

## Bibliographie

**Donald J. Albers—G. L. Alexanderson—Constance Reid, International Mathematical Congresses. An illustrated History 1893—1986, Revised Edition, 64 pages, Springer-Verlag, New York—Berlin—Heidelberg, 1987.**

This is a nice picture book covering the “World Congress” in Chicago, 1893, and all the International Congresses of Mathematicians beginning with the first in Zürich, 1897, and ending with the latest, the twentieth one in Berkeley, 1986. Each congress receives two pages (plus, in connection with his famous address in Paris, 1900, Hilbert himself an extra two) with three to five photos or drawings of illustrious mathematicians, alone or together, who played outstanding roles at the given congress or of characteristic buildings. It is the pictures that make the book nice. Not much can be said about the text. Using (sometimes fragmentary and irrelevant) citations, it tries to give a “feeling” of the given congress. The aspect of “the first American” of the three American authors pops up in an inordinate frequency. There are nine pages with the photographs of all Fields Medalists and a list of all the plenary lectures from 1893 to 1986.

The original edition of the book has been distributed during the Berkeley Congress. As it is made clear in Czesław Olech’s Opinion [*The Mathematical Intelligencer* 9 (1987), 36—37], the present revised edition has become necessary mainly because of protests, including his own in the same Opinion, against “an unfair description of the previous Congress in Warsaw” in the original edition.

*Sándor Csörgő (Szeged)*

**Hans Wilhelm Alt, Lineare Funktionalanalysis. Eine Anwendungsorientierte Einführung (Hochschultext), IX + 292 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.**

The material of this book is based on the lectures in linear functional analysis held some years ago at Bonn University for students in the fifth semester. The book consists of an introductory part, ten chapters and three supplements. The text gives an introduction to the study of Banach and Hilbert spaces, linear functionals and the most important classes of linear operators. The supplements deal with measures, integrals and Sobolev spaces. At the end of the book the spectral theory of compact normal operators can be found. All of the chapters end with exercises and their solutions.

The book is highly recommended to students who are interested in functional analysis and its applications in physics.

*L. Gehér (Szeged)*

**Analytic Theory of Continued Fractions**, Proceedings Pitlochry and Aviemore, Scotland, 1985. Edited by W. Y. Thron (Lecture Notes in Mathematics, 1199), III + 299 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1986.

In the last decades special attention is paid to the new results in the theory of continued fractions. The success of the workshop held in Loen, Norway in 1981 speaks for itself. Therefore a second workshop was arranged in Pitlochry and Aviemore, Scotland in 1985. This proceedings volume is thus the successor of Lecture Notes in Mathematics, Vol. 932.

This volume contains a survey article entitled "Schur fractions, Perron—Carathéodory fractions and Szegő polynomials" by W. B. Jones, O. Njåstad and W. J. Thron and thirteen original research papers. The introduction of the survey article presents historical comments with a limited list of references. Two main topics are treated in the research papers. The first one is the convergence theory of continued fractions, the second one is the investigation of various types of continued fractions useful in solving Stieltjes, Hamburger and trigonometric moment problems. In general the articles give applications from different branches of mathematics. Perhaps the volume would have been more interesting if some of the papers had contained open questions or conjectures in explicit form.

*L. Pintér (Szeged)*

**D. F. Andrews—A. M. Herzberg, Data: A Collection of Problems from Many Fields for the Student and Research Worker** (Springer Series in Statistics), XX + 442 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

The ultimate aim of Statistics is to provide methods and tools for the analysis of real data. In this very useful book the authors collected a great number of concrete real data sets. There are seventy-one concrete problems presented in the book with data sets given in 100 tables and 11 figures. For each data set the source or sources of the data are given with a description by the authors or by a contributor who supplied the data. No direct reference to any particular type of analysis is given: the student or researcher may try his/her arsenal of tools for the analysis. Some of the data sets are well known, such as the last century data on the number of deaths by horsekicks in the Prussian Army (the name of L. von Bortkiewicz, to whom the horsekicks belong, is written all four times erroneously as Bortkewitsch: the authors did not always go back to the original source), the Fisher *Iris* data (with which the collection starts), the Canadian lynx trappings data, the coal-mining disasters data, the Federalist Papers data, or the Stanford heart transplant data. The majority of the data sets, however, is relatively new and unknown to a wider statistical public and is very interesting. The authors have invested a great care into the organizational work and the uniformization of the presentation. The result is a splendid volume of great interest, completely unique in its kind and a great service to the international statistical community.

*Sándor Csörgő (Szeged)*

**Astrophysics of Brown Dwarfs**. Proceedings, Fairfax, 1985, Edited by M. C. Kafatos, R. S. Harrington and S. P. Maran, 276 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1986.

This book includes the scientific papers presented at a Workshop, held at George Mason University, Fairfax, Virginia, in 1985.

The term "brown dwarf" is a boundary class of stars that are partially supported by nuclear burning and partially by thermal cooling. These objects, "super-Jupiters", bridge the range of

masses between planets and normal stars. The mass of the brown dwarfs is less than 0.08 solar mass, and their surface temperatures are expected to be 1000—2000 K.

The book consist of two parts. In the first one the experimental works are presented. The observation of brown dwarfs is very difficult, owing to the faintness of these objects, and only one definite object has been found so far. The articles about the systematic search for a very nearby solar companion (“Nemezis” or “Shiva”) are especially interesting. In the second, theoretical part aspects of planetary interior physics are extended to higher densities and pressures. The brown dwarfs are of great importance in the stellar evolution theory.

Presently, in the topic of the brown dwarfs there are more theories than objects. However, with the help of space telescopes and infrared techniques the detection of numerous stars of this kind are likely to be discovered soon.

*K. Szatmáry (Szeged)*

**Werner Ballmann—Michael Gromov—Viktor Schroeder, *Manifolds of Nonpositive Curvature*** (Progress in Mathematics, Vol. 61), 263 pages, Birkhäuser, Boston—Basel—Stuttgart, 1985.

This book is based on four lectures given by Mikhael Gromov in February 1981 at the College de France in Paris. The presentation is due to Viktor Schroeder who made a coherent text by writing down all the proofs in complete detail. He also added some background material to Lecture I and exposed the basic facts on symmetric spaces needed for Lecture IV. The articles included in this book summarize the recent progress of the theory of manifolds of nonpositive curvature. The lectures are: I. Simply connected manifolds of nonpositive curvature, II. Groups of isometries, III. Finiteness theorems, IV. Strong rigidity of locally symmetric spaces. The further papers are: Manifolds of higher rank (by W. Ballmann); Finiteness results for nonanalytic manifolds, Tits metric and the action of isometries at infinity, Tits metric and asymptotic rigidity, Symmetric spaces of non-compact type (by V. Schroeder).

*Péter T. Nagy (Szeged)*

**J. L. Berggren, *Episodes in the Mathematics of Medieval Islam***, 97 figures and 20 plates, IX + 197 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1986.

Many people know today that some mathematical terminologies have their origin in the medieval Islamic (or Arabic) civilization, such as algebra and algorithm. It is also well known that several ancient Greek mathematical and philosophical works became known for the Renaissance Europe via Arabic translations, but we know very little about the original Islamic mathematics: “no textbook on the history of mathematics in English deals with the Islamic contribution in more than a general way” as the author writes. This is unfortunate because they made important contributions to the development of decimal arithmetic, plane and spherical trigonometry, algebra (e.g. solving cubic equations) and interpolation and approximation of roots of equations.

The aim of the present book is to make an attempt to fill this gap. In spite of that it is not and cannot be a “General History of Mathematics in Medieval Islam”, we are sure that this volume is a very important contribution to the subject.

In an introductory chapter the reader gets acquainted with the Islam’s reception of foreign science, the four most famous Muslim scientists: Al-Khwārizmī, Al-Bīrūnī, ’Umar al-Khayyāmī and Al-Khāshī, and the most important sources. The other chapters deal with arithmetic, geometrical

constructions, algebra, trigonometry and spherics. Each chapter is followed by a set of exercises and a bibliography.

We recommend this book primarily for students and teachers of mathematics, but everybody interested in the history of mathematics can read this well-illustrated book with joy.

*Lajos Klukovits (Szeged)*

**Arthur L. Besse, Einstein Manifolds** (Ergebnisse der Mathematik und ihrer Grenzgebiete, 3. Folge, Bd. 10), XII + 510 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

This book is intended to be a complete reference book of the differential theory of Einstein—Riemannian manifolds. In the author's opinion the Einstein metrics are the best candidates for nicest geometric structures on a given manifold which are very natural generalizations of Euclidean and classical non-Euclidean spaces. These manifolds have close relations with the geometries of constant curvature, but they have non-necessarily transitive isometry groups and thus their geometric properties can reflect characteristic non-homogeneous features. The indefinite semi-Riemannian analogies of these spaces are of basic importance in the modern physical space-time theory.

The book provides a self-contained treatment of many important topics of Riemannian geometry presented in a textbook for the first time, such as Riemannian submersions, Riemannian functionals and their critical points, the theory of Riemannian manifolds with distinguished holonomy group and Quaternion-Kähler manifolds. The central chapters of the book are devoted to the study of questions related to the Calabi conjecture made in 1954, whose solution given by S. T. Yau and T. Aubin in 1976 yields a large class of non-homogeneous compact Einstein manifolds. Corresponding to this conjecture the main problems treated in this book are related to the existence and uniqueness questions and principally to finding interesting examples of Einstein metrics. The book contains the formulation of the main open problems of this theory.

