Acta Sci. Math., 52 (1988), 215—230

Bibliographie

Ralph H. Abraham—Christopher D. Shaw, Dynamics — The Geometry of Behavior, Part 3: Global Behavior (The Visual Mathematics Library, 3), XI+123 pages, Aerial Press, Inc., Santa Cruz, California.

At the defence of a Ph. D. thesis on topological dynamics one of the referees criticized the author not presenting figures enough in his work. A sharp debate broke out about the question whether or not figures are necessary in articles or books on dynamics. Some people said "no" arguing that every drawing takes us in to some extent, it is in the way of the abstraction oversimplifying the circumstances. By the way, in his original work, *Mécanique analitique* Lagrange used no diagrams. Other people (including the reviewer) said that the geometrical ideas having been appeared in dynamics nowadays should be visualized in some way. Abraham's and Shaw's book shows that this purpose can be realized on a very high level. Their pictures do not restrict the abstraction, quite the contrary, they help the reader imagine and assimilate very abstract concepts and phenomena.

In talking among themselves mathematicians universally use the so called "dynamic picture technique": a picture is drawn slowly, line-by-line, along with a spoken narrative. The coordination between the phases of the picture and the narrative is very important in the process of comprehension. The book preserves the dynamics of the live presentation. If the final picture is sophisticated, the reader can find its intermediate phases with appropriate comments. A typical example is the section on the famous and mysterious Lorenz attractor, which is not so mysterious after having read and watched the section. Yes, the book has to be read and looked at alternately, and the interaction of reading and watching results a deep and quick understanding.

The book contains chapters and sections on attractors, separatrices, generic properties, structural stability, heteroclinic and homoclinic tangles, horseshoes and nontrivial recurrence. As an excellent supplement to the standard monographs in the field, it should be on the bookshelf of each student, user of mathematics or mathematician studying or teaching dynamics.

László Hatvani (Szeged)

A. N. Andrianov, Quadratic Forms and Hecke Operators (Grundlehren der mathematischen Wissenschaften, 286), XII+374 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

In the classical theory of quadratic forms remarkable multiplicative properties of the number of integral representations of integers by positive definite integral quadratic forms were discovered. To explain these properties, E. Hecke had introduced operators in 1973, which were named later after him. Hecke operators are classically linear operators acting on the space of modular forms of one variable. This concept may be generalized in a natural way to multivariate modular forms. Using this idea, many interesting multiplicative properties of the number of integral representations of quadratic forms of more than one variable by quadratic forms were discovered in the last 50 years.

The purpose of this book — as the author writes in the preface — is to present in the form of a self-contained text-book the contemporary state of the theory of Hecke operators on the spaces of holomorphic modular forms of integral weight (the Siegel modular forms) for congruence subgroups of integral symplectic groups.

The book is divided into five chapters. Three short appendices with the required knowledge about symmetric matrices, about quadratic spaces and about modules in quadratic fields make it complete.

The content of the book is briefly as follows. In Chapter 1 theta-series of positive definite quadratic forms are introduced and their automorphic properties are studied. Looking at all functions which satisfy similar transformations as the theta-series, the space of modular forms is defined in Chapter 2. This way makes it possible to study a lot of properties of theta-series using the nice analytic expansions of modular forms. Chapter 3 deals with Hecke rings. This concept is defined first abstractly, for pairs (Γ, S), where S is a multiplicative semigroup and Γ is a suitable subgroup of S. The special properties of the most interesting Hecke rings of the general linear groups, of the symplectic groups and of the triangular subgroup of the symplectic groups are studied in detail. Chapter 4 is devoted to the study of the multiplicative properties of the Fourier coefficients of modular forms. The most important tools to get such relations are Hecke operators, introduced also here. The last chapter deals with the action of Hecke operators on theta-series. Here, there are not proved final, general results on the multiplicative properties of the Fourier coefficients of theta-series but rather a possible way is shown to study this problem. So this book does not have a happy end, but I think, it will inspire further research on this topic.

This book is written in a clear, well readable style. I want to emphasize the few introductory sentences explaining the goal and methods before each section. I find the exercises another valuable component of the book.

This volume is designed for graduate students and researchers who wish to work in the arithmetic theory of automorphic forms.

