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Bibliographie

V. I. Arnold, Catastrophe Theory (Second, Revised and Expanded Edition), XIII+108 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

The strenuous interest in catastrophe theory, a mathematical tool of the description of jump transitions, discontinuities and sudden qualitative changes in the evolution of systems, has been continuous since its early development. The first edition of this excellent booklet has helped us form a judgement of the features and limitations of this theory and its applications, clear up the mysticism involved.

This first edition of the book was reviewed in these *Acta*, 47 (1984), 492—493. On the changes let us cite from the author's preface to this second English edition: "The present, most complete edition differs from the 1983 Springer edition at many points. A new chapter on Riemann surfaces, vanishing cycles and monodromy has been included from the second Russian edition, the abundant misprints of the first edition have been corrected and some recent new results are described (results on normal forms for the singular points of implicit differential equations and for slow motions in the theory of relaxation oscillations, results on boundary singularities and imperfect bifurcations, results on the geometrical meaning of the caustic of the exceptional group F_4 and on applications of the symmetry group H_4 of the 600-hedron in optimal control or calculus of variations problems)."

It is worth emphasizing that the new edition is a new translation from the Russian, which follows the standard mathematical terminology more than the first one.

The author wrote this book for a general public having minimal mathematical background, in order to clarify a new branch of mathematics which stands in the limelight. But mathematicians also have learnt a lot from this booklet, namely, the essence of the catastrophe theory. In addition, one can learn from it how to throw light upon a field of mathematics in an enjoyable formula-free way.

L. Hatvani (Szeged)

Banach Spaces. Proceedings, Missouri 1984. Edited by N. Kalton and E. Saab (Lecture Notes in Mathematics, 1166), VI+199 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

This book contains 22 papers presented at the conference "Factorization of linear operators and geometry of Banach spaces" held at the University of Missouri from 24 June to 29 June, 1984. The main themes of the papers are the weak topology, projections, the Radon—Nikodym theorem, Gateaux differentiability, Lie algebra of a Banach space and factorization of operators.

L. Gehér (Szeged)

T. S. Blyth—E. F. Robertson, Algebra through practice. A collection of problems in algebra with solutions, Books 1, 2 & 3, X+97+99+95 pages; Books 4, 5 & 6, X+104+101+100 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1984, 1985.

There are very few problem books in abstract algebra. This series of books meets a long-felt need in teaching and studying algebra, by providing about 640 exercises with complete solutions and almost 150 problems without solutions for those intending to test their proficiency. By means of these exercises the reader not only can practice himself in proving statements but also meets a number of concrete examples of algebraic structures. As the authors say they "have attempted, mainly with the average student in mind, to produce a varied selection of exercises while incorporating a few of a more challenging nature".

The series consists of six books. The exercises cover the most classical parts of algebra: sets, relations and mappings; linear algebra (matrices, vector spaces, linear mappings, Jordan forms, duality); group theory (subgroups, factor groups, automorphisms, Sylow theory, series, presentations), ring theory (ideals, divisibility in integral domains, unique factorization), field theory (extensions, Galois theory) and module theory (exact sequences, diagrams, chain conditions, Jordan—Hölder theorem, free and projective modules). Perhaps Galois theory receives less emphasis than it ought to.

At the beginning of each chapter the notions and results the reader is supposed to be familiar with are summarized. For the convenience of the reader, a list of widely used textbooks is included which the reader may consult for background material. It is indicated which chapters are most relevant to the chapters of the present books.

Mária B. Szendrei (Szeged)

B. Booss and D. D. Bleecker, Topology and Analysis. The Atiyah—Singer Index Formula and Gauge-Theoretic Physics, XVI+451 pages, Universitext, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

This text-book offers the interested reader a clear, easily accessible introduction to the Atiyah—Singer index formula, "one of the deepest and hardest results in mathematics" according to Hirzebruch, and also provides a survey on its applications of extremely wide range. Only a minimal amount of prerequisites is needed to read the book: some basic linear algebra and Hilbert space theory, elements from the theory of ordinary differential equations and the notion of a differentiable manifold.

The first three parts of the text give the translation (by D. Bleecker and A. Mader) of the original German edition (by B. Booss): Topologie und Analysis, Eine Einführung in die Atiyah—Singer—Indexformel, Springer-Verlag, 1977. The main points of Part I "Operators with index" deal with the properties of the index of Fredholm operators (e.g., the homotopy invariance of the index is established) and with the structure of the space of Fredholm operators on a Hilbert space. Part II "Analysis on manifolds" is devoted to the elements of the theory of partial differential equations, pseudo-differential operators, elliptic differential operators and boundary-value problems. Part III "The Atiyah—Singer index formula" is the heart of the book. After an introduction to K-theory and the index formula in the Euclidean case, here the embedding, the cobordism and the heat equation proofs of the index formula for an elliptic operator on a closed, oriented Riemannian manifold are all explained. Then a survey on applications including, for example, the cohomological formulation of the index formula, the theorem of Riemann—Roch—Hirzebruch and the Lefschetz fixed point formula is presented. Part IV "The index formula and gauge-theoretical physics" is written by D. Bleecker. Here the author first gives an account of the basic concepts in the geometrization of Yang—Mills theory. Then he expounds in detail how to use the index formula to instanton parameter counting in gauge theory and shows that the moduli space of irreducible self-dual connections (instantons) is naturally a manifold under suitable hypotheses.

The book under review gets the reader with a minimal background of knowledge and experience acquainted with one of the central theorems of differential topology and also with some of its "traditional" and most recent applications. So it can be recommended to everyone interested in the index theorem or in its applications, from students to active mathematicians and theoretical physicists.

L. Gy. Fehér (Szeged)

R. Carmona—H. Kesten—J. B. Walsh, École d'Été de Probabilités de Saint-Flour XIV—1984. Édité par P. L. Hennequin (Lecture Notes in Mathematics, 1180), X+439 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

This new Saint-Flour volume has a very high chance to become as much a widely referenced big success as the Saint-Flour IV—1974 volume (Lecture Notes in Mathematics, 480) which contained Fernique's famous 96-page paper" Régularité des trajectoires des fonctions aléatoires gussiennes". Note the (random?) play of the numbers: this was ten years and ten Saint-Fluor volumes, and exactly 700 Lecture Notes ago.

The present volume contains three long survey articles by three very illustrious mathematicians. The first paper is "Random Schrödinger Operators" by René Carmona (1—124 pages). It gives a rigorous development and an up-to-date overview of a relatively new and fascinating field of research, a field motivated by quantum mechanics and theoretical physics in general and belonging to both probability theory and functional analysis. There are many new results proved here for the first time and many open problems discussed. The second one is "Aspects of First Passage Percolation" by Harry Kesten (125—264 pages). This is also a fine introduction to the subject and a state-of-the-art survey of the field since the author's book (Percolation Theory for Mathematicians, Birkhäusler, Boston—Basel—Stuttgart, 1982) appeared, the Russian translation of which has just been published. The third paper, almost a monograph by itself, is "An Introduction to Stochastic Partial Differential Equations" by John B. Walsh. The author emphasizes that "this is an introduction, not a survey". However, it is his own introduction unifying two different kinds of approaches (one suited for noise with nuclear covariance, the other one for white noise) in a "(nearly) real variable setting" and also containing quite a number of new results. Looking through the foregoing lines, my feeling is that the "high chance" noted above is in fact probability one.

Sándor Csörgő (Szeged)

G. D. Crown, Maureen H. Fenrick, R. J. Valenza, Abstract Algebra (Monographs and Textbooks in Pure and Applied Mathematics, 99), vi+403 pages, Marcel Decker, Inc., New York—Basel, 1986.

The authors' aim was to write a self-justifying textbook on abstract algebra, which covers the fundamental concepts at a level appropriate to an upper-division undergraduate or first year graduate course.

To carry out this intention much space is devoted to solid, fundamental examples and correspondingly less space is available for advanced topics. The chapter headings are: Preliminaries (set operations, functions, partitions, equivalence relations, binary operations, integers) Groups, Group Actions and Solvable Groups, Rings, Factorization in Commutative Rings, Algebras, Modules and Vector Spaces, Field Extensions, Galois Theory. There are two short appendices on Zorn's Lemma, and categories and functors.

Each chapter is ended with a set of exercises which fall into three categories: instantiations of propositions and definitions (which help the reader in deeper understanding of the subject), routine combinatorial drills (the traditional mainstay of this type of text) and extended sequential exercises developing important supplementary topics.

This well-selected material will serve as a good textbook for both students and teachers.

Lajos Klukovits (Szeged)

L. Devroye, Lecture Notes on Bucket Algorithms (Progress in Computer Science, No. 6) 146 pages, Birkhäuser, Boston—Basel—Stuttgart, 1986.

At bucket algorithms data are partitioned into groups according to membership in equal-sized d-dimensional hyperrectangles, called buckets. In this book the connection between the expected time of various bucket algorithms and the distribution of the data is investigated. A variety of probability-theoretical techniques for analyzing various random variables (the average search time, the time needed for sorting, the worst-case search time etc.) related to the bucket structure is given. This is done in a very nice style: the author starts slowly on standard problems (one-dimensional sorting and searching) and moves on to multidimensional applications, in the areas of computational geometry, operations research and pattern recognition.

The book is recommended for people who are interested in computer algorithms.