This excellent book is warmly recommended to everyone interested in Riemannian geometry and its applications in mathematical physics.

*Péter T. Nagy (Szeged)*

**J. Bliedtner—W. Hansen, Potential Theory, An Analytic and Probabilistic Approach to Balayage** (Universitext), XIII + 434 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

Recently much attention has been paid to stochastic processes in modern analysis. The classical example is potential theory. Suppose we want to solve Dirichlet's problem in a domain  $U$  with smooth boundary  $\partial U$ , and  $f: \partial U \rightarrow \mathbb{R}$  is the continuous function we want to extend to  $U$  to a harmonic function. If  $\{X_t^{(x)}\}$  is a two-dimensional Brownian motion starting at  $x \in U$  and  $\eta = \eta(\omega)$  is the first time when  $\{X_t^{(x)}(\omega)\}$  hits  $\partial U$ , then  $u(x) = E(f(X_\eta^{(x)}))$ , the expectation of the random variable  $f(X_{\eta(\omega)}^{(x)}(\omega))$ , solves the problem. In fact, it follows from the Markov property of  $\{X_t\}$  that  $u$  possesses the mean value property in  $U$  and it is clear that if  $x \in U$  is close to  $y \in \partial U$  then  $\{X_t^{(x)}\}$  will hit a fixed neighbourhood (on  $\partial U$ ) of  $y$  with high probability, so  $u$  has  $f$  as its boundary function.

The same idea works in many other classical problems. The book under review is devoted to the study of general balayage theory which is at the heart of these applications. The central objects

are the so called balayage spaces which are certain closed function cones with the property that if  $u, v', v''$  are in the space and  $u \cong v' + v''$ , then  $u$  has a representation  $u = u' + u''$ ,  $u' \cong v'$ ,  $u'' \cong v''$ . These spaces occur in different chapters in different equivalent forms as families of harmonic kernels, sub-Markov semigroups and as Hunt processes (regularized Markov processes).

The authors were very careful to clarify the abstract notions and results through concrete examples such as classical potential theory, Riesz potentials, discrete potential theory, translation on  $R$  and heat conduction in  $R^n$ . These relax the abstract setting; still one may encounter the usual drawbacks of too much generality when trying to use the book as a "Universitext". I feel that it is more appropriate to recommend this work to those who have past experience with both classical potential theory and stochastic processes. Then the new examples and different approaches of the book can be refreshing.

For further orientation here is a characteristic list of section headings: Classical Potential Theory, Function Cones, Choquet Boundary, Laplace Transforms, Supermedian Functions, Semigroups and Resolvents, Hyperharmonic Functions, Harmonic Kernels, Minimum Principle and Sheaf Properties, Markov Processes, Stopping Times, Balayage of Functions and Measures, Dirichlet Problem, Partial Differential Equations, Bauer Spaces, Semi-Elliptic PDE, Elliptic-Parabolic Differential Operators.

Vilmos Totik (Szeged)

**Umberto Bottazzini, The Higher Calculus: A History of Real and Complex Analysis from Euler to Weierstrass**, 332 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1986.

It was just 300 years ago that Newton published his monumental work *Principia mathematica philosophiae naturalis*, in which he founded differential and integral calculus and revolutionized the science of functions, mathematical analysis. Thereafter an enormous development had started in this area of mathematics, whose summit was reached in the 19th century, often mentioned in the history of mathematics as the century of analysis. This excellent book gives a detailed account of the history of this splendid period.

Can a book on the history of mathematics be interesting for wide circles of readers? Having read Bottazzini's book it is easy to answer: yes, it can. The author does not restrict himself to dull reviewing the results but acquaints the reader with the outstanding mathematicians of the area as people with emotions. For example, we can read the letter of a 24-year-old mathematician to his old teacher written after his arrival at Paris in 1826: "Up to now I have only made the acquaintance of Legendre, Cauchy, and Hachatte, plus a few secondary but very able mathematicians, ... Legendre is an extremely amiable man, but unfortunately "as old as stones". Cauchy is crazy and there is nothing to be done with him, even though at the moment he is the mathematician who knows how mathematics must be done. His works are excellent, but he writes in a very confusing way. At first I understood virtually nothing of what he wrote, but now it goes better... Poisson, Fourier, Ampère, etc. etc. occupy themselves with nothing other than magnetism and other physical matters... Everyone works by himself without interesting himself in others. Everyone wants to teach and no one wants to learn. The most absolute egoism reigns everywhere." These are rather hard words, but the young man was named Abel. Nevertheless, the book is written mainly about mathematics itself and gives the milestones of the history of such big problems as the solution of the equations of vibrating string and of heat diffusion, expansion of functions into trigonometric series, the foundation of the theory of complex functions, etc. Special attention is paid to the development of the concept of a function. Who could think without reading the history of mathematics that the contemporary definition of a function, which is taught in every elementary school today and seems

born with us like our instincts, is a result of a long process of thinking and debates, and that Euler still defined a function as follows: "A function of a variable quantity is an analytic expression composed in any way from this variable quantity and from numbers or constant quantities."

This well-written book will be a very valuable and enjoyable reading not only for students and experts of the history of mathematics, but for every student learning calculus and for every researcher in mathematics, since Poincaré is absolutely right when saying: "The true method of foreseeing the future of mathematics is to study its history and its actual state."

*L. Hatvani (Szeged)*

**César Camacho—Alcides Lins Neto, Geometric Theory of Foliations, 205 pages, Birkhäuser, Boston—Basel—Stuttgart, 1985.**

The theory of foliations is a part of differential topology investigating the decompositions of manifolds into a union of connected, disjoint submanifolds of the same dimension. The origin of this subject is the geometric theory of differential equations that has begun with the works of Painlevé, Poincaré and Bendixson in the last century. The authors say in the introduction: »The development of the theory of foliations was however provoked by the following question about the topology of manifolds proposed by H. Hopf in the 1930's: "Does there exist on the Euclidean sphere  $S^3$  a completely integrable vector field, that is, a field  $X$  such that  $X \cdot \text{curl } X = 0$ ?" By Frobenius' theorem this question is equivalent to the following: "Does there exist on the sphere  $S^3$  a two-dimensional foliation?"«

The present book which is a translation of the original Portuguese edition published in Brasil in 1980 has the purpose to give an introduction to the subject. The first four chapters treat the basic notions and properties of foliations (Differentiable Manifolds, Foliations, The Topology of Leaves, Holonomy and the Stability Theorems). Chapter V discusses the relations between foliations and fiber bundles. Chapter VI is devoted to the proof of Haefliger's theorem about analytical foliations of codimension one. Chapter VII contains the proof of Novikov's theorem on the existence of a compact leaf of a  $C^2$  codimension one foliation on a compact three-dimensional manifold with finite fundamental group. Chapter VIII deals with foliations induced by group actions. There is an appendix containing the proof of Frobenius' theorem.

The book is highly recommended to anyone interested in differential topology and familiar with the material of standard courses on analysis, topology and geometry.

*Péter T. Nagy (Szeged)*

**Leonard S. Charlap, Bieberbach Groups and Flat Manifolds (Universitext), X+242 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1986.**

The theory of the Euclidean space form problem, treated in this book, is originated from Hilbert's famous 18th problem about the classification of discrete Euclidean rigid motion groups with fundamental domains, or what is the same, of crystallographic groups. The early solution of this problem, given by L. Bieberbach in about 1910, can be summarized in modern language in the following way. The fundamental group of a compact flat Riemannian manifold is a Bieberbach group, i.e. a torsionfree group having a maximal abelian subgroup of finite index which is free abelian. The manifolds with isomorphic fundamental groups are affinely equivalent, the number of their equivalence classes is finite.

The author of this book has developed the classification theory of Euclidean space forms based on the description of the linear holonomy group of flat Riemannian connections in the early 1960's. The purpose of the present treatment is to give a selfcontained introduction and at the same time a reference book on this topic. Chapter I contains the presentation of Bieberbach's classical theory. Chapter II gives an elementary introduction to Riemannian geometry including the notion and fundamental properties of linear holonomy groups. There is given a formulation of Bieberbach's results in the language of differential geometry. Chapter III deals with the algebraic classification of Bieberbach groups. It is finished with the proof of the Ansländer—Kuranishi theorem saying that any finite group is the holonomy group of a compact flat manifold. Chapter IV is devoted to the author's principal results about the space forms whose holonomy group has prime order. Chapter V discusses the properties of automorphism groups of flat manifolds.

The general results are illustrated with many examples. Open problems, conjectures, counterexamples and results related to the theory of nonflat manifolds are formulated throughout. This very nice book is really interdisciplinary, it uses tools of differential topology and geometry, algebraic number theory, cohomology of groups and integral representations.

*Péter T. Nagy (Szeged)*

**Coherence, Cooperation and Fluctuations**, Edited by F. Haake, L. M. Narducci and D. F. Walls (Cambridge Studies in Modern Optics, 5), VIII+456 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1986.

In 1963 Roy Glauber laid down the fundamentals of quantum optics by introducing the quantum concept of coherence and the coherent states of the radiation field. This fact, which is already a part of the history of physics, justifies the decision to devote one of the volumes of the Cambridge Studies in Modern Optics to the works honouring the 60-th birthday of R. Glauber.

