Bibliographie

M. Barr—C. Wells, Toposes, Triples and Theories (Grundlehren der mathematischen Wissenschaften, Band 278), XIII+345 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

This book provides an introduction to the three concepts of topos, triple and theory, and describes the connections between them. Among the three topics, topos theory is central in the book. That reflects the current state of development and the importance of topos theory as compared to the other two.

A topos is a special kind of a category defined by axioms saying roughly that certain constructions one can make with sets can be done in the category. However, the notion of topos originated as an abstraction of the properties of the category of sheaves of sets on a topological space. Later, mathematicians developed the idea that a theory in the sense of mathematical logic can be regarded as a topos. In this book a topos is regarded as being defined by its elementary axioms, saying nothing about set theory in which its models live. One reason for this attitude is that many people regard topos theory as a possible new foundation for mathematics.

Chapter 1 is an introduction to category theory which develops the basic constructions in categories needed for the rest of the book. A reader familiar with the elements of category theory including adjoint functors can skip nearly all of these. Chapters 2, 3 and 4 introduce the three topics of the title, respectively, and develop them independently up to a certain point. Each of them can be read without the other two chapters. Chapter 5 develops the theory of toposes further, making use of the theory of triples. Chapter 6 covers various fundamental constructions which give toposes, with emphasis on the original idea that toposes are abstract sheaf categories. Chapter 7 provides the basic representation theorems for toposes. Theories are then carried further in Chapter 8, making use the representation theorems. Chapter 9 develops further topics in triple theory, and may be read independently after Chapter 3.

The book provides a fairly thorough introduction to topos theory covering topologies and representation theorems but omitting the connection with algebraic geometry and logic. Each chapter ends with a list of exercies. The exercises provide examples or develop the theory further.

The book can be read by graduate students with a familiarity with elementary algebra.

Gy. Horváth (Szeged)

Kenneth L. Bowles—Stephen D. Franklin—Dennis J. Volper, Problem Solving Using UCSD Pascal. Second Edition with 106 illustrations, XI+340 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

This book is designed both for an introductory course in computer problem solving and for individual self study. This is a revised and extended edition of one of the most successful introductions to Pascal, "Microcomputer Problem Solving Using Pascal", Springer-Verlag, 1977.

In practice, it seems best to learn programming first using Pascal, and then to shift over to one of the other languages. Pascal is clearly the best language now in widespread use for teaching the concepts of structured programming at the introductory level. Structured programming is a method designed to minimize the effort that the programmer has to spend on finding and correcting logical errors in programs. Put more positively, it is a method designed to allow a program to produce correct results with a minimum amount of effort on the part of the programmer. The key to structuring in Pascal is the procedure. Therefore, this book introduces and emphasizes procedures from the very start. Procedures for graphics are introduced in the first chapter and the first programming assignment requires the student to use them. By the second chapter, students are writing and using their own procedures. The early and continuing emphasis on procedures is the major organizational difference between this book and most others which, in contrast, often do not introduce procedures until a third or even half way through their coverage of Pascal.

The subject matter has been chosen to be understandable to all students at about the college freshman level, with almost no dependence on a background in high school mathematics beyond simple algebra. At an introductory level, the basic methods of programming and problem solving differ very little between applications in engineering and science and applications in business, arts or humanities. It is possible to motivate and teach students across this entire spectrum using problem examples with a non-numerical orientation: the manipulation of text strings and graphics.

Chapter 1 of the book introduces the students to the use of the UCSD software system for Pascal. Chapter 2 through 7 present basic tools for programming and for expressing algorithms. Chapter 8 through 12 add tools for working with data transmitted to the computer from external devices and for working with complex data. Chapters 13—15 provide illustrations of complex problems of types that are frequently encountered by virtually all programmers. The appendices at the end of the book are included for reference purposes and survey some additional features of the Pascal language. Each chapter starts with a statement of the objectives that the students should attain before proceeding to the next chapter.

If you have a personal computer, the UCSD software system for Pascal, and this book in addition, you can easily learn to write computer programs and will master the bases of computer problem solving. The book is equally suitable as a textbook of a course.

K. Dévényi (Szeged)

Differential Geometric Methods in Mathematical Physics, Proceedings, Clausthal, August 30—September 2, 1983. Edited by H. D. Doebner and J. D. Hennig (Lecture Notes in Mathematics, 1139), VI+337 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

The aim of this series of conferences is to promote the application of geometrical, analytical and algebraic methods and their interplay for the modelling of complex physical systems. This volume contains 20 papers submitted by participants of the conference (the 12th in the series) organized by H. D. Doebner, S. I. Andersson (Clausthal) and G. Denardo (Trieste) at the Institute for Theoretical Physics A, Technical University of Clausthal, F.R.G. The main topics treated in the proceedings are described by the following key words which are also the titles of the chapters: Momentum Mappings and Invariants — Aspects of Quantizations — Structure of Gauge Theories — Non-Linear Systems, Integrability and Foliations — Geometrical Modelling of Special Systems.

The volume is dedicated to the memory of the outstanding mathematician and mathematical physicist Steven M. Paneitz, who died in a tragic accident while attending this conference at the age of 28.

Péter T. Nagy (Szeged)

Dynamical Systems and Bifurcations (Proceedings, Groningen 1984). Edited by B. L. J. Braaksma, H. W. Broer and F. Takens (Lecture Notes in Mathematics, 1125), 129 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

Readers paying attention regularly to the review section of these *Acta* have certainly realized that nowadays more and more monographs, texts and conferences are devoted to the modern theory of differential equations and especially to bifurcation theory. This is of course due to the interest in the topic, both in pure and applied mathematics. The present volume is the proceedings of the International Workshop on Dynamical Systems and Bifurcations, organized by the Department of Mathematics of Groningen University, April 16—20, 1984.

Almost all of the articles are concerned with the geometric theory of dynamical systems (papers by M. Chaperon, R. Dumortier, A. Floer and E. Zehdner, J. Palis and R. Roussavie, F. Takens, G. Vegter). One paper (written by R. Dieckerhoff and E. Zehdner) deals with the boundedness of the solutions of a second order timedependent nonlinear differential equation, and another one (by J. A. Sanders and R. Cushman) is devoted to global Hopf bifurcations.

L. Hatvani (Szeged)

E. Grosswald, Representations of Integers as Sums of Squares, XI+251 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

There are a number of branches of mathematics which seem to have no use in practical applications and in which the problems are merely investigated for their own beauty. The first of these is clearly the theory of numbers, called "The Queen of Mathematics" by Gauss. Everybody knows something about integers. Here many questions or problems can be formulated within the capacity of the man of the street. These questions are sometimes so interesting as the best puzzles having surprising results.

One of the oldest problems is the finding of Pythagorean triangles, right triangles whose sides are integers. The next question is which of the integers can be represented as a sum of two or more squares? Is it true that every integer can be written as a sum of four squares? These are curious questions in themselves and surprisingly they do have applications, for example in lattice point problems, in crystallography and in certain problems of mechanics. In the last few decades it was in fact practice that has raised several questions which can be answered by the aid of number-theoretic results.