J. Csirik (Szeged)

H. J. Eichler—P. Günter—D. W. Pohl, Laser Induced Dynamic Gratings (Springer Series in Optical Sciences, 50), XI+256 pages, Springer Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

The classical principle of linear superposition of electromagnetic fields does not hold in a medium which responds nonlinearly to the external perturbation. When two laser beams are arranged to interfere in a nonlinear medium they produce a transient periodic structure, a nonlinear diffraction grating. If a third wave is falling on this structure, in certain cases this wave will be reflected with reversed phase, i.e. the grating acts as a time reversal operator on the electromagnetic field. The effect has many important applications: restoration of distorted optical beams, optical data storing and processing, etc. Beyond the mathematical theory of dynamic gratings a detailed description of materials and practical arrangements are also presented in this book.

M. G. Benedict (Szeged)

Goodness-of-Fit Techniques. Edited by R. B. D'Agostino and M. A. Stephens (Statistics: Textbooks and Monographs, 68), XVIII+560 pages. Marcel Dekker, Inc., New York—Basel, 1986.

The editors write in their preface that when several of the nine authors first decided writing this book in 1976 they asked Egon S. Pearson, to whom the book is dedicated, if he would join them. "He declined and stated his view that the time was not yet ripe for a book on the subject." Judging from the vast amount of techniques covered, and a rather sizable amount that have been left out, initial look may leave a feeling to the contary: perhaps it was too late to write the first book on the subject.

Following a few-page overview by the editors, an overview of their own work and not of the field, and a chapter by D'Agostino discussing graphical plots of the empirical distribution function (EDF) and related functions of the sample (pages 7—62), a short description of χ^2 tests is given by

D. S. Moore (pages 63—95). This is perhaps the most intelligent chapter in the book. The most sizeable chapter by Stephens follows then (pages 97—193) on tests based on EDF statistics. These include tests for simple goodness of fit in general and for composite normal, exponential, Gumbel, Weibull, Gamma, Logistic, Cauchy, von Mises and some other hypotheses such as symmetry, the statistics themselves falling into one of the three boxes defined by the Kolmogorov—Smirnov, Cramér—von Mises and Anderson—Darling statistics. Chapter 5 (pages 195—234) by Stephens describes correlation or regression tests mainly for the uniform, normal, exponential and Gumbel distributions. Chapter 6 (pages 235—277) gives a clever review of transformation methods by C. P. Quesenberry, while in Chapter 7 (pages 279—329) K. O. Bowman and L. R. Shenton discuss techniques based on the sample skewness and kurtosis. The next three chapters (pages 331—366, 367—419, and 421—459) are on tests for the uniform, normal, and exponential distributious by Stephens, D'Agostino and again Stephens, respectively. The treatment of censored samples is insufficient throughout the book. The special chapter (pages 461—496) by J. R. Michael and W. R. Schucany devoted to this topic is disappointing. The last Chapter 12 (pages 497—522) by G. L. Tietjen on the analysis and detection of outliers is such as a chapter on this problem can be written presently. This is followed by an Appendix (pages 523—549) of selected tables and a not too helpful index.

There are many numerical illustrations involving simulated or real data in each chapter. Also the most needed tables for practical implementation are included in the text where they are first required. There are no proofs in the book. It is for the non-statistician practitioner who is supposed to use these tests.

Any professional statistician will find topics insufficiently treated on the level of importance he/she attaches to them, statements concerning the accuracy of this or that approximation which he/she will disagree with, or recommendations concerning the preference of this or that statistic for a given hypothesis. This is completely unavoidable in case of such a book, and this is perhaps what Egon Pearson had in mind: there are no Neyman—Pearson lemmas to make the picture clear, there are too many open problems and unexplored proposals and techniques, too many personal preferences. (It would be no point, therefore, to list my disagreements, part of which are based on my own personal preferences.) The situation may seem ideal for the researcher. In fact, the editors hope that "this book... will act as a base from which ... many questions can be explored". Considering the complete lack of theory in the book, this is perhaps too much to be hoped for. Be it as it may, Pearson's ripe time will probably always remain in the infinitely distant future.

If not every, but certainly many professional statisticians will miss one or two of their important papers or of their friends' from the reference lists presented at the end of the chapters. (Again, it would be fully needless to use space for my friends' missing-lists. However, I cannot refrain from mentioning one outside of that circle. It is the fine booklet "Omega—Square Tests" by G. V. Martynov, Nauka, Moscow, 1978; MR 80 g: 62028. It contains newly computed tables for all Cramér—von Mises and Anderson—Darling tests. Although it is in Russian, the tables of course use Arabic numbers.)

Was Egon Pearson right or not? He was, and he was not. Were the authors right to write this book? Yes, they were. Is it a good book or not? It is a useful book.

Sándor Csörgő (Szeged)

Wolfgang Hackbusch, Multi-Grid Methods and Applications (Springer Series in Computational Mathematics, 4), XIV+377 pages with 43 figures and 48 tables, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

Is there a gap between "pure" and "applied" mathematics? The public is certainly sure that such a gap exists. Generally "pure mathematics" has the image of being a subject which is not associated with practical problems. Perhaps the root of this widely held view is that "mind stands higher than matter". But the history of sciences proves that mathematics underlies in all kinds of scientific and technological developments. For example the theory of differential equations is in the closest connection with practice from the beginnings.

Boundary value problems arise in many fields of applications (in engineering, in physics, etc.). Therefore the solution of these problems is an important task of mathematics. The investigation is twofold: theoretical and practical, this is indeed one of the topics where theory and application are inseparable. The boundary value problems have been the starting points of some other branches of the theory. Then the development of the theory rendered possible not only to prove the existence (or non-existence) and the qualitative properties of the solutions of several problems but to solve them numerically, too. Although in our days we have faster and faster computers, new efficient numerical methods are required having fast convergence.

A certain amount of time is generally necessary to prove the efficiency of a new method. The multi-grid method was first described in the early sixties. Since the seventies a great number of articles verifies its manysided applicability. The main characteristic feature of this method is its fast convergence. The convergence speed, in contrast with some classical methods, does not deteriorate by refining the discretization.

This book gives a clearly written, up-to-date exposition of the subject including several applications, supplied with exercises and interesting comments. The author not only acquaints the reader with a very efficient method but he gives an overall view of the theme. The results which first appeared in various, sometimes hardly obtainable, journals are now made available by this work in a well-organized form for a wide range of interested people. The book will surely remind some readers of the following sentence of P. Halmos: "Pure mathematics can be practically useful and applied mathematics can be artistically elegant."

L. Pintér (Szeged)

Lars Hörmander, The Analysis of Linear Partial Differential Operators Vol. III. Pseudo-Differential Operators, Vol. IV. Fourier Integral Operators, (Grundlehren der mathematischen Wissenschaften, 274, 275), VIII+525 pages, VII+352 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

The first two volumes expand and update the author's book "Linear Partial Differential Operators" which was published in the same *Grundlehren* series originally in 1963 and since then became a standard reference for matematicians working in partial differential equations. The last decades produced an interesting development on the research fields overviewed there and, on the other hand, elegant new techniques appeared which became dominant recently. The author who contributed greatly to this changed fashion of the theory revised his 1963 monograph. The results are Volumes I and II published in 1983 as number 256 and 257 of the same series. Furthermore he has now added these two volumes which can be considered almost entirely new.

In this spirit Volumes III and IV cover the following main branches and their typical applications: Pseudo-Differential Operators and Fourier Integral Operators, Lagrangian Distributions with the underlying Simplectic Geometry, thus areas which became really fruitful only recently after Calderon's Uniqueness Theorem and the Atiyah—Singer—Bott Index Theorem inspite of an existing long tradition in the literature.

Volumes III and IV contain Chapters XVII—XXX of the complete work.

The introductory Chapter XVII, in contrast with all the later ones, displays second order differential operators treated by relatively classical means because of their independent geometrical importance. Chapters XVIII—XX discuss already the powerful machinery of pseudodifferential operataros. After a short heuristical motivation first the necessary basic tools (totally characteristic operators, Gauss transforms, Weyl calculus) and then the index theory of elliptic pseudo-differential operators on compact manifolds without boundary are reviewed which is followed by the treatment of elliptic boundary problems and closed by a motivating outlook to the existence theory of non-elliptic pseudo-differential operators. Based on index theorems, Chapter XXI treats symplectic geometry, a geometrical background essential for later purposes, which has deep roots in classical mechanics but which is now equally important for pure mathematics. Among others, the classifications of pairs of Lagrangian manifolds and of some other systems of relevant geometric systems of mappings are considered together with the symplectic equivalence of quadratic forms. The main aim of Chapter XXII is to illustrate the effectiveness of the methods based on the perturbation theory of pseudo-differential operators by examples of micro-hypoelliptic operator classes occuring naturally in physics and probability theory. Chapters XXIII—XXIV turn to the strictly hyperbolic Cauchy problems and mixed Dirichlet—Cauchy problems, respectively, applying the technique of energy integrals renewed by the calculus of pseudo-differential operators and some extensions of the material in symplectic geometry. Special attention is paid already at this point to the propagation of singularities of solutions. This latter theme for operators of principal type is the main goal of Chapter XXVI whose inclusion to Volume IV is naturally justified by the completeness of the results obtained.

The beginning of Volume IV is Chapter XXV where the author summarizes new arguments concerning Fourier integral operators which had old heuristical motivations in geometry, wave optics and classical and quantum mechanics but whose more systematic study emerged only after the 1960's. Chapter XXVII is devoted to subelliptic operators. In Chapter XXVIII the study of the Cauchy problem is resumed. Problems and tools suggested by Calderon's Uniqueness Theorem are discussed. Chapter XXIX presents very effective applications of the modern theory of Fourier integrals to the asymptotic behaviour of the eigenvalues and eigenfunctions of elliptic operators of higher order. The work is completed by long range scattering theory in Chapter XXX.