Attila Pethő (Debrecen)

M. Berger, Geometry I—II, (Universitext), XIII+428 pages, X+406 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

There are a lot of books on geometry but only few of them include all part of geometry and also written clearly, using modern terminology but do not lose in the labirinth of formalism. Here is an excellent book which certainly satisfies these conditions. It is the translation of the French book "Géométrie" originally published in five volumes. The book contains the detailed discussion of classical geometries and beside this it is a unified reference source for all the subfields of geometry. The author's aim was threefold as he writes: "to emphasize the visual, or 'artistic' aspect of geometry, by using figures in abundance; to accompany each new notion with as interesting a result as possible, preferably one with a simple statement but a non-trivial proof; finally, to show that this simple-looking mathematics does not belong in a museum, that it is an everyday tool in advanced mathematical research, and that occasionally one encounters unsolved problems at even the most elementary level".

It is hopeless to give even a short summary of the material discussed, so let us mention only some of the most delicate parts which usually omitted from textbooks: the classification of crystallographic groups, the classification of regular polytopes in arbitrary dimension, Cauchy's theorem on the rigidity of convex polyhedra, the discussion of polygonal billiards, Poncelet's theorem on polygons inscribed in a conic, the Villarceau circles on the torus, Clifford parallelism, the isoperimetric inequality in arbitrary dimension, the simplicity of the orthogonal group, the theorem of Witt and Cartan-Dieudonné.

In each chapter there are a great number of exercises which are usually more difficult than those in comparable books. The solutions of the most difficult ones and other exercises can be found in the companion volume "Problems in Geometry".

This book can be used in different ways. Teachers and students can use it for introductory course and some parts of it for higher-level course. It also serves as a handbook for researchers in geometry.

J. Kincses (Szeged)

T. Beth—D. Jungnickel—H. Lenz, Design Theory, 688 pages, Cambridge University Press, London—New York—New Rochelle—Melbourne—Sydney, 1986.

The main concepts and ideas of modern Design Theory are presented in this book.

Chapter I is a general introduction to the different topics of Design Theory. This part of the book provides those algebraic, geometric and parametric properties of certain incidence structures which are important for an advanced study of them. The second chapter is concerned with the techniques of deriving necessary parametric conditions which have to be fulfilled by an incidence structure of a given type. (Some titles from this chapter: Fisher's inequality for pairwise balanced designs, symmetric designs, generalizations for Fisher's inequality.) Since it is sometimes heplful to use the group of automorphisms of a design, the Chapter III deals with the connections between groups and designs. Separated chapter is devoted to Witt designs, which have been constructed with special Steiner systems and the Mathieu groups. (These are the only known finite t-transitive permutation groups with $t>3$, except for the symmetric and alternating groups.) For those readers who are familiar with non-elementary groups theory Chapter 5 is a nice application with the higly transitive groups. Further two chapters present the difference sets and the regular symmetric designs. Chapter 8 deals with various direct constructions of designs. In Chapter 9 some important recursive reconstruction methods are developed which will be applied to mutually orthogonal Latin squares and pairwise balanced designs. The next part provides more advanced existence and non-existence results for transversal designs. Separated chapter is devoted for the proof of Wilson's main theorem concerning the existence of an $S_\lambda(2, K, v)$. In the last chapter after returning to the discussion of automorphism groups an extensive literature is presented on characterisation problems.

An extensive bibliography of about 500 titles — all quoted in the previous sections — has been included.

The reader is expected to be familiar only with basic algebra but otherwise the work is self-contained. It is suitable for advanced courses and a reference book for private study, too. The proofs of several fundamental theorems have been simplified and many advanced results are presented. Last we notice that the book achived its aim: "to provide some of the necessary mathematical background for anyone working in Communication Engineering, Optimization, Statistical Planning, Computer Science and Signal Processing".

G. Galambos (Szeged)

Béla Bollobás, Combinatorics (Set systems, hypergraphs, families of vectors, and combinatorical probability), XII+180 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sidney, 1986.

Béla Bollobás has published formerly "Graph Theory", an introductory text, and two research monographs, "Extremal Graph Theory" and "Random Graphs". "Combinatorics" is a book whose main theme is the study of subsets of finite sets.

This book is an expanded account of a first-year graduate course in combinatorics but it contains considerably more material than one could reasonably hope to cover in a one semester course, this gives the lecturer ample freedom to slant the lectures to his taste.

The contents of the book (the list of section headings) present the topics very well: 1. Notation, 2. Representing Sets, 3. Sperner Systems, 4. The Littlewood — Offord Problem, 5. Shadows, 6. Random Sets, 7. Intersecting Hypergraphs, 8. The Turán Problem, 9. Saturated Hypergraphs, 10. Well-Separated Systems, 11. Helly Families, 12. Hypergraphs with a given number of Disjoint Edges, 13. Intersecting Families, 14. Factorizing Complete Hypergraphs, 15. Weakly Saturated Hypergraphs, 16. Isoperimetric Problems, 17. The Trace of a Set System, 18. Partitioning Sets of Vectors, 19. The Four Functions Theorem, 20. Infinite Ramsey Theorem.