This book contains the enlarged versions of lectures given by the author in 1980—1981 for an audience consisting of particularly gifted listeners with widely varying background. Therefore the theme of this work, starting at the most elementary level but pushing ahead as far as possible in some directions, seems to be ideal. Besides the elementary — but by no means simple — methods there appear various other notions from other branches of mathematics too, e.g. quadratic residues, elliptic and theta functions, complex integration and residues, algebraic number theory etc.

The list of mathematicians who have obtained results in the study of the problems in this book includes so illustrious names as Diophantus, Fermat, Lagrange, Gauss, Artin, Hardy, Littlewood, Ramanujan, Siegel and Pfister for instance.

The author made a serious effort to make the book accessible to a wide circle of readers. He follows roughly the historical development of the subject matter. Therefore, in the early chapters, the prerequisities are minimal. More advanced tools are necessary in the later chapters.

A number of interested problems is proposed at the end of many chapters, ranging from fairly routine to difficult ones. Also, open questions are mentioned.

Finally we cite the last sentence of the author's "Introduction": "The success of this book will be measured, up to a point, by the number of readers who enjoy it, but perhaps more by the number who are sufficiently stimulated by it to become actively engaged in the solution of the numerous problems that are still open."

L. Pintér (Szeged)

Erich Hecke, Lectures on the Theory of Algebraic Numbers (Graduate Texts in Mathematics, Vol. 77), Springer-Verlag, New York—Heidelberg—Berlin, 1981.

This is a translation of Erich Hecke's classic book, originally published in German in 1923. It is a welcome event that, after sixty years of its first edition, this excellent book became available to the English-reading public.

The first three chapters are of preparatory character: Chapter I contains the elements of the theory of rational integers, Chapter II summarizes the basic facts on Abelian groups that are needed later on, and Chapter III deals with the structure of the group $\mathfrak{R}(n)$ of the residue classes mod n, relatively prime to n. The theory of algebraic numbers starts in Chapter IV with a short introduction into the algebra of number fields (Galois theory is not discussed). Ideal theory, the core of algebraic number theory, is developed in Chapter V. Next, in Chapter VI, analytic methods are applied to the problem of the class number and to the problem of the distribution of prime ideals. Quadratic number fields are treated in detail in Chapter VII; in particular, the Quadratic Reciprocity Law is derived, and the class number of $k(\sqrt{d})$ is determined with, as well as without, the use of the zeta-function. Finally, Chapter VIII is devoted to the proof of the Quadratic Reciprocity Law in arbitrary algebraic number fields.

After that a book, like this one, had been in continuous use for sixty years, there is no doubt it will serve new generations of students and mathematicians who want to get acquainted with algebraic number theory. There are, however, two things which the present-day reader will miss: a subject index, and a short survey of what has happened in the field since 1923.

Unfortunately, in certain places the translation is not quite correct.

Ágnes Szendrei (Szeged)

Dan Henry, Geometric Theory of Semilinear Parabolic Equations (Lecture Notes in Mathematics, 840), IV+348 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

Nowadays the geometric theory of differential equations is an intensively developing branch of mathematics. The present book also deals with such general questions as existence, uniqueness, continuous dependence of solutions on the right-hand side, the semigroup properties of solutions, stability questions for nonlinear semigroups, Lyapunov functions, invariance principle, existence of equilibria and periodic solutions and the behaviour of solutions in their neighbourhood, questions of bifurcation, the application of the adjoint system, invariant manifolds and the behaviour of solutions in their small neighbourhoods, exponential dichotomies, orbital stability, perturbations of right-hand side, etc. The feature of the book is that the main results are formulated for general differential equations written in Banach spaces. The conditions of the theorems can be met by semilinear parabolic differential equations. Typically, the linear part of the right-hand side of the equation is a sectorial operator.

The questions studied are well-prepared by examples. The results are illustrated by interesting applications concerning nonlinear parabolic equations that arose in physical, biological and engineering problems. One of the chapters gathers the examples that accure in the book. The titles of the sections of this chapter show a wide range of the applications: Nonlinear heat equation, Flow of electrons and holes in a semiconductor, Hodgekin—Huxley equations for nerve axon, Chemical reactions in a catalyst pellet, Population genetics, Nuclear reactor dynamics, Navier—Stokes and related equations.

There are sections concerned with special problems of semilinear parabolic equations, e.g. the existence of travelling waves. Some exercises make this book interesting. They can be well comprehended after the study of the material, but the author also gives hints in more difficult cases.

This book is of interest to experts of the geometric and qualitative theories of differential equations. I feel it essential for researchers of parabolic partial differential equations. It gives a well-structured theoretical background and shows the directions of interesting applications.

J. Terjéki (Szeged)

E. Horowitz, Fundamentals of Programming Languages, Second Edition, XV+446 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

Traditional books on programming languages are like abbreviated language manuals. This book takes a fundamentally different point of view by focusing on a few essential concepts. These concepts include such topics as variables, expressions, statements, typing, procedures, data abstractions, exception handling and concurrency. By understanding what these concepts are and how they are realized in different programming languages, the reader arrives at a level of comprehension far higher than what can be achieved by writing programs in various languages. Moreover, the study of these concepts provides a better understanding of future language designs.

Chapter 1 is devoted to the study of the evolution of programming languages. Twelve criteria by which a programming language design can be judged are presented in Chapter 2. Subsequent chapters develop the concepts mentioned above. Much work has been done on imperative programming languages. The last three chapters cover non-imperative features such as functional programming, data flow programming and object oriented programming. A large number of exercises enriches the text.

This book is warmly recommended as a good text for a graduate course whose objective is to survey the fundamental features of current programming languages.

Gy. Horváth (Szeged)

O. A. Ladyzhenskaya, The Boundary Value Problems of Mathematical Physics (Applied Mathematical Sciences, 49), XXX+322 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

If an instantaneous state of a physical system is described by a function (e.g. string, membrane, temperature of a body, fluid stream) then the mathematical model of the system is often a second order partial differential equation. The purpose of mathematical physics is to study these equations using pure mathematical methods. The basic problems are the following: Under which conditions on the domain and on the functions involved in the equation does the equation have a solution with prescribed values on the boundary? Is the solution unique? How can the solutions be approximated by solutions of simpler problems?

The book is concerned with these problems for linear equations. The investigations for the existence and uniqueness of the solutions of boundary value problems are governed by a very natural principle. If a boundary value problem admits more solutions then it cannot be a good model of a uniquely determined physical process. So the first step is to find conditions for the uniqueness of the solutions. By experience, in case of linear problems the existence of the solutions is a consequence of their uniqueness (e.g. for systems of n linear algebraic equations in n unknowns). The author tries to establish this principle also for boundary value problems introducing various classes of generalized solutions.

There are other features of the book. The author considers elliptic, parabolic and hyperbolic equations alike. She studies also the smoothness of the generalized solutions and the connection between the generalized and classic solutions. To illuminate this, let us have a look at Chapter 2 devoted to the elliptic equation. First the solvability of the boundary value problems is established in the space $W_2^1(\Omega)$ consisting of all the elements in $L_2(\Omega)$ that have generalized first derivatives in $L_2(\Omega)$. The problems are reduced to equations with completely continuous operators and their Fredholm solvability is proved. Examples show that these solutions may not have second order derivatives (not even generalized derivatives either), and they salitisfy the conditions of the problem in some generalized sense. It is proved that under a minor improvement of the functions in the equation and a certain increase in the smoothness of the boundary, all generalized solutions in $W_2^1(\Omega)$ belong to $W_2^2(\Omega)$ and satisfy the equation in the ordinary sense for almost all $x\in\Omega$.