In spite of the series title, besides optics, there are papers on statistical physics and nuclear physics too, as Glauber himself contributed also to the development of the latter fields with essential and fundamental results. Among the authors of the invited papers we find L. Kadanoff, J. Langer, H. Feshbach, F. T. Arecchi, N. Bloembergen, S. Haroche, L. Mandel, R. Pike, M. O. Scully. The majority of the 33 papers deal with quantum optics, and reading them one may really learn what is in focus at present time in the field of optical coherence, cooperation and fluctuations. The two other topics are treated less comprehensively in this volume. To have at hand the roots of the ideas presented in the book, the editors included the reprints of the 4 classic papers of R. Glauber: the two about quantum coherence, the time dependent statistics of the Ising model and the one about the optical model of nuclear reactions.

The book is recommended mainly for research workers in the areas of nonrelativistic field theory and quantum optics.

*M. G. Benedict (Szeged)*

**M. Crampin—F. A. E. Pirani, Applicable Differential Geometry** (London Mathematical Society Lecture Notes Series, 59) 385 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1986.

Traditional courses in differential geometry contain first the elementary theory of curves and surfaces in a Euclidean space and thereafter the notion of a differentiable manifold and the theory of differential geometric structures on it. Such an approach has the disadvantage that the

notion of fibre bundles and the general theory of connections and Lie group actions can be treated only in lecture courses for final year graduate or postgraduate student audiences. But these techniques are needed in the modern applications of differential geometry in the foundation of mechanics, gauge field theories and gravitation theory.

The present book gives an introduction to these methods of differential geometry on the level of beginning graduate students. "The essential ideas are first introduced in the context of affine space; this is enough for special relativity and vectorial mechanics. Then manifolds are introduced and the essential ideas are suitably adapted; this makes it possible to go on to general relativity and canonical mechanics. The book ends with some chapters on bundles and connections which may be useful in the study of gauge fields and such matters." The treatment is illustrated with many examples motivated by the applications in mathematical physics. Each chapter is concluded with a brief summary of its contents.

The reviewer thinks that this excellent introduction will be especially useful if it is supplemented with parallel courses on analytical mechanics and relativity theory.

*Péter T. Nagy (Szeged)*

**Luc Devroye, Non-Uniform Random Variate Generation, XVI+843 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.**

The importance of this comprehensive work can hardly be overemphasized. A large amount of today's research in statistics, operations research and computer science depends upon large scale Monte Carlo computer simulation. Also, this is almost the sole means of investigation in certain applied fields in engineering, experimental and even theoretical physics and chemistry, the life sciences and technology, but such "esoteric pure mathematics" as number theory is not devoid of Monte Carlo experimentation either. Yet all these depend upon sequences of numbers or vectors generated on the computer which are to be viewed as independent realizations of a random variable or vector with a prescribed distribution. Then you apply one of the greatest things of Nature (or, put it with less euphemism, a trivial fact of probability theory), the law of large numbers, and, modulo the problem at hand, you are done.

Now, borrowing some expressions from the characteristically lively language of Devroye, the "story has two halves". Any machine that can be called a computer nowadays sports with a random number generator that is claimed to be capable to produce sequences of independent random variables uniformly distributed on  $(0, 1)$ . This will of course never be the case, and the theoretical and practical aspects of this problem belong to the circle of the deepest common puzzles of probability theory and algorithm theory. However, that machines do indeed have such generators is becoming more and more reasonable an assumption together with another, theoretically impossible assumption that the computer can store and manipulate real numbers.

Based on these two assumptions, the book is about "the second half of the story": how to generate random numbers with a prescribed non-uniform distribution most efficiently? The efficiency of a procedure is measured by the complexity of the algorithm which produces one such number. This notion is achieved by the author's third assumption that the fundamental operations in the computer (addition, multiplication, division, compare, truncate, move, generate a uniform random variate, exp, log, square root, arctan, sin and cos) all take one unit of time, and the complexity is simply the required time. The algorithms themselves are then investigated by providing lower and upper bounds for their expected complexity or for the tails of their distribution.

Following an introduction into a few basic probabilistic facts, Chapters 2 and 3 present the general principles of random number generation such as the inversion, the rejection, the composi-



tion, the acceptance-complement, alias and table look-up methods and their various combinations, while Chapters 4, 5, 8 and 14 describe a bewilderingly vast amount of specialized algorithms. The procedures are then applied in Chapters 7, 9 and 10 for the generation of random numbers from the most important continuous and discrete parametric families of distributions or from large families of distributions given by a qualitative property of the density such as log-concavity, monotonicity, or unimodality. The whole Chapter 11 is on random vector generation and Chapter 6 is devoted to the generation of the Poisson and related random processes. So these two chapters create special dependence structures already, and Chapters 12 and 13 go on further in this direction, to the generation of various sampling without replacement plans and to the generation of random permutations, binary and free trees, partitions and graphs. Finally, Chapter 15 presents the Knuth-Yao theory of discrete distribution generating trees in a random bit model in which, instead of Uniform  $(0, 1)$  random numbers; Binomial  $(1, 1/2)$  random numbers are available.

The hundreds of generation algorithms in the book are all written as PASCAL programs and are intelligible without knowing anything special about this language. In fact, the book is completely independent of today's computer and programming technology and I am sure that the author's hopes that "the text will be as interesting in 1995 as in 1985" are entirely well-developed.

The author outlines a course in computer science and another one in statistics that can be based on the book, moreover he proposes a "fun reading course on the development and use of inequalities". My own random fun reading course turned out to be most gratifying and enjoyable. Wherever opens, it is difficult to put down the book which is bound to become to be the basic reference in non-uniform random number generation. It contains a good number of new results and an enormous amount of knowledge from probability and statistics, computer science, operations research and complexity and algorithm theory, blended and arranged by masterly scholarship. Congratulations Luc!

*Sándor Csörgő (Szeged)*

**Differential Equations in Banach Spaces**, Proceedings of a Conference held in Bologna, July 2—5, 1985. Edited by A. Favini and E. Obrecht (Lecture Notes in Mathematics, 1223), VIII + 299 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1986.

When modelling the evolution in time of a physical system we have to decide how to describe the position of the system at an instant of time. For example, for a finite system of particles in classical mechanics a position is a point of  $R^n$ , while in the model of the vibrating string or the heat conduction problem we use to this end functions in  $C^2([0, l]; R)$ . Respectively, the model equation will be a system of ordinary differential equations and a partial differential equation. However, partial differential equations can be also considered as ordinary ones of the form  $\dot{u} = Au$  which are written in the Banach space  $C^2([0, l]; R)$  as a state space, and  $A$  is a differential operator in this space. This unification was inspired by the fact that the basic concepts and methods of the theory of ordinary differential equations (eigenvalue, Jordan form, exponents of a matrix, spectral theory, calculus of functions) have been developed for operators in Banach spaces by linear (and recently nonlinear) functional analysis. As it is also shown by these proceedings, the approach to differential equations as abstract equations in Banach spaces is a fruitful and very rapidly developing field. Among the topics discussed at the Conference are: regular and singular evolution equations, both linear and nonlinear, of parabolic and hyperbolic type, integro-differential equations, semi-group theory, control theory, wave equations, transmutation methods and fuchsian differential equations.

*L. Hatvani (Szeged)*

**B. A. Dubrovin—A. T. Fomenko—S. P. Novikov, *Modern Geometry — Methods and Applications: Part II. The Geometry and Topology of Manifolds*, 430 + XV pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.**

This is the English translation, by Robert G. Burns, of the original Russian edition, Nauka, Moscow, 1979. The present book is the second volume of a whole series in which the authors' main aim is the modernization of the teaching of differential geometry at universities. From this point of view this work can be considered as one of the best texts which acquaints the readers with a large part of modern differential geometry in a very clear and didactical style.

This second volume is devoted mainly to differential topology. After the elementary study of real and complex manifolds, Lie groups, homogeneous spaces, the exposition turns to the Sard theorem and related fields such as Morse theory, embeddings and immersions, the degree of mappings and the intersection index of submanifolds. Furthermore two chapters deal with the fundamental groups and homotopy groups of manifolds. After these the theory of fibre bundles, connections, foliations and dynamical systems is developed. The last chapter deals with general relativity and also with Yang—Mills theory whose comprehensible survey has been absent from the literature.

This well-written excellent monograph can be highly recommended to students, mathematicians and users interested in modern differential geometry.

*Z. I. Szabó (Budapest)*

**Sir Arthur Eddington, *Space, Time and Gravitation*. An outline of the general relativity theory (Cambridge Science Classics Series), XII + 218 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1987.**

This classic book on the general theory of relativity was published first in the exciting days of 1920, soon after the first objective tests of the new theory assumed historico-scientific values. The reader can understand how Sir Arthur Eddington, the creative participant of the development of this theory in mathematics, physics and philosophy, saw the problems of space, time and gravitation. This new reprint, which is the twelfth in a sequence, includes a foreword by Sir Hermann Bondi, describing the place of this book in its historical and scientific context. He says: "How does his writing strike us now, some sixty years after it first appeared in print? The beautiful English is as good as ever, the subject matter, the theories of relativity and gravitation, have not suffered relegation to the backburner, but are as integral a part of physics as in his day. Thus his book is still very good and very relevant." Everyone interested in the development of new ideas and viewpoints in the sciences will enjoy this book.

*Péter T. Nagy (Szeged)*

**K. J. Falconer, *The Geometry of Fractal Sets* (Cambridge Tracts in Mathematics, 85), XIV + 162 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1986.**

From the introduction of the author: "Recently there has been a meteoric increase in the importance of fractal sets in the sciences. Mandelbrot pioneered their use to model a wide variety of scientific phenomena from the molecular to the astronomical ... Sets of fractional dimension also occur in diverse branches of pure mathematics."