Each chapter begins with an about two pages long summary and ends with very detailed historical and bibliographical notes. The two volumes provide a comprehensive reference list of more than 450 items, and a detailed index and list of symbols aids their use as handbooks.

The style of the books can be charactized by their excellent organization which anables us to obtain a relevant insight into almost the whole of the enormous material of wide research areas.

The four volumes can be classified as professional reading. However, taking into consideration the range of direct and indirect applicability of the described results and methods they can be suggested as an indispensable collection of handbooks for all research teams in mathematics and theoretical physics even if their fields of interest are seemingly far from partial differential equations.

L. Stachó (Szeged)

Infinite Dimensional Groups with Applications, Proceedings, Berkeley, 1984. Edited by V. Kac (Mathematical Sciences Research Institute Publications, 4), X+380 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

The present book contains the Proceedings of the Conference on Infinite Dimensional Groups held at the Mathematical Sciences Research Institute, Berkeley, May 10—15, 1984. The Conference was concentrated on the following three of the most active directions in the theory of infinite dimensional groups: general Kac—Moody groups, gauge groups and diffeomorphism groups. These are in close connection with physical theories of current interest. Here the key-words are: solitons and instantons, completely integrable systems, Yang—Mills fields and string theory.

The best way to orient oneself is to have a look at the table of contents: 1. M. Adams, T. Ratiu and R. Schmid: The Lie group structure of diffeomorphism groups and invertible Fourier integral operators with applications. 2. E. Date: On Landau—Lifshitz equation and infinite dimensional groups. 3. D. S. Freed: Flat manifolds and infinite dimensional Kähler geometry. 4. R. Goodman: Positive-energy representations of the group of diffeomorphism of the circle. 5. M. A. Guest: Instantons and harmonic maps. 6. Z. Haddad: A Coxeter group approach to Schubert varieties. 7. V. G. Kac: Constructing groups associated to infinite-dimensional Lie algebras. 8. I. Kaplansky and L. J. Santharoubane: Harish—Chandra modules over the Virasoro algebra. 9. S. Kumar: Rational homotopy theory of flag varieties associated to Kac—Moody groups. 10. G. Lusztig: The two-sided cells of the affine Weyl group of type $\tilde{A}_n$. 11. A. Pressley: Loop groups, Grassmannians and KdV equations. 12. P. Slodowy: An adjoint quotient for certain groups attached to Kac—Moody algebras. 13. K. Ueno: Analytic and algebraic aspects of the Kadomtsev—Petviashvili hierarchy from the viewpoint of the universal Grassmann manifold. 14. B. Weisfeiler: Comments on differential invariants. 15. H. Yamada: The Virasoro algebra and the KP hierarchy.

This collection of high level papers gives an up-to-date overview on the present status of the theory of infinite dimensional groups and its applications and so it is recommended to everyone interested in the subject.

L. Gy. Fehér (Szeged)

Gabriel Klambauer, Aspects of Calculus (Undergraduate Texts in Mathematics), X+515 pages. Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

It is interesting that among new textbooks in mathematics there are a great number of books on calculus and real analysis (e.g. 15 among the first 50 volumes of the series Undergraduate Texts in Mathematics). Why? Probably because calculus is needed by almost all sciences, it is the most important tool of applied mathematics, and many other branches of mathematics use analysis. Therefore, it has to be tought on very different levels, for which different texts are necessary. However, a new book can be of interest only if the reader can find topics in it shown by the author from a new point of view, topics or examples which are unusual on the level of an introduction. The reader now gets such a book with two special features. One is that it is written for those who know more than "real" beginners, hence have the techniques of the manipulation of formulas, and at the same time wish to find out the deep roots of the basic concepts of analysis. In many texts such a purpose is often carried out by using a too high level of abstraction, which may result in that the reader cannot connect the new concepts with his/her earlier knowledge. But in this book — and this is the other feature — the author succeds in his purpose using very nice instructive examples and exercises. He includes numerous worked-out examples and concludes every chapter by a lot of exercises, the more difficult of which are accompanied with helpful hints of outlined solutions.

The first chapter is a brilliant geometrical introduction to the logarithmic and exponential functions based upon the specific relation between the hyperbolic segment and the logarithmic function. This approach quickly leads to the evaluation of certain important limits (e.g. $n(b^{1/n}-1)\to\to\ln b$ $(n\to\infty;\ b>1)$ can be obtained easily). The second chapter deals with limits and continuity. Chapters 3 and 4 are concerned with differentiation and its applications. A special section is devoted to the inequality between the arithmetic and geometric means. In Chapter 5 the concept of the Riemann integral is prepared by the quadrature of the parabola (by Archimedes), of the cycloid (by Roberval) and of the function $y=Ax^c$ (by Fermat). The last chapter on infinite series, more than 130 pages in extent, gives the most novelties in the book with its instructive examples and propositions.

To sum up, it is the attractive, interesting and useful examples and exercises that make this text very valuable for students being in transition from elementary calculus to rigorous courses in analysis, and indispensable for those teaching calculus and analysis.

L. Hatvani (Szeged)

W. R. Knorr, The Ancient Tradition of Geometric Problems, ix+441 pages, Birkhäuser, Boston—Basel—Stuttgart, 1986.

In the ancient Greek geometry, to raise a geometric problem was a request for working out a construction of a figure corresponding to a specific description. There were three famous problems: cube-duplication (the Delian-problem), angle-trisection and circle-quadrature. This book is a survey of the efforts made by several ancient Greek mathematicians, e.g., Hippocrates of Chios, Eudoxos, Archytas, Archimedes and Apollonius.

The author emphasizes the mathematical and historical aspects of the ancient writings taking into consideration not only the works of the mathematicians but Greek and Arabic commentaries, too. The final chapter includes aspects of philosophical interest as well.

The chapter headings are the following: Sifting History from Legend, Beginnings and Early Efforts, The Geometers in Plato's Academy, The Generation of Euclid, Archimedes—The Perfect Eudoxean Geometer, Successors of Archimedes in the 3rd Century, Apollonius — Culmination of the Tradition, Appraisal of the Analitic Field in Antiquity.

This valuable and beautiful book, which includes 400 geometric drawings and photographs, is recommended to anyone who wants to get acquainted with the ancient Greek geometry, in particular with its three famous problems.

Lajos Klukovits (Szeged)

Hermann König, Eigenvalue Distribution of Compact Operators, (Operator Theory: Advances and Applications, Vol. 16), 262 pages, Birkhäuser Verlag, Basel—Boston—Stuttgart, 1986.

The classical Riesz theory provides a qualitative description of the spectra of compact operators. Namely, it claims that every non-zero spectrum point of a compact operator on a complex Banach space is an isolated eigenvalue of finite multiplicity. The asymptotic behaviour of the sequence $\{\lambda_n(T)\}_n$ of the eigenvalues of compact Hilbert space operators T was characterized by H. Weyl in terms of the s-numbers of T introduced by Schatten and von Neumann.

This monograph gives an introduction to the theory of eigenvalue distributions of compact operators acting on general complex Banach spaces. The author's contribution to this rapidly developing theory was decisive. The subject is divided into four chapters.

The first chapter contains a brief account of the Hilbert space case. The generalizations of s-numbers to Banach spaces: the approximation-, Gelfand-, Weyl- and entropy-numbers are treated. Furthermore, the definitions and elementary properties of Lorentz spaces and of different operator ideals, among others the class of p-summing operators, are given. The main theorems, the generalized Weyl inequalities on eigenvalue distributions of operators are proved in the second chapter. In the third chapter applications to integral operators with kernels satisfying summability conditions or belonging to Sobolev and Besov spaces are discussed. The last chapter provides further applications to the trace formula and to projection constants.

The book is written in a clear style. It is almost self-contained, only a basic knowledge in functional analysis is needed. This excellent work can be warmly recommended to everyone who wants to get acquainted with this fascinating subject.

L. Kérchy (Szeged)

Serge Lang, A First Course in Calculus (Undergraduate Texts in Mathematics) Fifth Edition XV+624+A99+13 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

Nowadays the teaching of mathematics is much more widespread and varied than it was fifty years ago. Various fields in science and many of the professions demand a certain mathematical knowledge. The main problem is what mathematics should be taught and how? Calculus is certainly one of the topics which we would like almost all students to know to a certain degree.

The subject of any first course in the calculus consists of the basic notions of derivative and integral and some basic techniques and applications accompanying them. A solution of the problem of "how to present them" is what gives the characteristic differences between the books on calculus. This is a teaching problem primarily.

Lang's present book is a source of interesting ideas and brilliant techniques. The main question in this topic is the introduction of the notion of limit. The author's opinion is that any student is ready to accept as intuitively obvious the notions of numbers and limits and their basic properties. Epsilon-delta should be entirely left out of ordinary calculus classes. From the mathematical point of view this is not without danger. But in the reviewer's opinion here the mathematical and methodological difficulties are avoided in a masterly manner.

Let us mention another important feature of the book. It is well-known all over the world that students' facility in speaking and writing is less and less sufficient. The author writes in his Foreword: "I have made great efforts to carry the student verbally, so to say, in using proper mathematical language. It seems to me essential that students be required to write their mathematics paper in full and coherent sentences."

Many of the well-chosen problems and exercises are useful for both students and instructors.