The lengthy final chapter is devoted to the Method of Finite Differences. Introducing grids in the basic domain, the method reduces various problems for differential equations to systems of algebraic equations in which unknowns are the values of grid functions at the vertices of the grids. The limit process obtained when the lengths of the sides of the cells in the grid tend to zero is also examined here. The author is one of the pioneers of proving convergence in $L_2(\Omega)$-norm in case of initial-boundary value problems for hyperbolic equations.

The original Russian edition is supplied in the present translation by "Supplements and Problems" located at the end of each chapter. They are useful to awaken the reader's creativity by providing topics for independent work.

This excellent monograph is highly recommended to everyone interested in the theory of partial differential equations and its applications.

L. Hatvani (Szeged)

S. Lang, Complex Analysis, VII+367 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

The first edition of this book was published in 1977. The author has rewritten many sections, added some new material and made a number of corrections.

The book consists of two parts. Part I (Basic Theory) is intended as an introduction to complex analysis for use by advanced undergraduates and beginning graduate students. Part II (Various Analytic Topics) contains more advanced topics, the chapters here are titled as follows: Applications of the maximum modulus principle; Entire and meromorphic functions; Elliptic functions; Differentiating under an integral; Analytic continuation; The Riemann mapping theorem.

The author has made the necessary material on preliminaries as short as possible to get quickly to power series expansions and Cauchy's theorem. In every book on complex functions, one of the most interesting questions is the presentation of Cauchy's theorem. Perhaps the author's words about this problem enlighten the basic attitude of the book: "I have no fixed idea about the manner in which Cauchy's theorem is to be treated. In less advanced classes, or if time is lacking, the usual hand waving about simple closed curves and interiors is not entirely inappropriate. Perhaps better would be to state precisely the homological version and omit the formal proof. For those who want a more through understanding, I include the relevant material." He has included both Artin's proof and the more recent proof of Dixon for the theorem.

Several solved examples illustrate the results making easier the understanding and carefully selected exercises are proposed.

The chapters of Part II are logically independent and can be read in any order. Here we mention only one of the most striking applications, generally omitted from standard courses, the problem of transcendence: Given some analytic function f, describe those points z such that $f(z)$ is an algebraic number.

The most attractive feature of the book for the reveiwer is the successful composition of the classical and modern methods and results.

L. Pintér (Szeged)

László Leindler, Strong Approximation by Fourier Series, 210 pages, Akadémiai Kiadó, Budapest, 1985.

Shortly after L. Fejér had proved his summability theorem, Hardy and Littlewood noticed that it was true in a stronger sense. This was the first instance when strong means were considered but, except for a few feeble attempts, no one had analysed the question further until the late 1930's when J. Marcinkiewicz and A. Zygmund verified that (with the usual terminology)

$$(*)\qquad \frac{1}{n+1}\sum_{k=0}^{n}|s_k(f,x)-f(x)|^p \to 0 \quad (p>0)$$

almost everywhere. It had taken another quarter of a century before G. Alexits and D. Králik turned to the analogous approximation problem. What they found was quite surprising: for Lip α $(0<\alpha<1)$ functions the left-hand side of $(*)$ with $p=1$ has order $\{n^{-\alpha}\}$, i.e. the classical estimate of S. N. Bernstein holds true in the strong sense, as well. This result inevitably brought up the question about the mechanism behind these — literally — strong results and a rapid development in this field was predictable. At this crucial stage appeared L. Leindler on the scene and began to systematically study various kinds of strong means. His papers in the period 1965—1980 exerted dominating influence on the course of events and his one is the lion's share in the fact that today we have the *theory* of strong approximation at all. By the end of the 1970's the results of Leindler and several other authors began to take the shape of a more or less complete theory and the time was ripe for a monograph collecting and organizing the various results.

The book under review is an up-to-date summary of results about strong means. It can and certainly will serve as a reference book. Its content can be recommended not only to the experts of the field but to anyone interested in trigonometric approximation. Leindler's work may also be useful for graduate students for it contains many important techniques of mathematical analysis together with intricate counterexamples which have already proven to be useful in various other branches of analysis. The organization of the material is clear, the whole book is well got up. It was a good choice to omit the proof of certain theorems when they would have required repetition of earlier used techniques. At the same time the reader can learn about the latest and most general results — the author paid special attention at least to announce them. The only criticism I make is that with a little more effort and some 20—30 extra pages the parallel theory of strong summability could have been incorporated into the book, giving thereby a complete account of results about strong means. (The seemingly analogous theory of strong summability of general orthogonal series requires distinctly different arguments.)

The book consists of four chapters. Chapter I deals with the order of strong approximation, i.e., with so-called direct theorems. One of the most general estimate of this type was proved in Leindler's very first paper on the subject and for every classical mean the estimate he obtained turned out to be sharp in many function classes. However, an inequality being sharp does not mean that it can be reversed, so the question of what can be said in the opposite direction, i.e. when we inquire about properties of functions provided the order of some kind of strong mean is known, is very interesting. Chapter II is devoted to this problem. Chapter III establishes various imbedding relations of function spaces yielding in many cases the identification of several of these spaces. In the last chapter some miscellaneous results in three new directions are treated.

Finally, it is appropriate to note that in spite of the fact that the theory presented in the book seems to be complete (e.g. without embarrassing gaps), some very exciting questions in connection with strong means have not or barely been touched upon until now (one of them is the saturation problem). It is the reviewer's sincere hope that László Leindler's work will stimulate interest in some of these problems.

V. Totik (Szeged)

Model-Theoretic Logic, Edited by J. Bairwise and S. Feferman (Perspectives in Mathematical Logic), XVIII + 893 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

This is the second monograph in the *Perspectives* series where Bairwise acts as an editor, between the two he was the editor of the successful Handbook of Mathematical Logic. He and S. Feferman are among those mathematicians who considerably contributed to the flourishing of abstract model theory in the 1970's. The present monograph is research-oriented, the expositions in it reflect intensive present-day investigations. "The aim ... would be to give an entry into the field for anyone sufficiently equipped in general model theory and set theory, and thereby to bring them closer to the frontiers of research". Accordingly, several open problems are mentioned and a bibliography of over a thousand items can serve as a reference for the experts and to-be-experts in the field. The book is divided into six parts:

1. Introduction, Basic Theory and Examples (written by Bairwise, Ebbinghaus, Flum).
2. Finitary Languages with Additional Quantifiers (Kaufmann, Schmerl, Mundici, Baudisch, Seese, Tuschik, Weese).
3. Infinitary Languages (Nadel, Dickmann, Kolaitis, Eklof).
4. Second-Order Logic (Baldwin, Gurevich).
5. Logics of Topology and Analysis (Keisler, Ziegler, Steinhorn), and
6. Advanced Topics in Abstract Model Theory (Väänänen, Makowsky, Mundici).