This widespread applicability aroused both scientists' and mathematicians' interest in fractals. The aim of this book is to give a rigorous mathematical treatment of the geometrical prop-

erties of sets of both integral and fractional Hausdorff dimension, and the author unites into a theory the complete collection of these results, which have previously been available only in technical papers.

The first chapter contains a very good general measure theoretic introduction, the definition of Hausdorff measure and dimension and basic covering results. There is an emphasis on the Vitali covering theorem which will be often used. In some "simple" cases the Hausdorff dimension and measure are calculated. The next three chapters discuss the density properties and existence of tangents. The notion of local densities are similar to the Lebesgue case but there is no analogue of the Lebesgue density theorem. It is proved that only the integral dimensional sets can be regular and in the integral case the regular "curve-like" sets and irregular "dust-like" sets are characterised. In the fifth chapter a very useful tool, comparable net measures, is presented and applied to construct a subset with finite  $s$ -measure of a set with infinite  $s$ -measure, and to calculate the Hausdorff measure of Cartesian products of sets. In the sixth chapter two fruitful theories from analysis, potential theory and Fourier transforms are applied to investigate the projection properties of  $s$ -sets. The next chapter discusses the interesting problem of Kakeya of finding a set with zero measure containing a line segment in every direction. It is demonstrated that the previously described theory is related by duality to Kakeya sets. The final chapter contains miscellaneous examples of fractal sets. Methods for constructing curves of fractional dimension and generating self-similar sets are presented and applications to number theory, convexity, dynamical systems and Brownian motion are shown.

Each chapter contains a problem set which complete the topics and may help the reader in understanding the basic methods. The book is recommended to pure mathematicians, but it may be useful to anybody interested in the application of fractals.

*J. Kincses (Szeged)*

**D. J. H. Garling, A Course in Galois Theory, VIII + 167 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1986.**

This book deals with Galois theory at a level or somewhat higher than it is customarily presented for undergraduates of mathematics. In fact, the book grew out of a course of lectures the author gave for several years at Cambridge University. Pages 1 through 36 give a concise account on the necessary prerequisites like groups, vector spaces, rings, unique factorization domains and irreducible polynomials. The rest of the text is devoted to the theory of fields and Galois theory. Besides the classical topics including the problems of solvability the general quintic and geometric constructibility, some extra material, rarely discussed in teaching activity, is also added. For example, Lüroth's theorem, the normal basis theorem and a procedure for determining the Galois group of a polynomial is included. The reader is challenged by more than 200 exercises.

This book is warmly recommended mainly for instructors and students and also for everyone interested in its topic.

*Gábor Czédli (Szeged)*

**Geometrical and Statistical Aspects of Probability in Banach Spaces. Proceedings, Strasbourg 1985. Edited by X. Fernique, B. Heinkel, M. B. Marcus and P. A. Meyer (Lecture Notes in Mathematics, 1193), II + 128 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.**

The volume starts with a short description of the significant work of the young Strasbourg probabilist Antoine Ehrhard, who died less than two weeks before the meeting, by C. Borell.

A short note by S. Guerre deals with almost exchangeable sequences, another one by M. B. Schwarz with mean square convergence of weak martingales. B. Heinkel is on the strong law of large numbers in smooth Banach spaces, while M. Ledoux and M. B. Marcus are on the almost sure uniform convergence of Gaussian and Rademacher infinite Fourier quadratic forms. The papers by M. Ledoux and J. E. Yukich present results for the central limit theorem in a Banach space in two different directions. The paper of P. Doukhan and J. R. Leon deals with the central limit theorem for empirical processes indexed by functions based on stationary, strongly mixing random elements and for the local time of Markov processes with an application to testing uniformity on a compact Riemannian manifold, while the comprehensive 37-page article by P. Massart is on the rate of convergence in the central limit theorem for general empirical processes indexed by functions satisfying certain entropy conditions.

*Sándor Csörgő (Szeged)*

**Mikhael Gromov, Partial Differential Relations** (Ergebnisse der Mathematik und ihrer Grenzgebiete, 3. Folge, Band 9), IX+363 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1986.

The purpose of this book is to give a systematic and selfcontained treatment of analytical, topological and differential geometric methods of the theory of undetermined partial differential equations or differential relations and of its applications to imbedding and immersion problems of Riemannian and symplectic manifolds. This theory has been developed in the last 20 years mainly by the author's initiatives and activity.

Part 1 contains a survey of the basic problems and results giving the most important motivations for the theory. Part 2 is devoted to the study of a construction method which is a homotopic deformation of a jet-section solution into a differentiable map satisfying the differential relation. Part 3 deals with the investigation of  $C^\infty$  isometric immersions of Riemannian, Pseudo-Riemannian and symplectic manifolds.

The author writes in the Forward: "Our exposition is elementary and the proofs of the basic results are selfcontained. However, there is a number of examples and exercises (of variable difficulty), where the treatment of a particular equation requires a certain knowledge of pertinent facts in the surrounding field." But in the reviewer's opinion the reader is presupposed to be familiar with the techniques of singularity theory, differential operators, differential geometry and topology on higher than an elementary level.

The book includes many new results and yields a good overview of this developing field of mathematics.

*Péter T. Nagy (Szeged)*

**John Guckenheimer—Philip Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields** (Second Printing, Revised and Corrected; Applied Mathematical Sciences, 42), XVI+459 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

At the early stage of the history of mechanics the oscillations were studied as "small oscillations". It means that one considers the linearized equations of motion around the equilibrium or the periodic orbit investigated. But these linear equations can describe the behaviour of the motions only locally. For example, within this theory it is impossible to handle the interaction between two or more isolated equilibria or cycles, which are very common in differential equation models. The global description of trajectories demands the study of the original nonlinear models.

However, the theory of nonlinear differential equations, as contrasted with that of linear equations, is far from being complete. Here the qualitative approach is the most important and fruitful, which has been developed by marrying analysis and geometry.

Over the past few years there has been increasing a widespread interest in the engineering and applied science communities in such phenomena as bifurcations, strange attractors and chaos. The rigorous study of these phenomena needs a wide and deep mathematical background and this is provided by the modern theory of dynamical systems. This book gives an excellent introduction to this fairly sophisticated theory for those who do not have the necessary prerequisites to go directly at the research literature.

Chapter 1 provides a review of basic results in the theory of dynamical systems and differential equations. Chapter 2 presents four examples from nonlinear oscillations: the famous oscillators of van der Pol and Duffing, the Lorenz equations and a bouncing ball problem. By the aid of these examples the reader can get acquainted with the chaotic behaviour of solutions and the concept of the strange attractor: an attracting motion which is neither periodic nor even quasiperiodic. Chapter 3 contains a discussion of the methods of local bifurcation theory, including center manifolds and normal forms. Chapter 4 is devoted to the method of averaging, perturbation theory and the Kolmogorov—Arnold—Moser theory. In Chapter 5 the famous horseshoe map of Smale is discussed in a nice and intelligible way. Chapter 6 is concerned with global homoclinic and heteroclinic bifurcations. In the final chapter the degenerated local bifurcations are treated.

It is easy to understand that the second edition of this valuable text-book had become necessary. It has to be on the bookshelf of every mathematician and of every user of mathematics interested in the modern theory of differential equations, dynamical systems and their applications.

*L. Hatvani (Szeged)*

**James M. Henle, *An Outline of Set Theory* (Problem Books in Mathematics), VIII + 145 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1986.**

Set theory is full with charming exercises, brilliant constructions and problems which sometimes challenge even the experts. Henle's book misses them all and this is all the more unfortunate that there is no good published collection of (solved) problems in set theory (one should not count those feeble attempts that every now and then appear on the scene with completely trivial exercises).

More proper justice should however be given to this worthy book because its aim is different. I feel neither the "Problem Books" series nor the title are appropriate for this work, for this is not a problem book in the ordinary sense, nor it is about Set Theory. More appropriate title would be something like "Construction and properties of numbers", all sorts of numbers such as naturals, integers, rationals, reals, ordinals, cardinals, infinitesimals. The *spirit* is set theoretical and ultimately this is why I would rank very high Henle's book.

It introduces an outstanding pedagogical system: so called projects are assigned to students. These contain proofs, discoveries of theorems and concepts etc.; under the guidance of the teacher the students work alone, and through these projects they explore the field step by step like "real" researchers. They "experience the same dilemmas and uncertainties that faced the pioneers". Accordingly, the book consists of three parts, hints and solutions occupy the second and third ones.

In my opinion every good exercise book (cf. Pólya—Szegő's, Halmos's, Lovász's) should be based on similar principles; here however the method is applied to the very foundation and exposition of the subject. I saw the same method efficiently working at Ohio State University where selected high school students participated in university summer schools. Although I doubt that the ordinary

math major would successfully complete the projects in the book, the system is certainly applicable to the better ones.

It is unfortunate that the author mostly restricted himself to the dullest part of set theory: construction of numbers and operations between them (I must add, however, that due to J. von Neumann and A. Robinson, the construction of ordinals and infinitesimals is definitely an exception). This may be so because set theory is not too well adequate for the above method (after all infinite sets are not objects that you can experience with); number theory, geometry, elementary algebra etc. certainly are more suitable.

The main advantage of the book is that without disturbing formalism the author gets the students think and work in a way as a logician should do, and the presentation is extremely "clean" and accurate. For this reason I warmly recommend that every math major read the book even if they have already completed a course in set theory. The material can also be valuable for lecturers on set theory and logic.