The fact that the book has been reprinted and expanded after over twenty years says all that is needed to say.

L. Pintér (Szeged)

Serge Lang, Math! Encounters with High School Students, XII+138 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

It can be often heard that in most mathematical books, especially in school books, topics are treated in an incoherent way. Things are piled up without noticeable reason, technical details are accumulated endlessly, etc. Similar opinion is widely held about the teaching of mathematics, too.

Unfortunately, in many cases these opinions reflect the facts. Therefore it is especially important to see good books and to hear good lectures convincing people (at least some of them) that doing mathematics is a lively and beautiful activity. The production of such works does not depend on the intention of the author solely.

The author of this book experimented a risky undertaking while giving talks to students about 15—16 years old on various deeper problems of mathematics. The author is an experienced teacher (who has written about thirty books), a creative mathematician and these two qualities are inseparable. Therefore he has chances to achieve his aim.

This book contains seven talks (or rather dialogues) given by the author in various high schools in Canada and in France. The titles of the talks give an insight into the questions: What is pi?; Volumes in higher dimension; The volume of the ball; The length of the circle; The area of the sphere; Pythagorean triples; Infinities. Each dialogue is self-contained. The reviewer's favourite is the last one on infinities. The Postscript is a discussion concerning the teaching of mathematics interesting mainly for teachers.

I would recommend this book to students and teachers and I am sure that teachers will find several ideas and patterns helpful in the everyday work.

L. Pintér (Szeged)

Serge Lang, The Beauty of Doing Mathematics, Three Public Dialogues, XI+127 pages, Springer-Verlag, New York—Berlin—Hedigelberg—Tokyo, 1985.

Almost every Saturday from October to June the Science Museum in Paris welcomes and presents to the public eminent lecturers in all disciplines.

This book consists of three lectures given by Serge Lang. The audience was very diverse, ranging from young students to retired people, from housewives to engineers, but they were people curious enough. How can one convince them that mathematics is quite beautiful?

Serge Lang's aim was to show what pure mathematics is by examples, by doing mathematics with the audience. The first two lectures, Prime Numbers, and Diophantine Equations are in some sense near to a non-mathematical public. For example in the first part of the first lecture the author defined the prime numbers, the twin primes, proved that there are infinitely many prime numbers. He raised the question: Is there an infinite number of twin primes? The activity of the public was shown by several good questions.

The first two lectures are very interesting but the real surprise is the third. In this Professor Lang tries to explain a new geometrical result. I am sure that this lecture is a sensation for almost every reader. Therefore we don't say more. Read it!

L. Pintér (Szeged)

Lyapunov Exponents, Proceedings, Bremen, 1984. Edited by L. Arnold and V. Wihstutz (Lecture Notes in Mathematics, 1186), VI+374 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

By his famous thesis A. M. Lyapunov founded the stability theory of differential equations. He offered two methods of investigation: the first method was based upon the concept of the so-called exponents of solutions and the second or direct method used an auxiliary function of state variables and the time. Nowadays these exponents and functions bear his name.

The first method started with the following celebrated theorem. If the eigenvalues of A have negative real parts then the zero solution of the system $\dot{x}=Ax+f(t,x)$ $(x\in R^n,\ t\geqq 0,\ |f(t,x)|\leqq \leqq c|x|^{1+\alpha}$ for some $c,\ \alpha>0)$ is exponentially stable. Comparing the solutions of a linear equation $\dot{y}=A(t)y$ with the exponential functions $\exp[\lambda t]$ $(\lambda\in R)$, Lyapunov introduced the exponent of a solution and defined a spectrum for this kind of an equation, too, and was able to generalize the above theorem to the case of varying A.

The first method of Lyapunov has been widely applied and expansively developed further since his pioneering works. This volume, which contains 22 invited papers of the Workshop, gives a good flavour of these kind of results. The editors open the volume by an excellent survey article. In its first part the history and the classical results of the theory are reviewed. In the second part the authors write about the modern areas of the theory and characterize what the papers in the Proceedings contribute to them. The main fields are the following: 1. Products of random matrices and random maps; 2. Linear stochastic systems. Stability theory; 3. Random Schrödinger operators. Wave propagation in random media; 4. Nonlinear stochastic systems. Stochasitc flows on manifolds; 5. Chaos and phase transitions.

The volume is concluded by the complete list of the papers presented at the Workshop and a Subject Index.

L. Hatvani (Szeged)

Saunders MacLane, Mathematics: Form and Function, XI+476 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

One of the main problems of the teaching of mathematics is the fact that students learn relatively much in special branches of mathematics without acquiring a general image of the subject. For example after studying functions, their properties and applications in various fields the question "What is a function?" may be embarassing. Naturally enough, teaching cannot begin with this question but after some experience we must raise it nevertheless. After similar questions finally it is asked: What is the function of mathematics and what is its form? This is the main problem of this book which grew out of lectures given by the author and which is intended as a background for the philosophy of mathematics. Therefore the first task was to make clear what mathematics is. The book contains a survey of some basic parts of mathematics. In the course of the treatment the author tries to answer six questions: What is the origin of mathematics? What is the organization of mathematics? Are the formalisms of mathematics based on or derived from the facts: if not, how are they derived? How does mathematics develop? Is there an absolute standard of rigor and what are the correct foundations of mathematics? The most fundamental is a bundle of questions concerning the philosophy of mathematics: What are the objects of mathematics and where do they exist? What is the nature of mathematical truth? How is it that we can have knowledge of mathematical truth or of mathematical objects?

Let us enumerate the chapter headings: Origins of formal structure; From whole numbers to rational numbers; Geometry; Real numbers; Functions, transformations and groups; Concepts of calculus; Linear algebra; Forms of space; Mechanics; Complex analysis and topology; Sets, logic and categories; The mathematical network.

The book is very valuable for everybody who has some experience in mathematics. Several details are interesting in themselves. Perhaps the reader will not agree with the author in some of the answers but having read the book his/her own view will surely be more well-considered and endowed with new features in many cases. In my opinion this work is a source of important ideas especially for teachers. The style and presentation is fascinating. (Recently I went by train with one of my friends. I took this book along for the trip. My friend had a dip into it, then he grabbed my book and left me to bore myself with his newspapers till the end of the two hours' train ride.)

L. Pintér (Szeged)

P. C. Müller—W. O. Schiehlen, Linear Vibrations, (Mechanics: Dynamical Systems), X+327 pages, Martinus Nijhoff Publishers, Dordrecth—Boston—Lancaster, 1985.

Both in mechanics and engineering vibration analysis has been a central field since the very beginning. The theoretical methods in this field are based upon an exact mathematical description of the considered technical systems. This mathematical description leads to one or more differential equations; therefore, there is a very useful interaction between oscillation, vibration theory and the theory of differential equations. Many problems on differential equations arose in vibration theory and results of the theory of differential equations often open new directions of investigation in vibration theory. To illuminate this interaction it is enough to mention stability theory. The problem of stability of an equilibrium of a vibrating system appeared in mechanics long ago. Since 1892, when the great Russian mathematician and mechanician A. M. Lyapunov introduced the exact mathematical notion of a solution of a differential equation and discovered methods for their investigation, an enormous development can be observed also in stability theory in mechanics and engineering.

In the last decade two challenging phenomena inspired the further development in vibration analysis: increasing demands on precision and the growing use of electronic computers. Improvement in precision can be achieved by more accurate modelling of technical systems, which, first of all, means modelling mechanisms as systems with many degrees of freedom such as multibody systems, finite element systems or continua. The presence of big computers is also a motivation for making use models with more degrees of freedom, which could not be handled numerically earlier.

This book is a theoretical treatment of multi-degree-of-freedom vibrating systems. Part I gives a classification of these systems, which is in accordance with the classification of the modelling equations (time-variant or time-invariant systems; free, self-excited and forced vibrations; conservative-non-conservative systems). In Part II time-invariant vibrating systems are discussed. Besides the classical results vibrations excited by periodic functions are treated, which may display resonance, pseudoresonance or absorption (the last two phenomena can occur only in multi-degree-of-freedom systems). Random vibrations are also investigated by means of covariance analysis and spectral density analysis. Part III is devoted to time-variant systems including a detailed descussion on the parameter-excited vibrations and parameter-excited random vibrations. In the last part the mathematical prerequisites beyond matrix calculus are presented.

The book can be used as a text, too. Each section is accompanied with interrelated exercises and multiplechoice questions.

Applications of the results to some interesting and important models such as motor car, double pendulum, centrifuge, magnetically levitated vehicle, run through the book.

This excellently written and easily readable book is highly recommended to every scientist, engineer and student interested in vibration theory and its mathematical justification.

L. Hatvani (Szeged)

Peter J. Olver, Applications of Lie Groups to Differential Equations, (Graduate Texts in Mathematics, 107), XXVI+497 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

Mathematicians and physicists undoubtedly agree that the theory of continuous groups, now universally known as Lie groups, is one of the most important and powerful tool in modern mathematics and physics, engineering and other mathematically-based sciences. It is enough to mention its applications to such diverse fields as algebraic topology, bifurcation theory, numerical analysis, control theory, classical mechanics, quantum mechanics, relativity, continuum mechanics and so on. Nevertheless, probably only a few scientists know that this theory has its root in differential equations. In the last century the crucial problem of the theory of differential equations was to find more and more techniques to solve particular equations. Different types of equations were discovered which can be integrated such as separable, homogeneous and exact equations. It was Sophus Lie who pointed out that this method could be unified by a general integration procedure based on the invariance of the differential equation under a continuous group of symmetries. Later on the success of this discovery was overshadowed a little by the qualitative theory of differential equations, but nowadays research activity in this direction has been speeding up exceedingly. For example, it was recently pointed out that using the method of generalized symmetries (i.e. the method of including the derivatives of the relevant dependent variables in the transformations), initiated by E. Noether in 1918, one can view certain nonlinear partial differential equations as Korteweg—de Vries equations completely integrable.