V. Totik (Szeged)

Bernt Øksendal, Stochastic Differential Equations. An Introduction with Applications (Universitext), XIII + 205 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

"Unfortunately most of the literature about stochastic differential equations seems to place so much emphasis on rigor and completeness that it scares the nonexperts away. These notes are an attempt to approach the subject from the nonexpert point of new: Not knowing anything ... about a subject to start with, what would I like to know first of all? My answer would be: 1) In what situations does the subject arise? 2) What are its essential features? 3) What are the applications and the connections to other fields?"

The author, a lucid mind with a fine pedagogical instinct, has written a splendid text that achieves his aims set forward above. He starts out by stating six problems in the introduction in which stochastic differential equations play an essential role in the solution. Then, while developing stochastic calculus, he frequently returns to these problems and variants thereof and to many other problems to show how the theory works and to motivate the next step in the theoretical development. Needless to say, he restricts himself to stochastic integration with respect to Brownian motion. He is not hesitant to give some basic results without proof in order to leave room for "some more basic applications". The chapter headings are the following: Introduction, Some mathematical preliminaries, Itô integrals, Stochastic integrals and the Itô formula, Stochastic differential equations, The filtering problem, Diffusions, Applications to partial differential equations, Application to optimal stopping, Application to stochastic control. There are two appendices on the normal distribution and on conditional expectations, a bibliography of 71 items, a list of notation, and a subject index.

It can be an ideal text for a graduate course, but it is also recommended to analysts (in particular, those working in differential equations and deterministic dynamical systems and control) who wish to learn quickly what stochastic differential equations are all about.

Sándor Csörgő (Szeged)

Ordinary and Partial Differential Equations (Proceedings, Dundee 1984), Edited by B. D. Sleeman and R. J. Jarvis (Lecture Notes in Mathematics, 1151), XIV+357 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

These proceedings contain one half of the lectures delivered at the eighth International Conference on Ordinary and Partial Differential Equations which was held at the University of Dundee, Scotland, June 25—29, 1984.

The 36 lectures show a very wide spectrum. However, it is common in the articles that the authors investigate nonlinear differential equations, some of which are in fact concrete models of systems or processeses in the real world. The following main directions can be recognized among the themes. Many articles investigate the asymptotic behaviour (stability, oscillation etc.) of solutions of nonlinear second, third and n-th order ordinary differential equations. Differential equations with delays are treated, too. Special attention is paid to periodic problems both for ordinary and partial differential equations (e.g. the probleme of the existence of a periodic solution of prescribed period for Hamiltonian systems is studied). Stability and bifurcation problems are also considered for equations of several types.

The applications give a very valuable part of the proceedings. The reader can find interesting (mostly biological) models such as a hydrodynamical model of the sea hare's propulsive mechanism, models for a myelinated nerve axon, vector models for infectious diseases and multidimensional reaction-convection diffusion equations.

The collections can be recommended to those wishing to get a snapshot of the theory and applications of non-linear differential equations.

L. Hatvani (Szeged)

Richard S. Pierce, Associative Algebras (Graduate Texts in Mathematics, Vol. 88) Springer-Verlag, New York—Heidelberg—Berlin, 1982.

The study of associative algebras has long been, and still is, an area of active research which draws inspiration from and applies tools of many other branches of mathematics such as group theory, field theory, algebraic number theory, algebraic geometry, homological algebra, and category theory. The purpose of this book is twofold: to treat the classical results on associative algebras more deeply than most student-oriented books do, and at the same time to bring the reader to the frontier of active research by discussing some important new advances. The emphasis is put on algebras that are finite dimensional over a field.

The highlights of the classical part include the characterization of semisimple modules, the Wedderburn(—Artin) Structure Theorem for semisimple algebras, Maschke's Theorem, the Jacobson radical, the Krull—Schmidt Theorem, the structure and classification of projective modules over Artinian algebras (Chapters 1 through 6), the Wedderburn—Malcev Principal Theorem, Jacobson's Density Theorem, the Jacobson—Bourbaki Theorem (Chapters 11—12), and, within a detailed, systematic study of central simple algebras and the Brauer group of a field (Chapters 13 through 20), the Cartan—Brauer—Hua Theorem, Wedderburn's theorem that all division algebras of degree 3 are cyclic, the classification of central simple algebras over algebraic number fields, and Tsen's Theorem. Among the more recent developments of the subject presented in the book are the theory of representations of algebras, including a proof of the first Brauer—Thrall Conjecture and an exposition of the representation theory of quivers (Chapters 7 and 8), and Amitsur's Theorem on the existence of finite dimensional central simple algebras that are not crossed products (Chapter 20). Together with these results the reader is acquainted with the most powerful tools of the theory of associative algebras: tensor product, homological methods, cohomology of algebras, Galois cohomology, valuation theory, and polynomial identities.

Each of the twenty chapters ends with a note containing historical remarks and suggestions for further reading. In addition, every section is followed by a set of exercises; some of them are intended to help understanding the material, others call for working out details of proofs omitted in the text, and there are also exercises presenting important results which otherwise couldn't be included.

This excellent book is warmly recommended to students as well as established mathematicians who are interested in associative algebras. For those wishing to read only parts of the text, the subject index and the list of symbols are of great help.

Ágnes Szendrei (Szeged)

D. Pollard, Convergence of Stochastic Processes. XIV+215 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

Properties of empirical processes play an important role in mathematical statistics. Earlier results on this field, especially on the weak convergence of empirical measures can be found in the books of I. I. Gikhman and A. V. Skorokhod (Introduction to the Theory of Random Processes, Saunders, 1969 (in Russian 1965)), K. R. Parthasarathy (Probability Measures on Metric Spaces, Academic Press, 1967) and P. Billingsley (Convergence of Probability Measures, Wiley, 1968). These books have become classics and have generated a lot of work in the area. Recently R. M. Dudley (A Course on Empirical Processes, Lecture Notes in Math. vol. 1097, Springer-Verlag, 1985) and P. Gaenssler (Empirical Processes, IMS Lecture Notes vol. 3, IMS, 1983) have summarized the newest results on this topic and the book under review is another addendum to the literature. These three monographs cover nearly the same area of the theory of stochastic processes.

In Chapter I Pollard introduces notations of stochastic processes and empirical measures. Chapter II contains generalizations of the classical Glivenko—Cantelli theorem. He proves uniform convergence of empirical measures over classes of sets and classes of functions. The author gives a review on the weak convergence of probability measures on Euclidean and metric spaces in Chapters III and IV. Chapters V and VI deal with the weak convergence of empirical processes, properties of the limit processes and also contain some applications of the obtained results in mathematical statistics. Martingale central limit theorems are proven in Chapter VII and an application to the Kaplan—Meier (product-limit) estimator is sketched.

Pollard writes in his Preface: "A more accurate title for this book might be: An Exposition of Selected Points of Empirical Process Theory, With Related Interesting Facts About Weak Convergence, and Applications to Mathematical Statistics." It is certainly true that it is nearly impossible to write a book on empirical processes which would cover all the interesting and important theorems on these processes and also their applications. However, I think a research monograph must contain at least an up-to-date and rich enough list of references, and this cannot be replaced by saying "according to the statistical folklore..." (cf., for example, page 99). This is completely useless for a reader and hence I doubt that Pollard's monograph can be used as a reference book. On the other hand, the interested reader can certainly find some material collected together which was mainly published only in research papers earlier.