There is one more reason for this strong recommendation, and this is the Goodstein—Kirby—Paris theorem discussed in the last chapter (it is about an extraordinary number theoretical iterative process that seemingly produces larger and larger numbers but somehow it always reaches 0; and this can be proved *only* using infinite numbers). In the style of the quotations in the book: All's Well that Ends Well.

*Vilmos Totik (Szeged)*

**Homogenization and Effective Moduli of Materials and Media.** Edited by J. L. Ericksen, D. Kinderlehrer, R. Kohn and J.-L. Lions (The IMA Volumes in Mathematics and its Applications, Volume 1), X + 263 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

At the Institute for Mathematics and its Applications the year 1984—1985 was dedicated to the study of partial differential equations and continuum physics. This volume, the first one in a series, contains research papers presented at a workshop on homogenization of differential equations and the determination of effective moduli of materials and media. This up-to-date theme is interesting for mathematicians, physicists and for engineers equally well. The papers are well-organized. In general they contain the origin and the history of the investigated problem and after the discussion open questions are presented. The style is well-characterized by the following sentence taken from Luc Tartar's paper: "This mathematical model of some physical questions involving different scales will of course be questioned by some; it is natural that it be so but I hope that criticism will be made in a constructive way and so improve my understanding of continuum mechanics and physics (and maybe of mathematics)."

The titles of the papers are: Generalized Plate Models and Optimal Design. — The Effective Dielectric Coefficient of a Composite Medium: Rigorous Bounds From Analytic Properties. — Variational Bounds on Darcy's Constant. — Micromodeling of Void Growth and Collapse. — On Bounding the Effective Conductivity of Anisotropic Composites. Thin Plates with Rapidly Varying Thickness and their Relation to Structural Optimization. — Modelling the Properties of Composites by Laminates. — Wares in Bubbly Liquids. — Some Examples of Crinkles. — Mikrostructures and Physical Properties of Composites. — Remarks on Homogenization. — Variational Estimates for the Overall Response of an Inhomogeneous Nonlinear Dielectric.

*L. Pintér (Szeged)*

**Mark Kac—Gian-Carlo Rota—Jacob T. Schwartz, *Discrete Thoughts: Essays on Mathematics, Science, and Philosophy*.** Edited by Harry Newman (Scientists of Our Time), XII + 264 pages, Birkhäuser, Boston—Basel—Stuttgart, 1986.

the reason people so often lie  
is that they lack imagination:  
they don't realize that the truth, too,  
is a matter of invention.

This is how Rota starts the volume in his preface, translating nicely a three-line poem of Machado which summarizes the "prophetic warning" of the philosopher Ortega.

This fine composition of twenty-six essays should be read by every mathematician, statistician and computer scientist. It would be a little bit better still if every physicist, economist and historian of science could also read it, and the world, scientific or otherwise, would surely improve a trifle if in fact the whole intelligentsia read it. It is not just that three "gifted expositors of mathematics" came together as the jacket says, but these three illustrious thinkers really want to tell the truth, if not the whole truth, but nothing but the truth. And the world usually betters by telling the truth.

There are as many readings of such a text as readers. In the reviewer's reading the frame of this composition is constituted by the seven brilliant writings of the late Professor Kac (he died in the fall of 1984). These are: Mathematics: Tensions (essay No. 2), Statistics (4), Statistics and its history (5), Mathematics: Trends (8), Academic responsibility (13), Will computers replace humans? (18), Doing Away with Science (25). Heavier building blocks are brought by the six essays of Schwartz: The pernicious influence of mathematics on science (3), Computer science (7), The future of computer science (9), Economics, mathematical and empirical (10), Artificial intelligence (16), Computer-aided instruction (19) and by Rota's essay Combinatorics (6). The cohesion of and the paint on the structure is provided by Rota's shorter bookreviews and sketches: Complicating mathematics (11), Mathematics and its history (12), Husserl and the reform of logic (14), Husserl (15), Computing and its history (17), Misreading the history of mathematics (20), The wonderful world of Uncle Stan (21), Ulam (22), Kant (23) and Heidegger (24) and his Chapter 1 (Discrete thoughts) and Chapter 26 (More discrete thoughts).

The depth of thought, charm, experience and characteristic wit of Kac, the vehement cool logic of Schwartz and the broad knowledge and aphoristic penetration of Rota harmonize beautifully. The selection and ordering of the essays (presumably the work of the editor) to achieve the non-formalizable rhythm of thought in the book, a composition instead of a collection, will not be possible for any artificial intelligence. The whole thing is more continuous than discrete.

No, Uncle Mark, computers will never replace a man like you were.

*Sándor Csörgő (Szeged)*

**Serge Lang, *Linear Algebra*.** Third edition (Undergraduate Texts in Mathematics) IX + 285 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1987.

The text is divided into twelve chapters. The first four chapters introduce vector spaces over subfields of complex numbers, the space of matrices, matrices of linear equations, the notion of linear operators and show the connection between matrices and linear operators. In Chapter 5 scalar products and orthogonality are defined; and applications to linear equations, bilinear and quadratic forms are given. Chapter 6 is devoted to give the notion and elementary properties of determinants. Chapter 7 studies the important special cases of linear operators, symmetric, Her-

mitian and unitary operators. Chapter 8 defines eigenvalues and eigenvectors, the characteristic polynomial, and gives the method of computing eigenvalues by finding the maximum and the minimum of quadratic forms on the unit sphere. In Chapter 9 polynomials of matrices and linear operators are defined. The main purpose of Chapter 10 is to prove the existence of triangulation of linear operators and especially of the diagonalization of unitary operators. Chapter 11 deals with the factorization of polynomials and as an application of this concludes the Jordan normal form of linear operators. Chapter 12 is devoted to the study of convex sets and proves the finite dimensional case of the Krein—Milman theorem. At the end of the book an Appendix can be found dealing with complex numbers.

The book can be used as a handbook for learning linear algebra.

*L. Gehér (Szeged)*

**László Lovász—Michael D. Plummer, Matching Theory (North-Holland Mathematics Studies), XXXIII + 544 pages, Akadémiai Kiadó, Budapest and North-Holland, Amsterdam, 1986.**

Matching theory consists of only a part of graph theory, but one of its deepest and hardest parts. The history of matching is related to the four color problem, and since then it has been a focus of interest. Most of the general questions and methods of combinatorics are considered in matching theory, and many have a nice solution or application. Because of its complexity, matching theory really has the right to bear the title of theory.

The book starts with the most classical results. The first two chapters contain the very basic and very important results on bipartite graph matching and flow theory. The next three chapters deal with the structure of general graphs related to matching. The main result in this territory is the Edmonds—Gallai structure theorem which shows that from the point of view of matchings, every graph is built of several different kind of “bricks”. These bricks are well known, thanks to the two authors’ previous works. The next four chapters illuminate matching problems from different perspectives. The first one discusses the graph-theoretical consequences. The next chapter shows the very important connection with linear programming. The book discusses the description of a matching polytope, its facets and the dimension of the perfect matching polytope. This section includes the effect of the ellipsoid method on combinatorial optimization. There is one chapter on the related enumeration problems. Besides the basic results on permanents, pfaffians, and matching polynomials, there are some interesting applications of these results. One other chapter covers the algorithm-theoretic aspects of matchings, containing not only the important algorithms, but also their implementation. The final three chapters consider the generalizations of the question of matchings in graph theory and matroid theory. These discuss the problem of  $f$ -factors, vertex packing, hypergraph matching and matching of 2-polymatroids.

The book contains all of the important results which are in or related to matching theory. The many applications of this subject in other parts of combinatorics, and the wide variety of methods used ensure that the reader will not only learn about matching theory, but about most of the important parts of combinatorics. There are discussions of matroid theory, polyhedral combinatorics, enumerations, algorithm theory, and data structures. The corresponding chapters are not only good introductions to the fields but contain some of the most important results of current research. Sometimes the flow of results is interrupted by “boxes”. These boxes contain remarks which go beyond the scope of the book or sketch a main underlying idea. These parts are very useful to understand how to fit the actual results or methods into the main stream of research. The cited references are a big help to the reader whose appetite has been whetted. These short guides are also very helpful for the reader who is untrained in combinatorics. If the reader takes the effort and studies mathematics by solving problems (this is harder but more rewarding), then there are a lot of exercises



inserted in the text. Solving these exercises adds a lot of fun to the reading and gives good practice for the methods. If the reader requires more of a challenge, there are many open problems in the book. Both authors live and breathe matching theory. As a consequence, the chapters frequently suggest the most important directions of research, and the reader easily can find problems to think about.

Matching theory is only a small part of combinatorics. This might lead an outsider to think that this book is too specialized. (A very narrow, specialized subject gets no interest outside a small group.) This is not the case with this book. The relations between combinatorics and the classical fields, and the applications of combinatorics are an undiscovered part of the science. This book gives many examples of applications of combinatorics to other areas of mathematics and the physical sciences. Here are just some of them for appetizers; engineering, chemistry, physics, measure theory and topology.

Finally, the physical appearance of the book is pleasing, as it was typeset with the T<sub>E</sub>X system. The book is very important to any specialist in combinatorics. It is highly recommended to anybody who is interested in this new part of mathematics or who is working in a field which applies combinatorial methods.

*Péter Hajnal* (Szeged and Chicago)

**New Developments in the Theory and Applications of Solitons**, Proceedings of a Royal Society Meeting, London, 1984 November. Edited by Sir Michael Atiyah, J. D. Gibbon and G. Wilson, (Reprint from the Philosophical Transactions of the Royal Society, Ser. A. Vol. 315, p. 333—469), The Royal Society, London, 1985.

