theta_2}\operatorname{cov}V_i$, *where*

(16) $$V_i=(\tilde{X}_i^1)^\delta\ \textit{ or }\ V_i=\tilde{Y}_i,\quad i\in N_n,\quad \delta=400|D|^2(\Lambda_2+L_3)/\beta.$$

Then there exists Θ_1 *such that*

(17) $$(DAD)^{1/2}B(DAD)^{1/2}\in G(R_\beta/2,13),\quad \operatorname{tr}A\leqq 12R.$$

Proof. Let $x\in H$. Put $z=(DAD)^{1/2}x$,

$$\Lambda_2(\Theta)=\sum_{i\in\Theta}E|X_{i1}|^2,\quad L_3(\Theta)=\sum_{i\in\Theta}E|X_i^1|^3.$$

Note that

(18) $$E(\tilde{Y}_i,x)^2=2E(Y_i,x)^2.$$

From (18) and Lemma 3 it follows that

(19) $$(Bz,z)\geqq 2(A_0z,z)\lambda(\Theta_2)-4|z|^2\big(\Lambda_2(\Theta_2)+2L_3(\Theta_2)/\delta\big).$$

Furthermore

(20) $$|z|^2\leqq 4|D|^2|x|^2\lambda(\Theta_1).$$

By Lemma 4 the sets of the non-zero eigenvalues of the operators $(DAD)^{1/2}A_0(DAD)^{1/2}$ and $A_0^{1/2}DADA_0^{1/2}$ are the same. Put $y=DA_0^{1/2}x$. As in (19) we have

(21) $$(Ay,y)\geqq 2(A_0y,y)\lambda(\Theta_1)-4|y|^2\big(\Lambda_2(\Theta_1)+2L_3(\Theta_1)/\delta\big).$$

Moreover

(22) $$|y|^2\leqq 4|D|^2|x|^2\lambda(\Theta_2)\leqq 4|D|^2|x|^2.$$

Since under the assumptions of Lemma 5 $(DA_0)^2\in G(\beta,13)$, by Lemma 4 we have

(23) $$A_0^{1/2}DA_0DA_0^{1/2}\in G(\beta,13).$$

By Lemma 2 there exists a Θ_1 such that

(24) $$R\leqq\lambda(\Theta_1)\leqq 3R,\quad L_3(\Theta_1)/\lambda(\Theta_1)\leqq L_3.$$

Now from (15), (21), (22) and (24) we have for any $x\in H$

$$(A_0^{1/2}DADA_0^{1/2}x, x) \geqq 2(A_0^{1/2}DA_0DA_0^{1/2}x, x)R-16|D|^2|x|^2(\Lambda_2+2L_3(\Theta_1)/\delta) \geqq \tag{25}$$
$$\geqq 2(A_0^{1/2}DA_0DA_0^{1/2}x, x)R-\beta R|x|^2.$$

From (23), (25) and the results of §4, Ch. X in [10] it follows that

$$A_0^{1/2}DADA_0^{1/2}\in G(R\beta, 13). \tag{26}$$

Similarly from (15), (16), (19), (20), (24), (26) and from the simple inequality

$$\operatorname{tr} A \leqq 4\lambda(\Theta_1) \tag{27}$$

we get (17). Lemma 5 is proved.

Proof of Theorem. Put $\hat{S}_n^1=X_1^1+X_2^1+\dots+X_n^1$, $F(r)=P(S_n\in A_{a,r})$, $F_1(r)=P(\hat{S}_n^1\in A_{a,r})$, $b(r)=P(Z\in A_{a,r})$. Let $f(t)$ and $g(t)$ be the Fourier—Stieltjes transforms of F_1 and b respectively. Using the inequality

$$|b'(r)| \leqq \int_{-\infty}^{\infty} g(t)\,dt,$$

the symmetrization inequality (see Lemma 2.1 in [3]) and Lemma 2.4 in [5] we get

$$|b'(r)| \leqq c(|D|, \beta)(1+|a|^3). \tag{28}$$

It is easy to see that

$$|F(r)-F_1(r)| \leqq \sum_{i=1}^{n} P(|X_i| \geqq 1) \leqq \Lambda_2. \tag{29}$$

By Theorem 2, §1, Ch. 5 in [11], from (28), (29) we have for any $T>0$

$$|P(S_n\in A_{a,r})-P(Z\in A_{a,r})| \leqq \Lambda_2+c(1+|a|^3)/T+\int_{-T}^{T} |t|^{-1}|f(t)-g(t)|\,dt. \tag{30}$$