Generally an initial combinatorics textbook contains very little for these topics, but ones are as worthy of consideration as any, in view of their fundamental nature and elementary structure.

The sections are short summaries of the topics, with their main theorems and with elegant and beautiful proofs, those which may be called the gems of the theory.

The reader can consolidate his understanding of the material by tackling over one hundred exercises. If a researcher wants to know more about a special topic, he (or she) finds many articles on the basis of references.

Zoltán Blázsik (Szeged)

Detection of changes in random processes (Edited by L. Telksnys) Optimization Software, Inc. Publications Division, New York, 1986.

Changepoint problems have originally arisen in the context of quality control, where one typically investigates the output of a production line and would wish to signal deviation from an acceptable average output level while investigating the data. Such situations can usually be modelled by saying that we have a random process $\{X(t), 0 \leq t \leq T\}$ and we wish to detect whether the probabilistic behaviour of $\{X(t), 0 \leq t \leq \tau\}$ and $\{X(t), \tau \leq t \leq T\}$ is the same. Not surprisingly, changepoint problems have been studied by many researchers from theoretical as well as applied points of view.

The book under review is a new addition to the literature on this subject. It contains 25 papers on detection of changes in random processes. The papers give a nice summary on recent progress in the Soviet Union on these problems. The references reflect intensive activity in this field. The authors of the volume cite a lot of papers on changepoint problems published in the Soviet Union. However, they do not seem to be aware of results which have appeared in Western journals.

The translation of this collection makes results of researchers working in the Soviet Union readily available for a wider audience. This translation series of Optimization Software Inc. is a great service for the mathematical community.

Lajos Horváth (Ottawa, Canada)

Dietrich Braess, Nonlinear Approximation Theory (Springer Series in Computational Mathematics), XIV+290 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1986.

The monograph is based on the lectures given by the author to fourth year students at German universities. The material of these lectures is widened by additional one so that the book is a useful text not only for students but for researchers interested in approximation theory, too.

The prerequisites consist essentially of a good basic knowledge of analysis and functional analysis.

The book has been organized so that the sections recommended mostly for researchers (so as rational approximations, exponential sums, spline functions with free nodes) are independent of each other.

Let us give a short detail of the chapters pointing out just the main topic of them.

Chapter I is a review of well-known results from the linear theory. Chapter II contains the functional analytic approach (properties of Chebyshev sets; Kolmogorov criterion for suns). Chapter III is devoted to the methods of local analysis (critical points; nonlinear approximation in Hilbert spaces; Gauss—Newton method). Chapter IV is consisting of the methods of global analysis (the uniqueness theorem for Haar manifolds; concepts of the classification of critical points). In Chapter V the rational approximation is included (existence of best approximation; Chebyshev approximation by rational functions; rational interpolation; Padé approximation and moment problems; degree of rational approximation; the computation of best rational approximation). Chapter VI is devoted to the approximation by exponential sums (existence of best approximation; interpolation). Chapter VII contains Chebyshev approximation by γ-polynomials (Descartes family; approximation by proper γ-polynomials and by extended γ-polynomials; local best approximation). An finally Chapter VIII is dealing with the approximation by spline functions with free nodes (spline functions; Chebyshev approximation by spline functions; monosplines of least L_1, L_p and L_∞ norms).

The book is pretty well organized, its style is clear. Hopefully it can certainly be a very useful text for both researchers and students.

József Németh (Szeged)

Walter Dittrich—Martin Reuter, Selected Topics in Gauge Theories (Lecture Notes in Physics, 244), 315 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

This volume contains a collection of lectures and seminar talks given by the authors at Tübingen University and elsewhere. The material is organized into 16 chapters which are devoted to various aspects of chiral anomalies, topological objects like instantons and skyrmions, effective actions, background field methods and other topics of current interest in gauge theories. The material is presented in an unorthodox way: standard explanations (which can be found in textbooks) are omitted to a large extent, whereas computational details are completely given. The only general prerequisite is some grounding in quantum field theory, however, to get better acquainted with the background of the topics presented here, the reader should first consult some of the references cited at the end of each chapter.

The book is particularly recommended to those who are looking for a good introduction to topological aspects and chiral anomalies in gauge theories. The manner of presentation makes it ideally suited to the needs of graduate students.