The exponential growth of the number of papers about solitons has become somewhat less steep in the last few years, nevertheless they are still the subject of intensive study. Over and over again new delicate details of soliton theory are discovered by pure mathematicians, and the sudden appearance of solitons is not a rare event in any field of physics.

This situation is well documented in the introductory lecture of these proceedings: "A survey of the origins and physical importance of soliton equations" given by J. D. Gibbon. Here past and present of solitons are outlined, and this is the lecture that can be recommended both to the beginner and to the specialist, in order to see how wide this field really is. More detailed investigations of some of the branches of mathematics and physics, where solitons play important role can be found in the other 8 papers of this volume. Half of them have purely mathematical character, and show the connection of soliton theory with such classical problems as the integrability of ordinary differential equations, as well as with modern fields like algebraic geometry and Kac—Moody algebras. The rest of the articles communicate on experiments and applications of soliton theory in laser physics, biomolecules, magnetic monopoles, fluid dynamics etc.

*M. G. Benedict* (Szeged)

**New Directions in the Philosophy of Mathematics: An Anthology**, Edited by Thomas Tymoczko, XVII + 323 pages, Birkhäuser, Boston—Basel—Stuttgart, 1985.

The philosophy of mathematics has played an important role in philosophy going back to the ancient Greeks. This discipline has been radically changed about the turn of the century. The new dominant question (or the new paradigm, according to T. Kuhn's terminology) was: what is the foundation of mathematics?

Now, in the latest decades, as R. Hersh wrote: "We are still in the aftermath of the great foundationalist controversies of the early twentieth century. Formalism, intuitionism and logicism, each left its trace in the form of certain mathematical research program that ultimately made its own contribution to the corpus of mathematics itself."

In Part I, entitled *Challenging Foundations*, we can read five essays on the major perspectives on the philosophy of mathematics written by R. Hersh, I. Lakatos, H. Putnam, R. Thom and N. D. Goodman. They strongly criticize the foundationalist approach to the philosophy of mathematics. This part is followed by an interlude containing two writings of G. Pólya, who was the forerunner of quasi-empiricism in mathematics. The essays in the second part demonstrate quasi-empiricism which is an increasingly popular approach to the recent philosophy of mathematics.

Part II deals with the reexamination of mathematical practice. It contains three sets of essays. The first set explores some general issues in mathematical practice, starting with the concept of informal proof. The authors are: Hao Wang, I. Lakatos, Ph. J. Davis and R. Hersh. The second set of essays focuses on the growth of mathematical knowledge, the development or change in the essential aspect of informal proof. The authors are: R. L. Wilder, Judith V. Grabiner and Ph. Kitcher. The final set continues the theme of informal proof and discusses the change due to the use of computers in mathematical research. The authors are: T. Tymoczko, R. A. de Millo, R. J. Lipton, A. J. Perlis and G. Chaitin.

All essays of the second part argue the philosophical relevance of mathematical practice. According to the editor's view: "The crucial step in approaching them is our willingness to conceive of mathematics as a rational human activity, that is, as a practice."

Each part and almost all essays have an introduction written by the editor which helps the reader in better understanding and offers a short summary.

As a recommendation we cite the closing paragraph of the editor's Introduction: "Although this anthology does not completely represent the philosophy of mathematics, it does, I believe, gather together some of the more exciting essays published recently in the field. In this instance, the whole really is greater than the sum of all its parts; each essay reinforces the others. One purpose in bringing these essays together is to demonstrate their collective force. The collection will have succeeded if it stimulates the reader — mathematician or philosopher, professional, apprentice or amateur — to rethink his or her conception of mathematics."

*Lajos Klukovits (Szeged)*

**N. K. Nikol'skii, *Treatise on the Shift Operator: Spectral Function Theory*. With an Appendix by S. V. Hruščev and V. V. Peller. Translated from the Russian by Jeak Petree (Grundlehren der mathematischen Wissenschaften 273), XI+491 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.**

The title of the Introductory Lecture (chapters are called lectures) is: "What this book is about." A short, and thus by no means exhaustive, answer can be: about non-classical spectral theory in Hilbert space. The discussion is essentially based on the functional model for contractions due to Sz.-Nagy and Foiaş. This approach makes possible to use more function-theoretic tools, namely many properties of functions in Hardy classes, as in classical spectral theory. The central role of the shift operator in this model makes at once understandable why this work can be considered (again, not in its totality) as a "treatise on the shift operator".

Besides the introductory one the book contains eleven lectures. In the first parts of these lectures the shift operator in question appears as multiplication by the independent variable in the Hardy space  $H^2$  of scalar valued functions on the disc. These parts can be considered as an

introduction in an elementary fashion to the second ones, entitled "Supplements and Bibliographical Notes". These second parts contain more advanced studies extending the first parts in various directions and are written more condensedly and to a certain extent sketchily. Each lecture is ended by "Concluding Remarks" where a review of the literature and hints for unsolved problems complete the discussion.

It is hopeless to even try to sketch the rich contents of this book, the "Bibliography" lists about five hundred items! It may be informative to mention that the Carleson corona theorem plays a central role in the discussion. Another interesting method is the introduction of special Hankel and Toeplitz operators when studying the model operators.

The present book is not simply a translation of the original Russian one but it is an improved and considerably enlarged edition. Some parts have been revised and moreover, while the Russian original has contained only a single Appendix on the spectral multiplicity of operators of class  $C_0$ , the present edition contains four more ones. Appendix 2 presents the proof of all assertions on Hardy classes which are used in the text. Appendix 3 contains the modern proof of the Carleson corona theorem and its operator theoretic generalisation. Appendix 4 is devoted to Toeplitz and Hankel operators connected with the general orientation of the book. Appendix 5 entitled "Hankel operators of Schatten—von Neumann class and their application to stationary processes and best approximations" has been written by S. V. Hruščev and V. V. Peller. "List of Symbols", "Author Index" and "Subject Index" complete the book.

The reader needs to be familiar only with standard material in mathematical analysis taught usually in undergraduate courses. Because of the many interesting methods and the large material covered in this book, it can be warmly recommended to everybody who is interested in its topic. The special two-level structure of the discussion certainly helps the reader to orient himself. It is worth to glance through this edition even for those who know the Russian original well, because of the improvements and Appendices mentioned above.

*E. Durszt (Szeged)*

**Optimization and Related Fields**, Proceedings of the "G. Stampacchia International School of Mathematics" held at Erice, Sicily, September 17—30, 1984, Edited by R. Conti, E. De Giorgi and F. Giannessi (Lecture Notes in Mathematics, 1190), VIII+419 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

To find extreme values of functions is perhaps the most important problem of mathematics derived from practice. It is the simplest case of this problem when the maximum or minimum of a smooth function of several variables in a domain is to be found. But looking for the extreme values of smooth functions on a closed set with a piece-wise smooth boundary, which is common e.g. in econometrics already requires a lot of special methods that constitute mathematical (nonlinear) programming. Similarly, in the calculus of variations some new problems have appeared recently in which the control parameters vary on closed sets with boundaries. These problems gave rise to the "new calculus of variation", control theory or the theory of optimal processes. As it has turned out, functional analysis is suitable for investigating the deep common roots of these optimization problems seemingly independent at the first glance.

These lecture notes contain the invited talks of the meeting above, whose aim was to give an opportunity for promoting the exchange of ideas and for stimulating the interaction among various branches of optimization. The reader can find articles among others on gradient methods, homogenization problems in mechanics, Lagrange multipliers, equilibria in the theory of games, recent progress in the calculus of variations and optimal control problems and stability analysis in optimization.

*L. Hatvani (Szeged)*

**Oscillation Theory, Computation, and Methods of Compensated Compactness.** Edited by Constantine Dafermos, J. L. Ericksen, David Kinderlehrer, and Marshall Slemrod (The IMA Volumes in Mathematics and Its Applications, 2), IX+395 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1986.

This volume is the proceedings of the Workshop held under the same title in the Institute for Mathematics and its Applications (University of Minnesota). The Workshop was an integral part of the 1984—85 IMA program on Continuum Physics and Partial Differential Equations. The subject-matter of the conference was the treatment of nonlinear hyperbolic systems of conservation laws, which is the most important problem of continuum mechanics. Both the analytical and numerical sides were emphasized, and special attention was paid to the new ideas of compensated compactness and oscillation theory. The proceedings contain articles among others on the nonlinear Schrodinger equation, total variation diminishing schemes, the weak convergence of dispersive difference schemes, the Korteweg de Vries equation, nonlinear geometric optics, commutation relations, and the interrelationship among mechanics, numerical analysis, compensated compactness and oscillation theory.

*L. Hatvani (Szeged)*

**Pappus of Alexandria, Book 7 of the Collection,** Edited with translation and commentary by Alexander Jones, in two Parts, with 308 Figures (Sources in the History of Mathematics and Physical Sciences, 8), X+748 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

Pappus of Alexandria flourished about 320 A.D. and had the opportunity to read all the books of the preceding ages in the library. To help the forthcoming generations in studying the works of the famous Greek mathematicians and astronomers, he has written detailed commentaries. If he thought a proof of a theorem too difficult, then he inserted a lemma to make it easier, and if an author considered only one of the possible cases, then Pappus supplied with similar proofs the remaining cases. We know the content of several Greek works from his commentaries only.

One of his most famous works is the Collection, which contains eight books and preserved in a tenth-century manuscript, Vaticanus gr. 218. This is defective at the beginning and end. We have lost (in Greek) Book 1, the first part of Book 2, and the end of Book 8. The Collection has often been regarded as a kind of encyclopedia of Greek mathematics, a compendium in which Pappus attempted to encompass all the most valuable accomplishments of the past.