Now we estimate $|f(t)-g(t)|$. By Theorem 4.6 in [12] we have

$$|f(t)-g(t)| \leqq c(|t|+|t|^3)(1+|a|^3)(\Lambda_2+L_3)\varkappa(t), \tag{31}$$

where

$$\varkappa(t) = \sup_V \sup_M \inf_{\bigcup_{i=1}^{5} B_i=N_n\setminus M} \sup_{1\leqq q\leqq 4} \inf_{B\subset B_q} E^{1/4} \exp\{2it(\sum_{j\in B} DV_j, \sum_{j\in B_q\setminus B} V_j)\},$$

and M is any subset of N_n containing not more than one element, $V=(V_1, V_2, \dots, V_n)$, $V_i=\tilde{X}_i^1$ or $V_i=\tilde{Y}_i$, $i\in N_n$, $Y_1, Y_2, \dots, Y_n$ are independent Gaussian r.v.'s with zero means and covariance operators $A_1, A_2, \dots, A_n$, respectively.

Denote $\Lambda=\Lambda_2+L_3$. The left hand side of (2) is not greater than 1. Hence we may assume without loss of generality that

(32) $$\Lambda \leqq c_3,$$

where c_3 is a constant small enough. In fact, if (32) is not true then (2) is obvious with $c_1 \geqq 1/c_3$. We may also assume that

(33) $$\max_{1\leqq i\leqq n} \lambda_i \leqq \Lambda^{2/25}.$$

In fact, if there exists an i_0 such that $\lambda_{i_0}>\Lambda^{2/25}$ then $\operatorname{tr} A_{i_0}>\Lambda^{2/25}$. Since $\operatorname{tr} A_{i_0}= =E|X_{i_0}|^2$ we have $E|X_{i_0}|^2>\Lambda^{2/25}$. Moreover,

$$E|X_{i_0}|^2 = E|X_{i_01}|^2+E|X_{i_0}^1|^2.$$

Now we consider two possible cases.

Case 1. $E|X_{i_01}|^2 \geqq E|X_{i_0}^1|^2$. Then $E|X_{i_01}|^2>\Lambda^{2/25}/2$. Since $\Lambda \geqq E|X_{i_01}|^2$ we have $\Lambda>\Lambda^{2/25}/2$, that is $\Lambda>(1/2)^{25/23}$. This contradicts assumption (32).

Case 2. $E|X_{i_0}^1|^2>E|X_{i_01}|^2$. Then $E|X_{i_0}^1|^2>\Lambda^{2/25}/2$. We have

$$\Lambda \geqq \frac{(\Lambda^{2/25}/2)^{3/2}}{(E|X_{i_0}^1|^2)^{3/2}}\cdot\Lambda > \frac{\Lambda^{3/25}}{2^{3/2}}\cdot\frac{E|X_{i_0}^1|^3}{(E|X_{i_0}^1|^2)^{3/2}} \geqq \Lambda^{3/25}/2^{3/2}.$$

Hence $\Lambda>(1/2)^{75/44}$. This also contradicts assumption (32).

Furthermore, from the definition of $\varkappa(t)$, (33) and Lemma 1 it follows that if we denote

(34) $$\delta_1^4 = \inf_{\Theta_1, \Theta_2} E\exp\{2it(\sum_{j\in\Theta_1} DV_j, \sum_{j\in\Theta_2} V_j)\}, \quad \delta_1>0,$$

where $\Theta_1\cap\Theta_2=\emptyset$ and

(35) $$\sum_{i\in\Theta_1\cup\Theta_2} \lambda_i \geqq c,$$

then

(36) $$\varkappa(t) \leqq \delta_1.$$

Note that for any symmetric r.v. X, $z\in H$, $\delta>0$ we have

$$E\exp\{i(z,X)\} \leqq E\exp\{i(z,X^\delta)\}.$$

Therefore

(37) $$\delta_1^4 \leqq \inf_{\Theta_1, \Theta_2} E\exp\{2it(\sum_{j\in\Theta_1} DV_j', \sum_{j\in\Theta_2} V_j')\},$$

where $V_j'=V_j^\delta$ if $V_j=\tilde{X}_j^1$ and $V_j'=V_j$ if $V_j=\tilde{Y}_j$, $j\in\Theta_1\cup\Theta_2$. By Lemma 2.5 in [5] we get for all t and M that

$$4|D|\delta\varrho|t| \leqq 1, \quad 4|D|M\delta\max\{\gamma,\varrho\} \leqq 1,$$