Lajos Horváth (Ottawa)

Probability in Banach Spaces V, Proceedings of the International Conference Held in Medford, USA, July 1984. Edited by A. Beck, R. Dudley, M. Hahn, J. Kuelbs and M. Marcus (Lecture Notes in Mathematics, 1153), VI+457 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

The proceedings of the first four conferences on probability in Banach spaces have been published as Volumes 526, 709, 860, and 990 in these Lecture Notes series. The present collection containes 25 articles, almost all of which are very high level research papers. 15 papers deal with limit theorems (the central limit theorem, empirical processes, large deviations, ratio limit theorems for sojourns and other weak theorems; the law of large numbers and of the iterated logarithm, almost sure convergence of martingale-related sequences and other strong theorems), the rest address various problems in sample path behaviour, representation questions, moment and other bounds, stable measures and infinite divisibility. The illustrious list of the authors may give an impression of the contents: de Acosta; Alexander; Austin, Bellow and Bouzar; Berman; Borell; Bourgain; Czado and Taqqu; Dudley; Fernique; Frangos and Sucheston; Giné and Hahn; Heinkel; Hoffman—Jørgensen; Jain; Jurek; Klass and Kuelbs; LePage and Schreiber; Marcus and Pisier; McConell; Rosinski and Woyczynski; Samur; Vatan; Weiner; Weron and Weron; and Zinn.

Sándor Csörgő (Szeged)

Murray H. Protter—Charles B. Morrey, Jr., Intermediate Calculus (Undergraduate Texts in Mathematics), X+648 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

If you teach a course on analytic geometry and calculus at a college or university, your first task is to choose a text-book. This is not easy at all. The course usually consists of three semesters. Typically, the first two semesters cover plane analytic geometry and the calculus of functions of one variable, almost independently of the majors in the audience. The decision on the subject-matter of the third semester (and of the fourth one, if any) requires more care. One must take into account the further study of the students: do they go on in mathematics or do they need the calculus only for applications? This book has been written to help the instructors in this decision by providing a high degree of flexibility in structuring the course.

The first five chapters present the material of a standard third-semester course in calculus (the knowledge of plane analytic geometry and one-variable calculus are a prerequisite): 1. Analytic Geometry in Three Dimensions; 2. Vectors; 3. Infinite Series; 4. Partial Derivatives. Applications; 5. Multiple Integration. The further chapters give an opportunity for the instructor to include less traditional topics in the third semester and to design a fourth semester of analysis depending upon special needs: 6. Fourier Series; 7. Implicit Function Theorems. Jacobians; 8. Differentiation under the Integral Sign. Improper Integrals. The Gamma Function; 9. Vector Field Theory; 10. Green's and Stokes' Theorems.

The main feature of the book is its flexibility. Of course, this can be meant in several ways. On the one hand, the chapters are independent of each other. On the other hand, within each chapter readers can find material of different levels. E.g. in Chapter 10 Green's theorem is established first for a simple domain (this results is adequate for most applications), but it is followed by Green's and Stokes' theorems for the general cases proved by using orientable surfaces and a partition of unity.

The book is concluded with three appendices on matrices, determinants, on vectors in three dimensions, and on methods of integration. The sections contain many interesting problems (answers to the odd-numbered ones are available at the end of the book).

This excellent text-book will be very useful both for instructors and students.

L. Hatvani (Szeged)

Representations of Lie Groups and Lie Algebras, Proceedings, Budapest 1971. Edited by A. A. Kirillov, 225 pages, Akadémiai Kiadó, Budapest, 1985.

The present book contains the second part of the Proceedings of the Summer School on representation theory which was held at Budapest in 1971. The first part of the Proceedings was published by the Akadémiai Kiadó (edited by I. M. Gelfand) in 1975. The scientific program of the school was divided into two sections: advanced and beginner. Most of the lectures published here were read at the latter section but some of them are now rewritten or completed by new authors.

A. A. Kirillov's introductory lecture gives a summary of the basic notions, results and methods in the representation theory of finite and compact groups. The article by B. L. Feigin and A. V. Zelevinsky makes the reader acquainted with the theory of contragradient Lie algebras and their representations. In his paper D. P. Zhelobenko dissusses the constructive description of Gelfand—Zetlin bases for classical Lie algebras $\mathrm{gl}(n, \mathbf{C})$, $\mathrm{o}(n, \mathbf{C})$, $\mathrm{sp}(n, \mathbf{C})$. The method presented here is very effective in obtaining explicit formulas of the representation theory. The lecture by S. Tanaka is devoted to an instructive example; explicit description of all irreducible representations of the group $\mathrm{Sl}(2, F)$ where F is a finite field. The survey by I. M. Gelfand, M. I. Graev and A. M. Vershik "Models of representations of current groups" deals with a class of infinite dimensional Lie groups which is of great importance for physical applications. The authors describe in detail several classical as well as new models of the representations of current groups. G. J. Olshansky's article "Unitary representations of the infinite symmetric group: a semigroup approach" is devoted to the representation theory of another type of the "big" groups. The last lecture by G. W. Mackey is addressed first of all to physicists. It explains how the imprimitivity theorem and the concept of induced representations play an important role in quantum mechanics.

Most of the content of this book is accessible for beginners, but experts will also find some new and useful information in the articles published here.

L. Gy. Fehér (Szeged)

Representation Theory II, Proceedings, Ottawa, Carleton University, 1979, edited by V. Dlab and P. Gabriel (Lecture Notes in Mathematics, 832), XIV+673 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

The first volume of these proceedings in two volumes contains reports from the "Workshop on the Present Trends in Representation Theory" held at Carleton University, 13—18 August, 1979. The present volume contains the majority of the lectures delivered at the second part of the meeting "The Second International Conference on Representations of Algebras" (13—25 August, 1979), but some papers which were not reported at the conference are also published here. These two volumes together (the first volume contains also an extensive bibliography of the publications in the field covering the period 1969—1979) provide the interested reader with a comprehensive survey on the modern results and research directions of the representation theory of algebras. The list of the authors contributed to this volume is: M. Auslander—I. Reiten, M. Auslander—S. O. Smalo, R. Bautista, K. Bongartz, S. Brenner—M. C. R. Butler, H. Brune, C. W. Curtis, E. C. Dade, V. Dlab—C. M. Ringel, P. Dowbor—C. M. Ringel—D. Simson, Ju. A. Drozd, E. L. Green, D. Happel—U. Preiser—C. M. Ringel, Y. Iwanaga—T. Wakamatsu, C. U. Jensen—H. Lenzing, V. C. Kač, H. Kupisch—E. Scherzler, P. Landrock, N. Marmaridis, R. Martinez-Villa, F. Okoh, W. Plesken, C. Riedtmann, K. W. Roggenkamp, E. Scherzler—J. Waschbüsch, D. Simson, H. Tachikawa, G. Todorov, J. Waschbüsch, K. Yamagata.