This excellent book gives an introduction to the theory of Lie groups and its applications, to such important problems as the determination of symmetry groups, generalized symmetries and conservation laws, integration of ordinary and partial differential equations, reduction in order for systems in Hamiltonian form with emphasis on explicit examples and computations. Each chapter is concluded by further examples and exercises with a very wide range of difficulty (some of the exercises can be considered as research programs for a beginner).

This textbook can be warmly recommended to mathematicians, physicists and students interested in the theory and applications of symmetry methods.

L. Hatvani (Szeged)

Orders and their Applications, Proceedings, Oberwolfach, 1984. Edited by I. Reiner, K. W. Roggenkamp (Lecture Notes in Mathematics, 1142) X+306 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

The volume is the proceedings of a meeting on "Orders and their Applications" held in the Mathematische Forschungsinstitut Oberwolfach, 1984. The conference was organized around the following four topics: Galois module structure, non-abelian class field theory and analytic number theory of orders (6 titles due to J. Brinkhuis, C. J. Bushnell and I. Reiner, M. Desrochers, A. Fröhlich, L. McCulloh, M. Taylor); *K*-theory of orders and connection with algebraic geometry (6 titles due to M. Auslander, J. Brzezinski, J. F. Carlson, W. van der Kallen, R. Oliver, P. Salberger); Applications to group theory and group representations (2 titles; authors: F. de Meyer and R. Mollin, R. Sandling); and Classification of indecomposable lattices (4 titles of G. H. Cliff and A. R. Weiss, E. Dietrich, R. Guralnick, W. Rump). Additionally, the book contains survey articles from the main speakers, most of them mentioned above (H. Lenstra is the only exeption) and a historical note (due to R. Sandling).

The volume is interesting for researchers and experts on these four topics.

P. Ecsedi-Tóth (Budapest)

Probability Theory and Harmonic Analysis, Edited by J.-A. Chao and W. A. Woyczyński (Monographs and Textbooks in Pure and Applied Mathematics, 98), VIII+291 pages, Marcel Dekker, Inc., New York—Basel, 1986.

The volume presents fiteen uniformly high-level contributions by lecturers at the Mini-Conference on Probability and Harmonic Analysis, Cleveland, Ohio, May 10—12, 1983, and by speakers at other seminars of the Probability Consortium of the Western Reserve. The interaction between probability theory and harmonic analysis has been the subject of intensive research in the last decade or so and will obviously remain one for a period of time to come. Those who wish to join this stream will need the present collection without any doubt.

The authors have evidently been asked to call their contributions "chapters" to achieve an effect of unity. However, these "chapters" are fifteen individual expository, survey or research papers covering a range "from martingales, stochastic integrals, and diffusion processes on manifolds, through random walks and harmonic functions on graphs, and random Fourier series, to invariant differential and degenerate elliptic operators, and singular integral transforms". Nevertheless, in spite of the diverse nature of all these topics, there is indeed a kind of an effect of homogenity. Almost everyone working either in harmonic analysis or with probabilities on algebraic structures will find a paper or two in this volume indispensable for him or her. Entirelly subjectively, I single out three of them for special mention: Richard Durrett's review of reversible diffusion processes, Michael Marcus's discussion of infinitely divisible measures on the space of continuous functions induced by random Fourier series and transforms, and the 57-page article of Lajos Takács on the harmonic analysis of Schur algebras and its applications in the theory of probability. This last paper is in fact a prototype of the investigation of the interaction mentioned above.

Sándor Csörgő (Szeged)

Proceedings of the 4th Pannonian Symposium on Mathematical Statistics, Bad Tatzmannsdorf, Austria, 4—10 September, 1983.

Volume A: **Probability and Statistical Decision Theory,** Edited by F. Konecny, J. Mogyoródi and W. Wertz, XI+344 pages.

Volume B: **Mathematical Statistics and Applications,** Edited by W. Grossman, G. Ch. Pflug, I. Vincze and W. Wertz, VIII+321 pages.

Akadémiai Kiadó, Budapest and D. Reidel Publishing Company, Dordrecht, 1985.

The pleasant fate of conference series in their developing period appears to be that they expand and improve, provided of course that the necessary persistence and skill is invested continually into the organizing work including the acquirement of necessary funds to support sufficiently many good participants. Then anything can happen: the series ends abruptly (as was the case with the Berkeley Symposiums), it grows further (as the Vilnius conferences on probability do), or its level stagnates (your example). The Pannonian Symposiums have still a long way to go to be measured to the late Berkeley Symposiums, but they are getting better for sure. (This reviewer attended the first, third and fourth, and reviewed the Proceedings of the 3rd Symposium in these *Acta* 47 (1984), page 513). In fact, the big leap has been the 4th Symposium and the two volumes of its proceedings testify this adequately.

Volume A starts with three invited papers. These are a masterly survey of results on spacings by Paul Deheuvels with a number of new results and indications of the many-sided statistical applications; a comprehensive paper by Ulrich Müller-Funk, Friedrich Pukelsheim and Hermann Witting on locally most powerful unbiased tests for two-sided hypotheses; and an expository note of Pál Révész on the approximation of the Wiener process and its local time with many open problems and conjecturs all arising from the provoking observation that "nobody saw ever a Wiener path". These are followed by twenty-one contributed papers on really diverse problems. Out of these, with upmost subjectivity, we single out for special mention the paper on L_1 regression estimation by Luc Devroye and László Györfi, Norbert Herrndorf's note on invariance principles for strongly mixing sequences, and Detlef Plachky's paper on the converse of the Lehmann—Scheffé's theorem. Even this short list of six papers shows that many of the papers in Volume A could well have appeared under the title of Volume B, and this is completely true *vice versa*.

Volume B proudly boasts with the most enjoyable invited paper of Paul Erdős on probability methods in number theory which is one of his characteristic lists of open problems with many dollar-prizes offered by him for "prove or disprove...". Here we single out, perhaps even more subjectively, the note by Margit Lénárd on L_p spline approximation of stochastic processes and Harald Niederreiter's paper on quasi-Monte Carlo optimization.

Meanwhile the 5th and 6th Symposiums have already taken place. The reviewer was unable to attend these and can thus only hope that the trend continues to be upward.

Sándor Csörgő (Szeged)

P. Rabier, Lectures on Topics in One-parameter Bifurcation Problems, (Tata Institute of Fundamental Research Lectures on Mathematics and Physics, 76), VI+286 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

When models of systems and processes — algebraic, differential, functional equations — depend on a parameter, it frequently happens that there are certain values of the parameter such that small deviations of the parameter from these values cause significant changes in the qualitative behaviour of the solutions of the equation. The goal of bifurcation theory is to identify these bifurcation values of the parameter and to describe the nature of the system near such points.

These notes contain the subject-matter of a series of lectures delivered by the author at the Tata Institute of Fundamental Research Centre, Bangalore, in July and August 1984. The reader gets a good account on some interesting and very new ideas. For exemple, breaking with the traditional exposition of the Lyapunov—Schmidt method the author gives a new algorithm for finding the local zero set of a mapping in certain regular cases. The final chapter introduces a new method in the study of bifurcation problems in the degenerate case. Namely, it is shown how to find the local zero set of an $f\in C^\infty$ real valued function of two variables satisfying $f(0)=0$, $Df(0)=0$, $D^2f(0)\neq 0$ but $\det D^2f(0)=0$ (so that the Morse condition fails).

The book is concluded by some applications and remarks on further developments of the methods.

L. Hatvani (Szeged)

Recursion Theory Week, Proceedings, Oberwolfach, 1984. Edited by H. D. Ebbinghaus, G. H. Müller, G. E. Sacks (Lecture Notes in Mathematics, 1141), X+418 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

The proceedings of a conference on recursion theory that took place in the Mathematisches Forschungsinstitut Oberwolfach from April 15th to April 21st, 1984, include the following titles: Ambos—Spies, K. Generators of the recursively enumerable degrees; Blass, A. Kleene degrees of ultrafilters; Chong, C. T. Recursion theory on strongly Σ_2-inadmissible ordinals; Clote, P. Applications of the low-basis theorem in arithmetic; Dietzfelbinger, M., Maass, W. Strong reducibilities in α- and β-recursion theory; Fejer, P. A., Shore, R. A. Embeddings and extensions of embeddings in the r.e. tt and wtt-degrees; Friedman, Sy. D. An immune partition of the ordinals; Griffor, E. R. An application of π_2-logic to descriptive set theory; Hinman, P. G., Zachos, S. Probabilistic machines, oracles, and quantifiers; Homer, St. Minimal polynomial degrees of nonrecursive sets; Jockus, C. G. Jr. Genericity for recursively enumerable sets; Kechris, A. S. Sets of everywhere singular functions; Kucera, A. Measure, π_1^0-classes and complete extensions of PA; Lerman, M. On the ordering of classes in high/low hierarchies; Nerode, A., Remmel, J. B. Generic objects in recursion theory; Odifreddi, P. The structure of m-degrees; Sacks, G. E. Some open questions in recursion theory; Shinoda, J. Absolute type 2 objects; Simpson, St. G. Recursion theoretic aspects of the dual Ramsey theorem; Slaman, T. A. Reflection and the priority method in E-recursion theory; Wainer, S. S. Subrecursive ordinals.