Book 7 of the Collection is a companion to several geometrical treatises, which were supposed to equip the geometer with a "special resource" enabling him to solve geometrical problems. More precisely, they were to help him in a particular kind of mathematical argument called "analysis", which is a kind of reversal of the usual "synthetic" method of proof and construction.

In Part I we can read an Introduction: Pappus and the Collection, containing historical and textual remarks, an Introduction to Book 7, and the Greek text of Book 7 with a fresh English translation due to the editor.

Part II contains three essays on lost works that Pappus discusses: The Minor Works of Apollonius, Euclid's Porisms and The Loci of Aristaeus, Euclid and Eratosthenes. This part contains a general and a Greek index and the figures to the text.

We warmly recommend this valuable work to everybody who is interested in ancient mathematics.

*Lajos Klukovits (Szeged)*

H.-O. Peitgen—P. H. Richter, *The Beauty of Fractals*, XII+199 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

The theory of fractals is a rapidly developing part of mathematics nowadays. Mandelbrot's work indicated the turning point and his famous book aroused both scientists' and nonscientists' interest in fractals. The theory, besides the theoretical interest in itself has practical importance and the computer-generated colour pictures of fractals have aesthetic value. The authors aim was to unite these three aspects of fractals.

The book starts with the essay "Frontiers of Chaos" which, without any mathematical rigour, explains the background to the non-specialist. This is followed by eight special sections, each of which corresponds to a part of the essay and completes the topic considered there.

In the first special section the authors analyse the Verhulst dynamics which is a population growth model with one controlling parameter:  $x_{n+1} = (1+r)x_n - rx_n^2$ . Depending on the choice of the parameter  $r$ , the system may be convergent, periodic or, surprisingly, "chaotic". This "deterministic chaos" has become an important idea and directed the attention to aspects of complex analytical dynamical systems. Fatou and Julia extensively studied these processes during the first World War. In the second special section the definition and basic properties of Julia and Fatou sets are collected without proofs but with complete references for the interested readers. The works of Julia and Fatou "remained largely unknown, even to mathematicians, because without computer graphics it was almost impossible to communicate the subtle ideas". They characterized the Julia set, which is the set of initial values for which the process behaves chaotic, in two ways. These results make the computergraphical generation of Julia sets possible and their properties become easy by looking at these pictures. This enables us "thinking in pictures" and the experimental computer results can help in arriving at new discoveries and conjectures. The philosophical contents of this kind of unity of science and art is discussed in detail in the essay. The third special section contains Sullivan's famous classification theorem of the components of the Fatou set. The authors give several examples from physics, biology and other fields to show that the quadratic dynamical systems have special importance. (From the dynamical point of view these are equivalent to the processes generated by the polynomials  $p_c(z) = z^2 + c$ ). From the general theory of Fatou and Julia it follows that the Julia set in this case is either connected or a Cantor set. Mandelbrot defined and investigated the set of  $c$  values for which the corresponding Julia set is connected. This strange set is named after him today. The fourth special section is devoted to the Mandelbrot set and an up-to-date list of known results and related problems are presented. When  $c$  is wandering in a component of the interior of the Mandelbrot set then the corresponding Julia set does not change topologically but at branch points qualitative changes occur and crossing the boundary yields the most dramatic one, the Julia set becomes a Cantor set. This "transition from order into chaos" phenomenon is one of the central questions treated in the essay. In the fifth section the relationship between two-dimensional electrostatics and quadratic processes are discussed. Potential theory is applied to obtain additional information about the structure of fractals. The maps contained in the book were coloured by calculating equipotential lines. Calculation of field lines is usually hard but in the case of the Mandelbrot set an efficient method, the Hubbard trees, is presented. In the next three special sections the Newton method for the complex and for the real case and a discrete Volterra—Lotka system are analyzed from the dynamical point of view. Surprisingly, the pictures of fractals in the complex and real cases are different. In the authors opinion the former are in baroque style and the latter are more modern shapes, and they state that something must be hidden behind this fact.

The second essay "Magnetism and complex boundaries" is an intuitive outline of a possible

explanation of phase transition on the ground of the fractals. The next two special sections contain the physical and mathematical details.

The book contains papers of four invited contributors, the most distinguished experts of the field. B. Mandelbrot reports on the way that has led him to the discovery of the Mandelbrot set. A. Douady presents an outline of the known results and unsolved questions. The physicist G. Eilenberger describes the symbolic meaning of what the authors' pictures may have within the changing comprehension of nature. H. W. Franke, one of the pioneers of computer graphics, reports on his own experiences and draws a number of inferences from them.

There is a "Do it yourself" section at the end of the book. This contains some hints for interested readers who want to try to generate pictures on their own computer. The presentation of the book is nice, it contains 88 really beautiful pictures. The book is recommended to anybody, from pure mathematicians to the layman, who is interested in fractals.

*J. Kincses (Szeged)*

**R. Michael Range, Holomorphic Functions and Integral Representations in Several Complex Variables** (Graduate Text in Mathematics), XIX + 386 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

"The subject of this book is Complex Analysis in Several Variables. This text begins at an elementary level with standard local results, followed by a thorough discussion of the various fundamental concepts of" complex convexity "related to the remarkable extension properties of holomorphic functions in more than one variable. It then continues with a comprehensive introduction to integral representations, and concludes with complete proofs of substantial global results on domains of holomorphy and on strictly pseudoconvex domains in  $C^n$ ", including, for example, C. Fefferman's famous Mapping Theorem."

The book, written in a lucid style and offering the reader a wealth of material, is excellent for courses and seminars or for independent study. Much of this material was not readily accessible and the inclusion of such topics greatly enhances the value of the book. The most important prerequisites are: calculus in several real variables, complex analysis in one variable, Lebesgue measure and the elementary theory of Hilbert and Banach spaces and some basic facts of point set topology and algebra.

A good book has some characteristic features which run through it. In this work integral representations are the principal tools in developing the global theory. This presentation has several advantages. For example, as the author writes, it helps to bridge the gap between complex analysis in one and in several variables, it directly leads to deep global results and concrete integral representations lend themselves to estimations. The work presents the main developments of the last twenty years concerning integral representations. One of the other characteristic features of the book is the constant presence of historical comments. In the light of these comments the new notions and results become more natural and understandable. This is the most attractive peculiarity of the book for the reviewer. (One of the particularly valuable gems can be found at the end of Ch. IV. on the history of integral representations.) A further remarkable feature of this book is that it contains a relatively large number of exercises ranging from the routine to the very advanced ones. This is particularly important since the subject abounds in abstract theorems and has only a few worked examples.

*L. Pintér (Szeged)*

**Patrick J. Ryan, *Euclidean and non-Euclidean Geometry. An Analytic Approach*, XVII+215 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1986.**

Teachers of geometry can find nowadays some good text-books on Euclidean and non-Euclidean plane geometry which can serve as an introduction to combinatorial, algebraic or topological theories of transformation groups, to direct methods of non-Euclidean spaces and of the differential geometry of homogeneous manifolds. Since computational-analytical aspects of geometric theories have increasing importance in the applications in mathematical physics and computer graphics, there is a demand on an up-to-date analytical introduction to plane geometry. The present book gives a very well-written and useful treatment of this topic. It contains the fundamentals of Euclidean, spherical, elliptic and hyperbolic plane geometry using the methods of isometric, affine and projective transformation groups. At the same time it provides an arsenal of computational techniques and a certain attitude toward geometrical investigations. It aims to give an appropriate background for teachers of high school geometry and to prepare students for further study and research.

The book is self-contained for upper-level undergraduate mathematics students, the necessary knowledge is summarized in appendices. Only a familiarity with linear algebra and elementary transcendental functions is expected from the reader. The material is illustrated with many exercises, requiring specific numerical computations or supplying proofs that have been omitted. Some of them extend the results proved in the text.

The first main part is, of course, Euclidean plane geometry (Historical introduction, Plane Euclidean geometry, Affine transformations in the Euclidean plane, Finite groups of isometries of  $E^2$ ). The second part contains: Geometry on the sphere, The projective plane  $P^2$ , Distance geometry on  $P^2$ . The last chapter is: The hyperbolic plane.

The book gives a very good basis for high school geometry teaching and a good introduction for graduate work in differential geometry or computer graphics.

*Péter T. Nagy (Szeged)*

**Lewis H. Ryder, *Quantum Field Theory*, X+443 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1985.**

For a long period, quantum field theory had meant only the quantum theory of electromagnetic fields. Other forces of nature as the weak and strong nuclear interactions resisted the formalism that proved to be so successful in the description of electromagnetism. The principle of local gauge invariance has overcome the difficulties, and the prominent achievements of gauge field theories of the seventies have reached the textbook level by now.

To find a good balance however, in a single book, between the several parts of this huge subject is not an easy task. This is the more so if among the author's aims is that the presentation should be intelligible by a graduate student. The *Quantum Field Theory* by L. Ryder has solved this problem successfully. To read this volume it is enough to be familiar with quantum mechanics and special relativity. The text leads us with great pedagogical skill, step by step from elementary field theory to the renormalization of gauge fields. The emphasis of the presentation is on the path integral method. Besides introducing the fundamentals of quantum field theory, the author acquaints the reader with some modern mathematical tools as well. One may regret that certain more recent concepts (e.g. supersymmetry) are not found in the book, but the author probably wanted to include only those results that have more or less experimental basis.

The book is very well suited for teaching and studying this subject, and brings even the beginner close to present day field theory.