L. Gy. Fehér (Szeged)

Rings and Geometry, Proceedings of the NATO Advanced Study Institute held at Istambul, Turkey, September 2—14, 1984. Edited by R. Kaya, P. Plaumann and K. Strambach (NATO ASI Series C: Mathematical and Physical Sciences, Vol. 160), XI+567 pages, D. Reidel Publishing Company, Dordrecht—Boston—Lancaster—Tokyo, 1985.

This volume contains 11 lectures given at the summer school on Rings and Geometry.

Until quite recently, when looking for applications of ring theory in geometry, we could mainly think of abstract algebraic geometry which can be formulated in the language of commutative algebra. Of course the connections between ring theory and geometry cannot be circumscribed only by the subject of algebraic geometry. There are some old and new areas in mathematics, highly developed in the last decades, which are the results of the interaction of ring theory and geometry. The editors write in the Preface: "It is the aim of these proceedings to give a unifying presentation of those geometrical applications of ring theory outside of algebraic geometry, and to show that they offer a considerable wealth of beautiful ideas, too. Furthermore it becomes apparent that there are natural connections to many branches of modern mathematics, e.g. to the theory of (algebraic) groups and of Jordan algebras, and to combinatorics".

The book consists of four parts. In the first one the non-commutative analogue of the function field of an algebraic variety and the generalization of classical objects of algebraic geometry to the case of non-commutative coordinate field are studied. (P. M. Cohn, Principles of non-commutative algebraic geometry; H. Havlicek, Application of results on generalized polynomial identities in Desarguesian projective spaces.) The second part is devoted to the study of topological, incidence-theoretical, algebraic and combinatorial properties of Hjelmslev planes. (J. W. Lorimer, A topological characterization of Hjelmslev's classical geometries, D. A. Drake—D. Jungnickel, Finite Hjelmslev planes and Klingenberg epimorphisms.) The third part contains the treatment of the theory of projective planes over rings "of stable rank 2". (J. R. Faulkner—J. C. Ferrar, Generalizing the Moufang plane; F. D. Veldkamp, Projective ring planes end their homomorphisms.) The authors of the papers in the fourth part study the possibility of the extension of the classical relations between projective geometry and linear algebra, investigate the structure of linear, orthogonal, symplectic and unitary groups and matrix rings over large classes of rings, the geometric structure of the units of alternative quadratic algebras and the coordinatization of lattice-geometry. (C. Bartolone—F. Bartolozzi, Topics in geometric algebra over rings; B. R. McDonald, Metric geometry over localglobal commutative rings, Linear mappings of matrix rings preserving invariants; H. Karzel—G. Kist, Kinematic algebras and their geometries; U. Brehm, Coordinatization of lattices.) An appendix describes some concepts of geometry indispensable for mathematics. (C. Arf, The advantage of geometric concepts in mathematics.)

This book is warmly recommended to anybody interested in the interaction of algebra and geometry. It can be used as an up-to-date reference book "in that area of mathematics where the ring theory accuring outside of algebraic geometry is harmonically unified with geometry".

Péter T. Nagy (Szeged)

Murray Rosenblatt, Stationary Sequences and Random Fields, 258 pages, Birkhäuser, Boston—Basel—Stuttgart, 1985.

The first systematic book on time series analysis was "Statistical Analysis of Stationary Time Series" by Ulf Grenander and Murray Rosenblatt published in 1957 by Wiley. Since then a number of excellent monographs of the field have appeared reflecting the fast growth of knowledge. Now, 28 years after his first book, here is a second look of one of the foremost researchers of the topic. As he writes in the preface: "This book has a dual purpose. One of these is to present material which selectively will be appropriate for a quarter or semester course in time series analysis and which will cover both the finite parameter and the spectral approach. The second object is the presentation of topics of current research interest and some open questions".

Chapter I presents basic results on the Fourier representation of the covariance function of a weakly stationary process and the harmonic analysis of the process itself, that is the Herglotz and Cramér theorems. Chapter II formulates the problem of linear prediction, or linear mean square approximation, gives the basic relations between moments and cumulants, fully discusses the linear prediction problem for autoregressive and moving average (ARMA) processes, describes the Kalman—Bucy filter, and investigates the identifiability of the phase function of a non-Gaussian linear process (one of the several novelties of the book, to be fully taken up in Chapter VIII). Of particular interest to a probabilist is Chapter III. It first gives Gordin's device to obtain central limit theorems for partial sums of strictly stationary sequences from those for a martingale difference sequence, then this is used to prove asymptotic normality for general covariance estimators and quadratic forms including the periodogram under cumulant summability conditions. A nice discussion of strong mixing, one of the many important spiritual children of the author, is given together with a basic central limit theorem of the author for strongly mixing sequences. Exotic behaviour under long-range dependence is also discussed here. Chapter IV gives the estimation theory for ARMA schemes with new results, based partly on those in Chapter III, on asymptotic normality in which, besides strong mixing, only the finiteness of a few moments are assumed. Chapters V and VI deal with second and higher order cumulant spectral density estimation, respectively, assuming strong mixing and requiring again only the finiteness of a few moments in the asymptotic normality results. The brief Chapter VII is on kernel density and regression function estimates under "short range" dependence. The final Chapter VIII is on various questions of estimating the phase or transfer functions and moments of non-Gaussian linear processes and fields with the associated deconvolution problems. A short appendix gethers some facts of measure theory, of Hilbert and Banach spaces and Banach algebras. Author and subject indices help the reader.

The last sections of the chapters discuss how the results for sequences extend for fields, what are the difficulties, limitations, etc. There are Monte Carlo simulations for the comparison of the deconvolution procedures and various concrete applications involving turbolence and energy transfer are scattered through the book beginning with the fourth chapter. Occasional figures illustrate the text and the arising computational questions receive sensitive discussions. Each chapter ends with a set of problems that can be given to students, followed by a set of notes providing further examples, explanations, references to and descriptions of the literature and many open problems for further research are discussed at some length.

The text goes very smoothly, it is thoughtful, sympathetic and (sometimes overly) modest. You feel as if you would be inventing what you read and forget that you hear everything from the horse's mouth. Both the mathematics and the discussion concentrates on the essence of the problem at hand, heuristics constitute an integral part of the matter, and unnecessary details are elegantly sidestepped. The main achievment of the book is, I think, that it can be read enjoyably and profitably on several different levels of understanding, with various backgrounds. Thus the author has considerably surpassed his "dual purpose".

The reviewer was fortunate enough to have the opportunity of sitting in a course "Time Series Analysis" that Professor Rosenblatt gave for upper undergraduate and early graduate students in the spring quarter of 1985, at the University of California, San Diego, using the galley proofs of the present book. He covered most of the material in Chapters I, II, IV, and V, motivating the necessary results from Chapter III and from the elements of probability theory and Fourier analysis on heuristic grounds. (This is what he advises to do in his preface.) The course, attended besides the 15—20 students by other senior guests with the Department of Mathematics and a few research workers from some applied departments of UCSD, was a real success.

The book will be inevitable for students, instructors, regular users, occasional appliers, and researchers of time series analysis as well.