L. Gy. Fehér (Szeged)

Beno Eckmann Selecta, Edited by M. A. Knus, G. Mislin and U. Stammbach, XII+835 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1987.

The edition of 65 selected papers of Beno Eckmann is in honor of his work on the occasion of his seventieth birthday. The volume contains the representatives of his research papers. Some of his survey articles have also been included, which are exceptional in their art of presenting mathematical ideas to non-specialists. Professor Eckmann writes in his Biographical notes: "Under the wonderful guidance of Heinz Hopf I then got my doctoral thesis work. It was characteristic of Hopf's views on our science that this meant not only learning algebraic topology — then a very young field — but also getting acquainted with group theory, differential geometry, and algebra in the 'abstract' sense of the Emmy Noether school. The combination of these fields, considered at that time to be largely separated from each other, remained a constant challange during all my later work." Really, it is the characteristic feature of the fundamental results and all scientific activity of Beno Eckmann that the mentioned fields represent a unified, organically connected subject in mathematics. The most competent classification of the directions of his research can be formulated by the titles of his comments to the selected papers: Homotopy groups and fiber spaces; Continuous solutions of linear equations; Cohomology of groups; Homological algebra, transfer; Duality in homotopy theory; Duality groups, Poincaré duality.

Péter T. Nagy (Szeged)

A. T. Fomenko—D. B. Fuchs—V. L. Gutemacher, Homotopic Topology, 310 pages, Akadémiai Kiadó, Budapest, 1986.

This book is a translation of the Russian original which based on the lectures held at the Moscow University. The authors' main aim "was to dig a tunnel for the ignorant from the basic terms to the 'height of heights' — the Adams spectral sequence, and it was a lucky chance that this tunnel led through a few reefs of gold". This aim is completely fulfilled.

The first chapter contains the basic ideas of homotopy theory. First the general constructions are presented: natural group structures on the sets $\pi(X, Y)$, homotopy groups, covering spaces, fibratons and homotopy sequences and then the homotopy of CW-complexes are studied in details. The second chapter introduces the general homology theory. This is started with singular homology and cohomology of topological spaces, especially the computation of the homology groups of CW-complexes and then the connections between homology and homotopy groups are studied, namely Hurwitz's theorems are proved. The chapter ends with the obstruction theory. The third chapter deals with the construction of the spectral sequences of filtered spaces and with their applications to the calculation of homology groups. The subject of the fourth chapter is the discussion of cohomology operations. After the general constructions some particular but very important cases are presented namely the Steenrod squares and Steenrod algebras. Finally the fifth chapter is fully devoted to the Adams spectral sequence and to its applications.

The presentation of the material is clear, the proofs, even of the most abstract theorems, are as geometric as possible. The book is fully illustrated by A. Fomenko's pictures which are organic part of it. Each of them gives an intuitive insight into a complicated construction or shows the main point of a proof. The book contains also a great number of exercises which help to understand the main concepts and extend the theory.

The book is recommended to everybody interested in homotopy theory but it can be useful for researchers in topology and related fields.

J. Kincses (Szeged)

George K. Francis, A Topological Picturebook, XV+194 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1987.

This book is about how to draw mathematical pictures. Many mathematicians and teachers would like to draw pictures, but they belive that they can not do it. No this book teaches everybody to draw, but gives some method how one can imagine and draw some figures in mathematics.

The author believes that: "There are some rules, based on differential geometry, which can be distilled into practical routines for 'calculating' how to draw a picture." He proves his idea using many examples from different objects of mathematics.

It is noteworthy that each chapter is a "picture story", i.e. tells a topological story matching the picture.

This very nice book with 87 illustrations is warmly recommended to all teachers of mathematics and mathematicians who would like to illustrate their lectures.

Árpád Kurusa (Szeged)

Felix R. Gantmacher, Matrizentheorie, 654 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

This book is the German translation of the Russian original edition appeared in 1966.

The text is divided into two parts, the fist of which (chapters 1—10) deals with general theory of matrices and the second one is devoted to special questions and applications. Chapters 1—8 give the theory of matrices in general finite dimensional vector spaces. Chapters 9 and 10 investigate special matrices, linear operators, quadratic and Hermitian forms in inner product spaces. Chapters 11—14 deal with complex symmetric, antisymmetric and orthogonal matrices, matrices with non-negative elements, regularity criteria and localization of characteristic roots. Chapter 15 presents applications of the theory of matrices for systems of linear differential equations. The last chapter is devoted to Routh—Hurwitz problem and joined questions.