The volume is recommended to experts and students on advanced level in recursion theory.

P. Ecsedi-Tóth (Budapest)

J. A. Sanders—F. Verhulst, Averaging Methods in Nonlinear Dynamical Systems, (Applied Mathematical Sciences, 59), X+247 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

The averaging method is the most important asymptotic method of perturbation theory.

Most differential equations admit neither an exact analytic solution nor a complete qualitative description. However, there are some special classes of equations (linear equations, some autonomous systems, ...) the asymptotic behaviour of whose solutions is known. Perturbation theory is the collection of methods for the study of equations close to equations of these special forms. These latter equations are called unperturbed and their solutions are assumed to be known. Briefly speaking, perturbation theory studies the effect of small changes in the differential equations on the behaviour of solutions.

Model equations often contain a small parameter ε, and the size of the perturbation is characterized by ε. If we investigate the effect of the perturbation over a fixed bounded interval of time independent of ε, we can use the variational equation along the unperturbed solution. However, the investigation of the behaviour of solutions over a large time interval, e.g. of order $1/\varepsilon$, is much more complicated. This is the subject of the so-called asymptotic methods of perturbation theory.

The averaging method gives estimates on the difference between a solution of a nonautonomous equation containing a small parameter and the solution of the autonomous equation obtained by replacing the right-hand side by its integral mean. The method has been used to determine the evolution of planetary orbits under the influence of the mutual perturbation of planets since the time of Lagrange and Laplace, often intuitively. Even nowadays, many physicists and astronomers consider averaging a natural and obvious procedure which need not be justified. However, as is shown in the book by examples and counterexamples, it is important to establish a rigorous approximation theory. The problem of strict justification of the method is still far from being solved.

The first two chapters of the book are of introductory character. The third chapter contains the basic theory of averaging with special emphasis on periodic and almost periodic systems. Chapter 4 deals with the cases when either the original or the averaged equation has an attractor. Chapter 5 is devoted to averaging over spatial variables which allows us to handle systems with slowly varying coefficients. In Chapter 6 the normal forms are considered. Chapter 7 is concerned with Hamiltonian systems in the various resonance cases. Here the method of averaging is used to determine periodic orbits and invariant tori. The book is concluded by many appendices with interesting examples, applications and supplements.

This monograph will be very useful for mathematicians, physicists, astronomers and other users of mathematics interested in qualitative aspects of asymptotic methods.

L. Hatvani (Szeged)

D. H. Sattinger—O. L. Weaver, Lie Groups and Algebras with Applications to Physics, Geometry, and Mechanics (Applied Mathematical Sciences, Vol. 61), IX+215 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

The authors undertake to give an exceedingly brief and at the same time consistently constructed introduction to the theory of Lie groups and Lie algebras. They are commanded by the aim that readers interested in applications (first of all analysts or physicists) could go deeply into the subject by connecting it with well-known structures and concepts. However the geometer or algebraist can also appreciate the wide-ranging applications of the theory and can get acquainted with the physical motives behind a number of questions belonging to the topic.

The book has the virtue that, while explaining the results in a homogeneous treatment, the authors bring great care to present their historical development as well. The way in which the authors combine their modern attitude with the explanation of the classical development of the basic results on Lie algebras and Lie groups is interesting for the geometer. A similarly significant feature of the book is its descriptiveness. It shows the essence of the structure of Lie groups by investigating the ones that are significant from the physical and geometrical point of view. This descriptiveness is typical for the investigations of the connection between Lie groups and Lie algebras.

Owing to keeping in view the applications, the representations of Lie algebras play an exceedingly important role. After that the reader learns the general structure of Lie algebras (solvability, nilpotency, Cartan's criteria) and structure of semi-simple algebras (Cartan subalgebras, root space), a whole part deals with the representation theory of semi-simple Lie algebras and the very important spinor representations. The same view also explains the reasons why the authors study the real and complex Lie groups and Lie algebras more comprehensively. The last part completes the material with some important applications such as completely integrable systems; the Kostant—Kirillov symplectic structures and spontaneous symmetry breaking.

There is a number of good exercises at the end of each section.

József Kozma (Szeged)

Winfried Scharlau, Quadratic and Hermitian Forms (Grundlehren der mathematischen Wissenschaften, 270) X+421 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

The purpase of this book is to give a glimpse into the theory of quadratic and Hermitian forms from an essentially algebraic point of view. The material is divided into ten chapters and at the end of the book an Appendix can be found. The first chapter introduces the basic concepts which will be used in the first seven chapters: quadratic forms and symmetric bilinear forms over fields of characteristic unequal to 2. In Chapter two the basic methods and results of the algebraic theory of quadratic forms can be found. In Chapter three a short introduction into the relations between quadratic forms and ordered fields is given. The subjects of the fourth chapter are a deeper investigation of the algebraic theory of quadratic forms and the theory of Pfister forms. Chapters five and six deal with the number-theoretic aspect of the theory of quadratic forms: Instead of the integers and the rational fields more generally an arbitrary algebraic number field and its ring of algebraic integers are considered. Chapter seven is devoted to a general and abstract foundation for the important concepts in connection with bilinear, hermitian and quadratic forms. Chapter eight contains basic results about finite dimensional simple algebras and many interesting connections between the theory of quadratic and hermitian forms on the one hand and the theory of simple algebras and involutions on the other. In Chapter nine the theory of Clifford algebras is developed in an elegant *ad hoc* presentation. Chapter ten returns to hermitian forms and continues the investigations begun in the seventh chapter. The appendix contains some examples.

L. Gehér (Szeged)

Thomas B. A. Senior, Mathematical Methods in Electrical Engineering, VIII+272 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1986.

This textbook contains the subject-matter of the one-semester course taught by the author at University of Michigan for students in electrical and computer engineering. At every university such a course has to include the Laplace and Fourier transforms and their applications and some basic knowledge on linear systems. Instructors of a course like this can hardly find a good textbook which is suitable also for undergraduate students not having the basic ideas of complex-variable theory. This book has filled the gap.

Chapter 1 gives a short introduction to complex numbers. Chapter 2 acquaints the reader with the Laplace transform and its applications to differential equations. Chapter 3 deals with the basic concepts and methods of linear-systems theory paying attention equally both to the physical and the mathematical aspects of the subject. The same feature characterizes Chapter 4, which is devoted to Fourier series. Chapter 5 is of more mathematical character, in which the reader gets a good introduction with rigorous theorems and proofs to the analysis of functions of a complex variable. Chapter 6 deals with Fourier transforms. The final Chapter 7 is a short discussion on the connection between Laplace and Fourier transforms. Each chapter contains a number of worked examples and ends with exercises.

This textbook will be very useful for undergraduate students who have a firm background in calculus and differential equations and for their teachers as well.

L. Hatvani (Szeged)

Joseph H. Silverman, The Arithmetic of Elliptic Curves (Graduate Texts in Mathematics), XII+400 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

If a mathematician speaks on mathematics to a non-mathematician public the topic is often taken from number theory, especially from diophantine equations. From the beginnings number theory has formed a great part of mathematics. But its role changed surprisingly in the late forties. Number theory, a characteristic branch of "pure mathematics", has a lot of practical applications in our days. The theory did not loose its attractive features, its influence on mathematics is deeper than ever. An important new result in number theory may arouse the interest of mathematicians all over the world. Consider, for example, the proof of the famous Mordell conjecture by G. Faltings from the last years. As originally formulated the conjecture said that any irreducible polynomial $f(x, y)$ with rational coefficients, having genus greater than or equal to two, has at most a finite number of pairs $x_i, y_i \in Q$ with $f(x_i, y_i)=0$.

The aim of this book is to present an essentially self-contained introduction to the basic arithmetic properties of elliptic curves. Although the author presented approximately half of the material of what he hoped to include, what he wrote is a clear well-organized text offering a good survey of the subject. As prerequisites a first course in algebraic number theory and rudiments of complex analysis are supposed. The reader will find in the first two chapters an introduction to the algebraic geometry of varietes and curves with references. There are numerous interesting exercises at the end of the chapters, some of them are unsolved problems. Similar work in this area has not been published yet which, considering the wast amount of research done in the last decades, is a little curious.

The author says in his Preface: "It is certainly true that some of the deepest results in this subject, such as Mazur's theorem bounding torsion over Q and Faltings' proof of the isogeny conjecture, require many of the resources of modern "SGA-style" algebraic geometry. On the other hand, one needs no machinery at all to write down the equation on an elliptic curve and to do explicit computations with it; and so there are many important theorems, whose proof requires nothing more than cleverness and hard work. Whether your inclination leans toward heavy machinery or imaginative calculations, you will find much that remains to be discovered in the arithmetic theory of elliptic curves. Happy hunting!"

L. Pintér (Szeged)

C. Smoriński, Self-Reference and Modal Logic (Universitext) xii+333 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

The volume is devoted to the investigation of self-reference using modal logic. The aim of this investigation is to clarify that "Gödel's Theorem is not artificial; the use of self-reference has not been obsoleted by recursion theory or combinatorics; and self-references is not that mysterious. This monograph reports on the beginnings of a coherent theory of self-reference and incompleteness phenomena, ...".