Sándor Csörgő (Szeged)

James G. Simmonds, A Brief on Tensor Analysis, VIII+92 pages, Under-Graduate Texts in Mathematics, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

The book is divided into four chapters. The first chapter introduces the concept of vectors and defines the dot product and cross product of vectors. A second order tensor is defined as a linear operator that sends vectors into vectors. Chapter 2 is devoted to the description of tensors by using general bases and their dual bases and covariant and contravariant coordinates of vectors. Chapter 3 deals with the Newton law of motion, introduces moving frames and the Christoffel symbols. The last chapter presents a short glimpse into vector and tensor analysis, introducing the notions of a gradient operator, divergence and covariant derivatives. All the chapters end with a set of exercises. The book may serve as a text for first or second-year undergraduates.

L. Gehér (Szeged)

Gary A. Sod, Numerical Methods in Fluid Dynamics: Initial and Initial Boundary-Value Problems, IX+446 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1985.

In fact there is practically very little about fluid dynamics in this book, intended to provide the foundations for Volume II, which — according to the author's preface — will deal exclusively with the equations governing fluid motion. This volume is rather a text on numerical methods for solving partial differential equations and it is directed at graduate students.

The book deals with the finite difference method, the other, not less powerful method of finite elements is not discussed. Nevertheless it is very useful for everyone, wishing to actually apply the results of numerical analysis of partial differential equations in any field.

The book consists of the following chapters: I. Introduction, II. Parabolic Equations, III. Hyperbolic Equations, IV. Hyperbolic Conservation Laws, V. Stability in the Presence of Boundaries. It is written from the physicist's and engineer's point of view, which does not mean that it would lack mathematical rigor. It is simply detailed enough to be comprehensible for the mathematically less trained reader. It emphasises the concepts and contains all the necessary definitions and theorems about convergence and stability, indispensable for those too who want only good recipes. Several methods for solving parabolic and hyperbolic equations are introduced and analysed, and the problem of boundaries is dealt thoroughly. The more sophisticated mathematical theorems and proofs are avoided, some of them are left to the 4 appendices.

Reading the book, all the natural questions arising in the reader are answered by the author, except for one: Why this title? It certainly calls the attention of practitioners in the field of fluid dynamics, but one is afraid that the set of potential readers will unnecessarily be restricted. One can warmly recommend this book to anybody who just wants to find a good introduction to the finite difference method for solving parabolic and hyperbolic equations.

M. G. Benedict (Szeged)

Edwin H. Spanier, Algebraic Topology, XIV+528 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

This is the second edition of the original text first published in 1966 by McGraw-Hill. It starts with an introductory part summarising the basic concepts of set theory, general topology and algebra. The material is divided into three main parts each of which consists of three chapters. The first three chapters introduce the concepts of homotopy and the fundamental group and apply these in the study of covering spaces and polyhedras. The second three chapters are devoted to homology and cohomology theory. The general cohomology theory and duality in topological manifolds are studie here, too. For each new concept, applications are presented to illustrate its utility. In the last three chapters homotopy theory is studied; basic facts about homotopy groups are considered, some applications to obstruction theory are presented and in the last chapter some computations of homotopy groups of spheres can be found.

No prior knowledge of algebraic topology is assumed but some faimiliarity of the reader in general topology and algebra is preassumed.

L. Gehér (Szeged)

Stability Problems for Stochastic Models, Proceedings of the 8th International Seminar Held in Uzhgorod, September 1984. Edited by V. V. Kalashnikov and V. M. Zolotarev (Lectures Notes in Mathematics, 1155), VI+447 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

Although the 23 papers collected in this volume represent somewhat more diverse research activities than the 22 papers in the predecessor collection under the same title (Lecture Notes in Mathematics, 982), almost everything that I wrote about that one (these *Acta*, 47 (1984), page 260) is valid for the present proceedings. Characterization problems and related questions still constitute a strong topic in the volume, but there is a noticable shift towards limit theorems and convergence-rate problems.

Sándor Csörgő (Szeged)

Stochastic Space-Time Models and Limit Theorems. Edited by L. Arnold and P. Kotelenez (Mathematics and its Applications), XI+266 pages, D. Reidel Publishing Company, Dordrecht—Boston—Lancaster, 1985.

This is the first volume of a new series launched by the D. Reidel Company. In his preface Series Editor M. Hazewinkel emphasizes "the unreasonable effectiveness of mathematics in science" (to quote one of his six mottoes, this one due to E. Wigner) and outlines the aims of this new programme.

The book presents the 13 invited papers presented at a workshop held at the University of Bremen, FRG, in November 1983, describing the state of the art of the mathematics of space-time phenomena, a specific combination of the modern theory of stochastic processes and functional analysis. These widely applicable models describe motions which change with time and are randomly distributed in space, and are usually given by stochastic differential equations. The first seven surveys (by Albaverio, Høegh—Krohn and Holden; Da Prato; Dettweiler; Ichikawa; Kotelenez; Krée; and Ustunel) are devoted to the existence, uniqueness and regularity problems of solutions of stochastic partial differential equations, and in general to stochastic analysis and Markov processes in infinite dimensions. The other six papers (by Van den Broeck; Grigelionis and Mikulevičius; Metivier; Pardoux; Rost; and Zessin) deal with various limit theorems where stochastic processes describing a space-time model emerge as limits of sequences of stochastic processes on lattices or of positions of finitely many particles with several kinds of interaction.

An intelligent unifying introduction by Kotelenez puts the whole contents in a broad perspective and a subject index helps orientation.

If the editors will be able to maintain the high level set by this very carefully compiled first volume, this series will no doubt be a success.

Sándor Csörgő (Szeged)

Symposium on Anomalies, Geometry and Topology, Proceedings, Chicago, United States, 1985. Edited by W. A. Bardeen and A. R. White, XVIII+558 pages, World Scientific, 1985.

This book contains the proceedings of the Symposium on Anomalies, Geometry and Topology, which took place at the Argonne National Laboratory of the US and at the University of Chicago on March 28—30, 1985.

The 56 papers published here report on the recent progress made in field theory and particle physics mostly by means of the application of sophisticated geometrical-topological methods and results. One of the main concerns pursued in the lectures is the new superstring physics—hopefully the right candidate to become a real "Theory of Everything". The other subject which dominated the talks is the mathematical structure of anomalies. Anomalies, originally appeared in perturbative calculations, are now related to properties of Dirac operators and to the connected index theorems of Atiyah and Singer, as well as to homotopy, cohomology and complex structure of manifolds in four and higher dimensions. The investigation of anomalies is an important part of the consistency analysis of quantum field theories. For example, anomaly cancellations played a crucial role in the recent breakthrough achieved in superstring theory. In addition to the main topics of the Symposium, anomalies and superstrings, there were special sessions devoted to charge fractionalization and compactification issues and to the method of effective Lagrangians in various models. Beyond the mentioned ones, the book also contains articles concerning several subjects somehow related to the above. Just to give examples, there are papers addressed to the quantum mechanics of black holes, or dealing with the global structure of supermanifolds. Amongst the authors are M. F. Atiyah, M. Green, D. Gross, R. Jackiw, V. Kac, J. R. Schrieffer, J. Stasheff, G.'t Hooft, E. Witten, S. T. Yau, B. Zumino and many other outstanding scientists. This list clearly proves how close is the connection between pure mathematics and theoretical physics these days.