*M. G. Benedict (Szeged)*

M. Shirvani—B. A. F. Wehrfritz, *Skew Linear Groups* (London Mathematical Society Lecture Note Series, 118), 253 pages, Cambridge University Press, Cambridge—New York—Melbourne, 1986.

Skew linear groups arise naturally as a generalization of linear groups, by omitting the requirement of the commutativity of the corresponding field. One of the main problems in passing from linear groups to skew linear groups is that, at least presently, division rings are rather difficult to handle. The investigation of skew linear groups is a fairly young branch of algebra, in comparison with the theory of linear groups, however, in recent years it has expanded very rapidly. This book is the first monograph providing a systematic treatment of a number of results that were available, till now, in research papers only.

In Chapter 1 the basic concepts such as irreducibility, absolute irreducibility and unipotence are reviewed in the context of skew linear groups, and some groups with faithful skew linear representations are constructed. Chapter 2 discusses finite (and locally finite) skew linear groups, including the description of finite subgroups of division rings of characteristic zero, and the theorem that finite skew linear groups over division rings of characteristic zero have large metaabelian normal subgroups. Chapter 3 is devoted to skew linear groups over locally finite-dimensional division algebras, with the emphasis laid on nilpotence and solubility. In Chapter 4 the authors consider skew linear groups over division rings generated by a central subfield and a polycyclic-by-finite subgroup. Chapter 5 contains a detailed study of normal subgroups of absolutely irreducible skew linear groups. In Chapter 6 the book concludes with an application showing how the theory of skew linear groups may shed light on some known results on group rings.

To help the reader, the authors give a list of prerequisites for each chapter, a detailed notation index, and author and subject indices. This monograph is warmly recommended as a textbook for those wishing to get acquainted with the subject, and as a reference book as well.

*Ágnes Szendrei (Szeged)*

Michael Shub, *Global Stability of Dynamical Systems*, XII+150 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1986.

The most characteristic property of an equilibrium position in a mechanical system is its stability or instability. The equilibrium position is stable if during its motion the system remains arbitrarily near the equilibrium state provided that it was near enough at the initial moment. Obviously, only the stable equilibria can be realized in practice, so the research for conditions of stability have started at the early stages of mechanics and mathematics. Later on the investigations have been extended to general dynamical systems and have created the Lyapunov Stability Theory. In this theory it is always assumed that the system itself is under ideal circumstances, i.e. it cannot be disturbed by outside effects. However, each system is under the action of certain small undefinable perturbations. Therefore, it is clear that one can expect only those properties of the model to be realized in reality which are not too sensitive to small changes in the model. In 1937 Andronov and Pontryagin introduced the concept of robustness or roughness of a system (nowadays it is called structural stability), which means that the topological structure of the trajectories does not change under small perturbations of the system. If we observe the trajectories only in a neighbourhood of a point then we talk about local stability. If the trajectories are observed on the whole manifold then the stability is global.

The central objective of the modern theory of Dynamical Systems is the description of the orbit structures of vector fields on a differentiable manifold. There exist, however, fields with



extremely complicated orbit structures, thus one has to restrict the study to a subset of the space of vector fields. It is desirable that this subset should be open and dense, or as large as possible, and it should consist of structurally stable vector fields with simple enough orbit structure so that one could classify them. Due to the celebrated Hartman—Grobman Theorem, this program has been completely solved if the stability and the equivalence are meant locally.

To complete the above program in the global sense is much more difficult. As it was proved by Smale, on manifolds of dimensions higher than two the structurally stable fields are not dense, and the structure of the trajectories and their limit sets even for the stable fields can be extremely complicated. Their description is still an active area of research.

Shub's book gives an excellent account on the results of this area, most of which were available only in articles so far. The reader can get acquainted the central concepts, theorems and examples of the global theory of dynamical systems such as filtration, hyperbolic invariant sets, change recurrence, stable and center manifold theorems, Smale's Axiom A, symbolic dynamics, Markov partitions,  $\Omega$ -stability theorems, Smale's horseshoe and the solenoid. It is highly recommended to anyone interested in dynamical systems and stability theory.

*L. Hatvani (Szeged)*

**Gábor J. Székely, Paradoxes in Probability Theory and Mathematical Statistics** (Mathematics and its Applications), XII+250 pages, Akadémiai Kiadó, Budapest and D. Reidel Publishing Company, Dordrecht, 1986.

This book is very unusual and, as far as I know, is unique in its kind. It endeavours "to show how the rapidly progressing and widely used branch of knowledge of the mathematics of randomness has developed from paradoxes". While this is a bit too much to be hoped for as it flaunts, and would be the greatest paradox of all had the author succeeded in doing so, his paradoxical vision is certainly a valid one and interesting. The result is a most enjoyable reading which is worth much more than two dozens of half-thought dishonest "introduction to probability and statistics" books published in so great a number nowadays.

Chapter 1 contains the discussion of 12 paradoxes or families of paradoxes from classical probability theory, while Chapters 2, 3 and 4 expose and treat 12, 6 and 12 paradoxes or families of paradoxes in mathematical statistics, the theory of stochastic processes and the foundations of probability theory, respectively. The discussion of each paradox is divided into five parts: the history, the formulation and the explanation of the paradox, remarks and references. Furthermore, the four chapters end, respectively, with 15, 16, 8 and 8 of what the author calls quickies which either did not fit into the main line of thought of the book to be discussed in such detail as the numbered paradoxes, or are adjacent curiosities, strange facts, gems.

The history and remark sections and the quicky passages contain a lot of interesting historical and cultural information, narrated in the easy, chatting style of the author, together with stories, anecdotes and gossip. Instead of fooling around for pages on end to say the same thing politely, for example, he is not afraid of very simply stating that "R. A. Fisher hated K. Pearson". Or, while discussing the paradox of the almost sure eventual extinction of a critical Galton—Watson process, he proposes an interesting system for the inheritance of family names to avoid the replacement of "nice old family names" by "more common dull ones like Smith, etc."

The book makes an easy and recreational reading but can be used more seriously as a supplementary reading to almost any course in probability and statistics. In fact, my math major students like the original Hungarian edition, of which the present English one is a revised and updated version.

*Sándor Csörgő (Szeged)*

**Audrey Terras, Harmonic Analysis on Symmetric Spaces and Applications I**, 341 + XII pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1985.

Harmonic analysis is one of the most useful areas of mathematics which made a deep influence on several other fields of mathematics and physics. The book demonstrates exactly this usefulness by presenting many applications in number theory, statistics, medicine, geophysics and quantum physics. This is the first volume of a series dealing with the harmonic analysis of the three classical geometries (euclidean, spherical and hyperbolic).

Besides the standard development of euclidean Fourier analysis we learn in the first chapter how to use this theory to the solution of the heat equation, to the examination of crystals, as well as zeta functions of algebraic number fields. In Chapter 2 spherical Fourier analysis is applied for the study of the hydrogen atom, for the sun's magnetic field and also for group representations and Radon transforms. The last chapter is devoted mainly to the fundamental domains of discrete subgroups of hyperbolic isometries, the Reolche—Selberg spectral resolution and the Selberg trace formula.

We recommend this excellent text-book to every mathematician, engineer, scientist and applied mathematician who is interested in harmonic analysis and in its applications.

*Z. I. Szabó (Buda pest)*

**The Craft of Probabilistic Modelling: A Collection of Personal Accounts**, Edited by J. Gani (Applied Probability. A Series of the Applied Probability Trust), XIV + 313 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

This is the first volume of the new series in the braces above with series editors J. Gani and C. C. Heyde. The beginning is indeed very nice. The volume contains nineteen essays from leading probabilists who, among other things, have distinguished themselves in applied probability model building. Each of the essays are preceded by a short biography. Some of the essays concentrate on the models themselves that the authors have built, others are entirely autobiographical, while the rest is a combination of the two. Some of the writings are very dry, some are exceptionally lively. I don't single out any of the essays for special mention here because more than half of them are very close to my heart for one reason or other. In the grouping of the editor, the contributors are the following. Early craftsmen: D. G. Kendall, H. Solomon, E. J. Hannan, G. S. Watson; The craft organized: N. T. J. Bailey, J. W. Cohen, R. Syski, N. U. Prabhu, L. Takács, M. Kimura, P. Whittle, R. L. Disney; The craft in development: M. F. Neuts, D. Vere-Jones, K. R. Parthasarathy, M. Iosifescu, W. J. Ewens, R. L. Tweedie.

The book is a very enjoyable reading.

*Sándor Csörgő (Szeged)*

**John B. Thomas, Introduction to Probability** (Springer Texts in Electrical Engineering), X + 247 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

This is a textbook designed for an introductory one-term course for undergraduate or beginning graduate students majoring in engineering, the social sciences or business administration. The only prerequisite is a solid standard calculus course. Contrary to the practice followed by dozens of texts with the same aim, the present one introduces the basic notions and formulates the corresponding theorems with very great care and rigour. Of course, not all the proofs can be given by formal arguments in this framework. These are sometimes substituted by very nice heuristic explana-

tions. There is a great number of well-chosen, illustrative examples and the eight chapters (Introduction and preliminary concepts, Random variables, Distribution and density functions, Expectations and characteristic functions, The binomial, Poisson, and normal distributions, The multivariate normal distribution, The transformation of random variables, Sequences of random variables) each end with a good set of homework problems. The trend is towards engineering applications. Six short appendices on integration and matrix theory help the student. Instructors of courses of the type noted above will like the book. A clean and honest work.

*Sándor Csörgő (Szeged)*