The book is quite self-contained: Chapter 0 collects almost all the background material required in further chapters. Chapters 1—3, the beginning sections of Chs. 4 and 6 form the core of the material. Chapter 1 develops some of the syntactical tools for Modal Logics (Basic Modal Logic and Provability Logics) while Chapter 2 deals with their model theory in the style of Kripke. Chapter 3 is devoted to questions of arithmetic interpretations of Provability Logics by establishing Solovay's Completeness Theorems stating that Provability Logic is the modal logic of provability in Primitive Recursive Arithmetic. The whole material is generalized to bi-modal logics in Chapter 4. The next chapter deals with Lindenbaum fixed point algebras, and the so called diagonalizable algebra in order to obtain some representation theorems. Chapter 6 treats Rosser sentences. Finally, Chapter 7 is devoted to presenting some applicational oriented material.

The book is clearly written and in a good style. It is recommended to anyone interested in Gödel's Incompleteness results and provability.

P. Ecsedi-Tóth (Budapest)

Frederick H. Soon. Student's Guide to Calculus by J. Marsden and A. Weinstein, vol. I—III, 888 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985—1986.

When Calculus I—III by Marsden and Weinstein came out, we expected a good deal out of it (see the review in these *Acta*, 50 (1986), pages 242—243). Relying on our experiences so far, now we can establish that all of our expectations are fulfilled. The book has been proved especially useful for those students who are willing to choose the best way of learning calculus (and any mathematical subject): attempting to solve problems on their own. The present supplement to the textbook can make this method even more effective.

The sections are of the same structure. Each of them is started with Prerequisites, Prerequisite Quiz, Goals and Study Hints. The prerequisite quiz helps the reader decide if he/she is ready to continue. The goals serve as guidelines during the study of the section emphasizing the most important points. The study hints point out what is worth memorizing, and what is not, from the topic.

Each section provides the detailed solutions to every other odd numbered exercise in the corresponding section of the textbook. Since most of the exercises in the book are written in pairs, the solutions can also be used as a guide to solving the corresponding even numbered exercises.

The sections are accompanied with quizzes, at least one of which is a word problem, for the reader to evaluate his/her mastery of the material. Finally, answers can be found to both the prerequisite and section quizzes.

The chapters are concluded by review sections with questions and answers which may appear on a typical test. The three-hour comprehensive exams, included after every third chapter, help the reader prepare for the midterms and final examinations.

This guide — together with the textbook of Marsden and Weinstein — will be welcomed by students who wish to make their study of calculus easier and enjoyable.

L. Hatvani (Szeged)

Stochastic Analysis and Applications, Proceedings, Swansea, 1983. Edited by A. Truman and D. Williams (Lecture Notes in Mathematics, 1095), III + 199 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

This is a collection of thirteen research papers, eight of which were read at the Workshop on Stochastic Analysis, Swansea, 11—15 April, 1983, and the rest are some more recent contributions by the Swansea school itself. As the editors write, "the applications include such diverse topics as stochastic mechanics and the Titius—Bode law (for the distances of the planets from the sun), non-standard Dirichlet forms and polymers, statistical mechanics, quantum stochastic processes, the applications of local time to proving path-wise uniqueness of solutions of stochastic differential equations and its application to excursion theory, Bessel processes and pole-seeking Brownian motion, queues, potential theory and Wiener—Hopf theory". The central theme of investigation is of course Brownian motion from which most of the more general processes required by the above applications take their origin. There are many new results for Brownian motion in this collection. However, beside probabilists, the volume may be of interest to theoretical physicists as well.

Sándor Csörgő (Szeged)

The analysis of concurrent systems, Proceedings, Cambridge, 1983. Edited by B. T. Denvir, W. T. Harwood, M. I. Jackson (Lecture Notes in Computer Science, 207), VII+398 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

The volume contains the Proceedings of a Workshop organised by the Standard Telecommunication Laboratories Limited, held in 1983. The four tutorials give expositions of different approaches to the analysis and description of concurrent systems. They are the well known algebric, net theoretic, temporal logic and axiomatic approaches discussed by prominent authors in these topics. However, the most interesting part of the book is the set of ten problems on concurrency and their solutions. The problems are briefly documented and the various solutions of the participants of the Workshop are described in detail. The problems having both theoretical and practical interest are: two-way channel, network service, firing squad, railway, array processor, packet network, parallel reduction of function combinators, mixing synchronous and asynchronous input, cash-point service and matrix switch. Each problem has more solutions based on different theoretical backgrounds due to different authors.

Á. Makay (Szeged)

The book of *L*, Edited by G. Rozenberg and A. Salomaa, XV+471 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

The book is dedicated to Aristid Lindenmayer who introduced language-theoretic models in biology referred to as *L* systems. It contains about 40 articles showing a continuous interest in the topic. Most of them are up-to-date research papers concerning different classes of *L* systems (e.g. 0L, D0L, DT0L) from formal language theoretical point of view.

"A 0L scheme *is a pair* (X, σ) *with* X *a finite alphabet and* σ *a finite substitution of* X *into the* free monoid X^*. It is deterministic (a D0L scheme) if $\sigma(a)$ is a singleton set for each $a \in X$, and in this case σ can be considered an endomorphism of X^*. ... A 0L system is a triple (X, σ, ω) such that (X, σ) is a 0L scheme and $\omega \in X^*$ is the axiom of the system. For a 0L system $G=(X, \sigma, \omega)$ one considers the languages

$$L_i(G) = \begin{cases} \{\omega\} & \text{if } i = 0 \\ \{\sigma^i(\omega)\} & \text{if } i > 0 \end{cases}$$

The language of G is the set $L(G) = \bigcup_{i=0}^{\infty} L_i(G)$." (H. Jürgensen, D. E. Metthews)

People interested in applications of the *L* systems find articles in developmental biology, transplantation and software technology.

Á. Makay (Szeged)

The Influence of Computers and Informatics on Mathematics and its Teaching. ICMI Study Series, IV+155 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sidney, 1986.

Willy-nilly one must be interested in the computer. It enters increasingly in everyday life and of course in mathematics, in research, in the process of applying mathematics as well as in teaching. In mathematics the computer is not only a new tool, it is itself the source of new areas of research. As any new tool it comes with advantages and disadvantages, it can be used well or poorly, it can be overemphasized or ignored.

The plan of the International Commission on Mathematical Instruction (ICMI) is to present a series on topics of mathematical education. The first study deals with the influence of computers on mathematics and its teaching. A discussion document was sent to all national delegates of ICMI. It looks in particular at the three themes: 1. How do computers and informatics influence mathematical ideas, values and the advancement of mathematical science? 2. How can new curricula be designed to meet changing needs and possibilities? 3. How can the use of computers help the teaching of mathematics? Contributions written in response formed the basis of discussions at a symposium held in Strasbourg in 1985.

This book contains the above mentioned report and a selection of papers contributed to the Symposium. Let us enumerate some of them: M. F. Atyah: Mathematics and the Computer Revolution (This is one of the most inspiring lectures in the book having sub-titles: A historical perspective, Mathematics and theoretical computer science, Computers as an aid to mathematical research, The intellectual dangers, Economic dangers, Educational dangers, Conclusion). L. A. Steen: Living with a New Mathematical Species; N. G. de Bruijn: Checking Mathematics with the Aid of a Computer; J. Stern: On the Mathematical Basis of Computer Science; H. Murakami and M. Hata: Mathematical Education in the Computer Age; H. Burkhardt: Computer-aware Curricula: Ideas and Realization.

Perhaps even this short list shows that this book is an interesting collection of different opinions and propositions in a theme standing in the limelight of every mathematician and teacher of mathematics.

L. Pintér (Szeged)

Theoretical Approaches to Turbulence, Edited by D. L. Dwoyer, M. Y. Hussaini and R. G. Voigt, XII+373 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

Observations of turbulence, which is the most natural mode of fluid motion, are very old. One can find references to it already in the Bible; Leonardo da Vinci sketched it in circa 1500. The modern scientific study of turbulence or chaos, which dates from the late 1800s with the work of Osborne Reynolds, can be devided into three distinct movements: the earliest statistical movement is of a strong nondeterministic character, the structural movement is predominantly observational, and the most recent one based upon the results of modern theory of dynamical systems is known as the deterministic movement. In spite of the efforts, the phenomenon of turbulence can be considered as one of the oldest and most difficult open problems of physics.

This book contains the subject-matter of the lectures of the recognized leaders (fluid dynamicists, mathematicians and physicists) in the field of turbulence delivered in a workshop during October 10—12, 1984, which was sponsored by The Institute for Computer Applications in Science and Engineering and NASA Langely Research Center. According to the categories of the theoretical approaches to modelling turbulence, the lectures can be devided into four groups: (1) analytical modelling, (2) physical modelling, (3) phenomenological modelling, (4) numerical modelling.