(38) $$\delta_1^4 \leqq c\inf_{\Theta_1, \Theta_2}\left(\exp(-\varrho^2/(\operatorname{tr} A+\delta\varrho))+\exp(-\gamma^2/\operatorname{tr} B)+\varphi(s)\right),$$

where $s=\min\{2|t|, M\}$, $\varphi(s)=E\exp\{\text{is}(DU, V)/2\}$, U, V are independent Gaussian r.v.'s with zero means and covariance operators $A=\sum_{j\in\Theta_1}\operatorname{cov} V_j'$ and $B=\sum_{j\in\Theta_2}\operatorname{cov} V_j'$, respectively. From (1.4) in [3] it follows that

(39) $$\varphi(s)=\prod_{j=1}^{\infty}(1+s^2\beta_j/4)^{-1/2},$$

where $\beta_j, j=1, 2, \dots$ are the eigenvalues of $(DAD)^{1/2}B(DAD)^{1/2}$.

Now we estimate the right hand side of (38) for different values of t.

Case 1. $|t|\leqq c/(\Lambda\ln(1/\Lambda))$. Put $\gamma=c\ln^{1/2}(1/\Lambda)$, $\varrho=c\ln(1/\Lambda)$, $\delta=c\Lambda$, $M=c/(\Lambda\ln(1/\Lambda))$. From (1), (27) we get

(40) $$\operatorname{tr} A+\operatorname{tr} B\leqq 4.$$

Hence for the above values of $\gamma, \varrho, \delta, M$ we have

(41) $$\exp\left(-\varrho^2/(\operatorname{tr} A+\delta\varrho)\right)+\exp\left(-\gamma^2/\operatorname{tr} B\right)\leqq c\Lambda^c.$$

By Lemma 5 and (32), (33), (35) there exist Θ_1 and a constant c such that

(42) $$(DAD)^{1/2}B(DAD)^{1/2}\in G(c\beta, 13).$$

From (39), (42) we have

(43) $$\varphi(s)\leqq c(1+t^{13})^{-1}.$$

Case 2. $c/(\Lambda\ln(1/\Lambda))\leqq|t|\leqq c/\Lambda$. Put $\gamma=c\ln^{1/2}(1/\Lambda)$, $\varrho=c$, $M=c/(\Lambda\ln(1/\Lambda))$, $\delta=c\Lambda$. By Lemma 5 and (32), (33), (35) there exist Θ_1 and a constant c such that

(44) $$(DAD)^{1/2}B(DAD)^{1/2}\in G(c\Lambda^{2/25}\beta, 13),\quad \operatorname{tr} A\leqq c\Lambda^{2/25}.$$

From (39), (40), (44) we have

(45) $$\delta_1^4\leqq c\left(\Lambda^c+(1+\Lambda^{1/25}/(\Lambda\ln(1/\Lambda)))^{-13}\right)\leqq c\left(\Lambda^c+\Lambda^{312/25}\ln^{13}(1/\Lambda)\right).$$

Furthermore, put $T=c/\Lambda$, $T_1=c/(\Lambda\ln(1/\Lambda))$. From (31), (34), (36), (37), (41), (43), (45) we get

(46) $$\int_{-T}^{T}|t|^{-1}|f(t)-g(t)|\,dt\leqq c(1+|a|^3)\Lambda\int_{-T}^{T}(1+t^2)\varkappa(t)\,dt\leqq$$
$$\leqq c(1+|a|^3)\Lambda\int_{-T}^{T}(1+t^2)\delta_1\,dt\leqq c(1+|a|^3)\Lambda\left(\int_{-T_1}^{T_1}+\int_{T_1}^{T}+\int_{-T}^{-T_1}\right)(1+t^2)\delta_1\,dt\leqq$$
$$\leqq c(1+|a|^3)\Lambda\left(1+\int_{-\infty}^{\infty}t^2(1+t^{13/4})^{-1}\,dt+\Lambda^{0.12}\ln^{13/4}(1/\Lambda)\right)\leqq c(1+|a|^3)\Lambda.$$

Estimates (30) and (46) imply (2). The theorem is proved.

Proof of Corollary 1. This follows from the theorem. To this end it is enough to put $h=0$, $D=I$, the identity operator, and to note that if $A_0 \in G$ $(\beta, 13)$, then $A_0^2 \in G(\beta^2, 13)$.

The proof of Corollary 2 is obvious.

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