In conclusion, this collection of high level papers is warmly recommended to everybody interested in the exciting developments reported in it.

L. Gy. Fehér (Szeged)

Universal Algebra and Lattice Theory (Proceedings, Charleston 1984). Edited by S. D. Comer (Lecture Notes in Mathematics 1149), VI+282 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

This volume contains research papers on universal algebra and lattice theory, mostly based on lectures presented at the conference in Charleston, 1984. "In keeping with the tradition set by its origin, it is notable that there are a number of papers that deal with connections between lattice theory and universal algebra and other areas of mathematics such as geometry, graph theory, group theory and logic." The list of the papers is as follows.

M. E. Adams and D. M. Clark: Universal terms for pseudo-complemented distributive lattices and Heyting algebras; H. Andréka, S. D. Comer and I. Németi: Clones of operations on relations; M. K. Bennett: Separation conditions on convexity lattices; A. Day: Some independence results in the co-ordinization of Arguesian lattices; Ph. Dwinger: Unary operations on completely distributive complete lattices; R. Freese: Connected components of the covering relation in free lattices; B. Ganter and T. Ihringer: Varieties of linear subalgebra geometries; O. C. Garcia and W. Taylor: Generalized commutativity; A. M. W. Glass: The word and isomorphism problems in universal algebra; M. Haiman: Linear lattice proof theory: an overview; D. Higgs: Interpolation antichains in lattices; J. Jezek: Subdirectly irreducible and simple Boolean algebras with endomorphisms; E. W. Kiss: A note on varieties of graph algebras; G. F. McNulty: How to construct finite algebras which are not finitely based; R. Maddux: Finite integral relation algebras; J. B. Nation: Some varieties of semidistributive lattices; D. Pigozzi and J. Sichler: Homomorphisms of partial and of complete Steiner triple systems and quasigroups; A. F. Pixley: Principal congruence formulas in arithmetical varieties; A. B. Romanowska and J. D. H. Smith: From affine to projective geometry via convexity; S. T. Tschantz: More conditions equivalent to congruence modularity.

The book gives a good account on several directions of the current research, and it is warmly recommended to research workers interested in the subject.

Gábor Czédli (Szeged)

Vertex Operators in Mathematics and Physics, Proceedings, Berkeley, 1983. Edited by J. Lepowsky, S. Mandelstam and I. M. Singer (Mathematical Sciences Research Institute Publications, 3), XIV+482 pages, Springer-Verlag, New York—Berlin, Heidelberg—Tokyo, 1985.

This volume includes proceedings from the Conference on Vertex Operators in Mathematics and Physics, held at the Mathematical Sciences Research Institute, Berkeley, November 10—17, 1983.

Among the subjects of highest current interest in physics and mathematics are the superstring, supergravity theories on the one hand and the theory of infinite dimensional Lie algebras on the other hand. In the last few years interesting connections have been found between the dual-string theory and the affine Kac—Moody algebras, through the use of vertex operators that originally appeared in the interaction Hamiltonians of the string models. The central role of the mentioned theories in contemporary mathematics and physics made it possible to explore interrelations between seemingly so remote topics as string models, number theory, sporadic groups, integrable systems, the Virasoro algebra and so on. To explore further connections — this was the purpose of the Conference. Most informatory for an expert can be the table of contents of the book under review, so we give it below.

Section 1 — String models. S. Mandelstam: Introduction to string models and vertex operators; O. Alvarez: An introduction to Polyakov's string model; T. Curtright: Conformally invariant field theories in two dimensions.

Section 2 — Lie algebra representations. P. Goddard and D. Olive: Algebras, lattices and strings; J. Lepowsky and R. L. Wilson: Z-algebras and the Rogers—Ramanujan identities; J. Lepowski and M. Primc: Structure of the standard modules for the affine Lie algebra $A_1^{(1)}$ in the homogeneous picture; K. C. Misra: Standard representations of some affine Lie algebras; A. J. Feingold: Some applications of vertex operators to Kac—Moody algebras; M. Jimbo and T. Miwa: On a duality of branching coefficients.

Section 3 — The Monster. R. L. Griess: A brief introduction to finite simple groups; I. B. Frenkel, J. Lepowsky and A. Meurman: A Moonshine Module for the Monster.

Section 4 — Integrable systems. M. Jimbo and T. Miwa: Monodromy, solitons and infinite dimensional Lie algebras; K. Ueno: The Riemann—Hilbert decomposition and the *KP* hierarchy; Ling-Lie Chau: Supersymmetric Yang—Mills fields as an integrable system and connections with other non-linear systems; Yong-Shi Wu and Mo-Lin Ge: Lax pairs, Riemann—Hilbert transforms and affine algebras for hidden symmetries in certain nonlinear field theories; L. Dolan: Massive Kaluza—Klein theories and bound states in Yang—Mills; I. Bars: Local charge algebras in quantum chiral models and gauge theories; B. Julia: Supergeometry and Kac—Moody algebras.

Section 5 — The Virasoro algebra. C. B. Thorn: A proof of the no-ghost theorem using the Kac determinant; D. Friedan, Zongan Qiu and S. Shenker: Conformal invariance, unitarity and two dimensional critical exponents; A. Rocha—Caridi: Vacuum vector representations of the Virasoro algebra; N. R. Wallach: Classical invariant theory and the Virasoro algebra.

J. Lepowsky writes in the Introduction: "The excitement of discovering surprising connections between different disciplines has become a rule in the subject of this volume". There is no doubt that the present interaction between mathematicians and physicists continues and that this book will be very useful for all experts involved in it.

L. Gy. Fehér (Szeged)

M. Zamansky, Approximation des Fonctions (Travaux en Cours), V+91 pp., Hermann, Paris, 1985.

A more appropriate title would be "Approximation of periodic functions" because Zamansky's small treatise is entirely devoted to periodic functions of a single or several variables. The author belongs to that prominent group of mathematicians who nursed the theory of harmonic approximation through its infancy and in this book he twisted again on the usual course of events, namely he provided us with a rather unconventional treatment of the subject. Today it is uncommon, though it may be refreshing, to put multipliers into the heart of the subject. The notations are also peculiar which may cause some headache for the expert who wants to use the book as a reference. Nevertheless, the topics covered are familiar and they provide an independent introduction to harmonic approximation in L^p and C spaces. Both the book's strength and its weakness lies in its concise form. On the one hand this enables the author to consider several spaces and approximation methods simultaneously, on the other hand too general statements blur the necessary distinction between important and unimportant (methods, for example) and hide the differences between certain spaces (e.g. L^1 and L^p, $p>1$) that play(d) so important role in mathematics.

Very briefly the contents: Chapter 1: Approximation of functions of a single variable (Fourier series, moduli of smoothness, generalizations of Jackson's estimate and its converse, conjugate series, derivatives of functions, saturation with many examples); Chapter 2: Convolutions (equivalence of convolution processes, saturation, direct and converse theorems); Chapter 3: Multidimensional case. A 9 page historical account closes the book.

V. Totik (Szeged)