In the preface the editors give an excellent preparatory summary and evaluation on each article included. The 19 titles of the book are as follows: Dennis M. Bushnell, Turbulence sensitivity and control in wall flows; Gary T. Chapman and Murray Tobak, Observations, theoretical ideas, and modelling of turbulent flows — past, present and future; Joel H. Ferziger, Large eddy simulations: its role in turbulence research; Jackson R. Herring, An introduction and overview of various theoretical approaches to turbulence; Robert H. Kraichnan, Decimated amplitude equations in turbulence dynamics; Marten T. Landhal, Flat-eddy model for coherent structures in boundary layer turbulence; B. E. Launder, Progress and prospects in phenomenological turbulence models; W. D. McComb, Renormalisation group methods applied to the numerical simulation of fluid turbulence; A. Pouquet, Statistical methods in turbulence; William C. Reynolds and Moon J. Lee, The structure of homogeneous turbulence; P. G. Saffman, Vortex dynamics; D. Brian Spalding, Two-fluid models of turbulence; E. A. Spiegel, Chaos and coherent structures in fluid flows; R. Temam, Connection between two classical approaches to turbulence: the conventional theory and the attractors; Hassan Aref, Remarks on prototypes of turbulence, structures in turbulence and the role of chaos; Jean-Pierre Chollet, Subgrid scale modelling and statistical theories in three-dimensional trubulence; John L. Lumley, Strange attractors, coherent structures and statistical approaches; Parviz Moin, A note on the structure of turbulent shear flows; S. B. Pope, Lagrangian modelling for turbulent flows.

Anyone who is not familiar with the history and basic ideas of turbulence and chaos, but wishes to get an excellent overview of them, must read the article of Chapman and Tobak. Of course, experts are also warmly recommended to have this book on their bookshelf.

L. Hatvani (Szeged)

Brian S. Thomson, Real Functions (Lecture Notes in Mathematics, 1170), VII+229 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

The book consists of seven chapters and an Appendix. Chapter one introduces a general structure called local system of sets by the aid of which a variety of general notions of limit, continuity, derivative etc. can be formulated. The second chapter gives a review of the classical material on real cluster points and develops some abstract presentation of this material. The purpose of the third chapter is to generalize the elementary notion of continuity, and to introduce the notion of continuity relative to a local system. Chapter four investigates the variation of a function. For each function and local system a measure can be defined which can be used to discover various differentiation properties of the function. Chapter five presents a systematic and detailed investigation of general classes of monotonicity theorems. Chapters six and seven are devoted to describe the relationships among different types of generalized derivatives. The text ends with an Appendix containing a variety of computations directly related to the notion of set porosity.

L. Gehér (Szeged)

Topics in the Theoretical Bases and Applications of Computer Science, Proceedings of the 4th Hungarian Computer Science Conference, Győr, Hungary, July 8—10, 1985. Edited by M. Arató, I. Kátai and L. Varga, X+514 pages, Akadémia Kiadó, Budapest, 1986.

The volume contains a selection of papers presented at the 4th Hungarian Computer Science Conference. The subject of the conference included various topics ranging from theoretical fields to practically motivated computer applications: formal languages, automata theory, Petri nets, program semantics, models of computation, mathematical algorithms, databases and information retrieval systems, distributed systems, expert systems and artificial intelligence. The following is a list of the invited papers: A. Salomaa: Grammar forms: A unifying device in language theory, W. Brauer and D. Taubner: Petri nets and CPS, R. Albrecht: Formal principles of computer architecture, F. Hossfeld: Parallel algorithms — beyond vectorization, Dj. Babayev and R. Babayev: Generating 0—1 integer programming test problems, Y. Matijasevich: A posteriori version of interval analysis.

Z. Ésik (Szeged)

Andrzej Trautman: Differential Geometry for Physicists *(Stony Brook Lectures).* Monographs and Textbooks in Physical Science, V+145 pages, Bibliopolis, Napoli, 1984.

Differential geometric methods are increasingly applied in modern physics, in particular in relativity theory and high-energy physics. Physicists may, however, have difficulty in reading the available (otherwise excellent) textbooks written by (and for) mathematicians. This gap is filled by Trautman's Stony Brook Lectures. Professor Trautman, a recognized authority who has made important contributions to the field, provides students and researchers with a comprehensive, physics-motivated introduction to the theory of differential manifolds, Lie groups and fibre bundle theory. He explains the use of these structures for gauge fields. The theory of characteristic classes and non-trivial fibre bundles is illustrated on the examples of monopoles and instantons. The well-written and nicely-printed book may be used for a one-semester introductory course for physics students.

P. A. Horváthy (Dublin)

S. M. Ulam, Science, Computers and People, From the Tree of Mathematics, edited by M. C. Reynolds and Gian-Carlo Rota, XXI+264 pages, Birkhäuser, Boston—Basel—Stuttgart, 1986.

This is a collection of 23 essays (originally published between 1946 and 1982) written by the famous Polish born mathematician Stanisław Ulam, whose influence on the development of mathematics and, in particular, the application of mathematics in unconventional areas can hardly be overestimated.

According his own view of Ulam — as we can read in a preface written by Martin Gardner — "I am the type that likes to start new things rather than improve or elaborate, ...". He wrote that "I cannot claim that I know much of the technical matherial of mathematics. What I may have is a feeling for the gist, or maybe only the gist of the gist".

In these sentences Ulam was too modest, he knew much about the technical side of mathematics as well, but in his way of seeking the gist he was able to open several new roads which often led to new branches of mathematics, e.g., the theory of cellular atomata (he proposed it to von Neumann), using the Monte-Carlo method in areas different from probability theory, and nonlinear-processes.

Most of the essays are dealing with physical problems (i.e., Ideas of Space-Time, Thermonuclear Devices), computational problems (i.e., A First Look at Computings, Computers in Mathematics, Computations in Parallel), problems on patterns of growth of figures and biological applications.

We can read three very interesting essays on John von Neumann and his work. Probably this is the unique source where these three masterpieces appeared in one volume. There are also four shorter writings on other eminent scientists (Gamow, Smoluchowski, Kuratowski and Banach).

Reading these essays we can enjoy the brilliant writting style of Ulam, which is a mix of crystal clear prose, subtle humor, and graceful phrasing, therefore this volume — which has three introductory chapters written by M. Gardner, Gian-Carlo Rota and Ulam's wife Francoise Ulam — can be warmly recommended to the whole mathematical community.

Lajos Klukovits (Szeged)

Jan-Cees van der Meer, The Hamiltonian Hopf Bifurcation, (Lecture Notes in Mathematics, 1160), VI+115 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

In these notes the author expounds his new theory that gives a complete description of the bifurcation of periodic solutions for the generic case of the Hamiltonian Hopf bifurcation. In fact, he originally was interested in a particular bifurcation of periodic solutions in the restricted problem of three bodies at the equilibrium L_4. In this problem a particle P of zero mass moves subject to the attraction of two other bodies P_1, P_2 of positive mass rotating in circles about their centre of gravity. Euler described the first three equilibrium points of the particle lying on the line of P_1 and P_2 (E_1, E_2, E_3). Later on, Lagrange found two further equilibria which form an equilateral triangle with P_1 and P_2 (L_4, L_5). If we are interested in the motion near L_4 then we have a Hamiltonian system with a so-called nonsemisimple $1: -1$ resonance. Because of the special properties of this resonance the existing methods had to be reformulated in order to deal with the specific nature of the problem. Applying the normal form theory and some ideas of Weinstein and Moser, the author has succeeded in giving a complete description of the behaviour of periodic solutions of short period in the bifurcation as the family of systems passes through the resonance. Such a bifurcation appears in the restricted problem of three bodies at L_4 when the mass parameter passes through the critical volue of Routh.

These well-written lecture notes must be read by every mathematician, physicist and astronomer interested in perturbation theory of Hamiltonian systems, celestial mechanics and, especially, in the three body problem.

L. Hatvani (Szeged)

Robert L. Vaught, Set Theory. An Introduction, X+141 pages, Birkhäuser, Boston—Basel—Stuttgart, 1985.

Vaught's book is intended to serve for a course at the undergraduate level. The author presents the material in the style of the originator of the subject, Georg Cantor. He writes: "For many years, the widely used introductory books on set theory all presented intuitive set theory. For the past two or three decades, the exact opposite has been true: all such books have given axiomatic set theory. But for the student, the trivial and irritating business of fooling around, as he begins to learn set theory with axioms (saying for example that $\{x, y\}$ exists!) discourages him from grasping the main, beautiful facts about infinite unions, cardinals, etc., which should be a joy."

The core of the material is presented in Chapters 1—7. The first five chapters give a good, intuitive introduction to such topics as sets, operations on sets, cardinal numbers, orders and order types, finite sets and number systems (of the integers, rationals and reals). Axioms appear first in the very short Chapter 6 (five pages only!). The next chapter is devoted to the study of well-orderings and the formal definition of cardinals and ordinals (in the manner due to von Neumann), topics (in particular, results on transfinite recursion) which seem to be "more easily grasped working axiomatically than intuitively".

Chapter 8 gives a short, easy discussion of the axiom of regularity. The next chapter presents results in logic which can be used in consistency and independence proofs. The material on logic is out of the scope of set theory, but concludes with formalisations of the ZFC set theory. Chapter 10 gives the relative consistency of the axioms of regularity and infinity following the "inner model" method of von Neumann. Finally, the last chapter returns to pure set theory and provides additional material on the arithmetic of cardinals and ordinals.

The volume is written in a clear and interesting style and is higly recommended to undergraduate students of mathematics as well as of philosophy.

P. Ecsedi-Tóth (Budapest)

Wolfgang Walter, Analysis I. (Grundwissen der Mathematik, 3) XII—385 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

This book contains essentially the material of an introductory analysis course in the first two semesters. The text is divided into three main parts. Part A is introductory. The main purpose of this is to give the notion of real numbers, the basic concepts of set theory, the notion of functions and some fundamental facts concerning functions. Part B introduces the notions of convergence and continuity and gives the usual elementary theorems. Part C is devoted to the introduction of the notions of differentiation and the Riemann integral and presents the classical theory.

The book is recommended to students in the first two semesters as a handbook.

L. Gehér (Szeged)