The book is recommended not only to mathematicians but to every specialist interested in application of mathematics.

László Gehér (Szeged)

M. B. Green—J. H. Schwarz—E. Witten, Superstring Theory, Volume 1: Introduction, X+469 pages; **Volume 2: Loop Amplitudes, Anomalies and Phenomenology,** XII+596 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1987.

Recently there has been an enormous and even growing interest in superstring theory. No wonder, superstring theory is the most promising candidate to reconcile general relativity with quantum mechanics and to unify the fundamental interactions. There is a widely felt need for a systematic exposition of the subject. This two volume text written by outstanding experts on string theory is intended to meet this need.

Volume 1 is a self-contained introduction to string theory. It starts off with an introductory chapter in which the authors explain what string theory is, present its historical background and general philosophy concentrating on bosonic strings. The next two chapters develop the theory of a free bosonic string in detail. All the four approaches (covariant, light cone, path integral and BRST) of quantization are presented here. Chapters 4 and 5 are devoted to questions concerning world-sheet and space-time supersymmetry in string theory, i.e. the fermionic degrees of freedom are introduced. In Chapter 6 the authors describe how gauge symmetries can be introduced in string theory. This is essential to make the link with the real world. Finally, this volume contains a detailed discussion of the evaluation of scattering amplitudes in the tree approximation.

Volume 2 contains a number of topics from current research papers. Chapters 8 and 9 deal with one-loop amplitudes in bosonic string and in superstring theory respectively. A large amount of space is given to questions concerning anomalies in effective field theory. The authors investigate the emergence of effective field theory and possible mechanisms of compactification of extra dimensions. The necessary differential and algebraic geometric background material is presented in cosiderable detail in separate chapters. In the final, 16^{th} chapter the authors illustrate how the machinery of algebraic geometry can be used to understand the properties of four dimensional models obtained from

$D=10$ effective field theory via compactification. They discuss how topological formulae can fix the number of generations, the couplings and symmetries of elementary particle interactions.

The authors write in the preface: "We hope that these two volumes will be useful for a wide range of readers, ranging from those who are motivated mainly by curiosity to those who actually wish to do research on string theory." There is no doubt that this excellent book will become a standard reference on string theory. It is a need for everybody interested in this very exciting subject.

László Gy. Fehér (Szeged)

E. Hairer—S. P. Norsett—G. Wanner, Solving Ordinary Differential Equations I. Nonstiff Problems (Springer Series in Computational Mathematics, 8), XIII+480 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

Nowadays many mathematicians dealing with pure mathematics also have a personal computer of big capacity and efficiency on their desks. So dealing with differential equations one is strongly tempted to get or firm conjectures via computer experiments. (The most exciting problem of the last decade in the theory of dynamical systems, the chaotic behavior has been discovered by such an experiment.) This activity needs precise and fast numerical methods of solving differential equations, so there is a great interest in them among mathematicians and users of mathematics. The present monograph will satisfy these demands.

The first chapter gives a survey of the "Classical Mathematical Theory" of differential equations from Newton and Leibniz to limit cycles and strange attractors. Fortunately, it does not repeat the standard way of recalling the basic theorems, it is written markedly by numerical analysts. The reader can find existence theorems using iteration methods and Taylor series, and the very first proof of the convergence of Euler's method due to Cauchy, which has recently been discovered on fragmentary notes and was never published in Cauchy's lifetime.

The second chapter contains the one step methods, i.e. the Runge—Kutta and extrapolation methods. Besides the classical ones, the modern procedures with practical error estimation and stepsize control are presented such as Dormand and Prince formulae, the embedded Runge—Kutta methods, the newest Nyström type methods for the second order equations, etc. Special section is devoted to delay differential equations and their applications (infectious disease modelling, enzyme kinetics, population dynamics, etc.).

The third chapter is concerned with the multistep methods and general linear methods. The order, stability and convergence properties are studied. The various available codes are compared by using numerical examples.

The book is concluded by an appendix containing the FORTRAN codes of some very new effective procedures treated in the book. They can be obtained from the Authors also on IBM diskette on payment of 15 Swiss Franks.

László Hatvani (Szeged)

Arthur Jones—Alistair Gray—Robert Hutton, Manifolds and Mechanics (Australian Mathematical Society Lecture Series, 2), IV+166 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1987.

We learned from the classical texts of mechanics (see e.g. P. E. Appel's and E. T. Whittaker's books) that the motions of a holonomic system with n degrees of freedom could be described by the Lagrange equation of second kind, in which the Lagrangian function is defined and differentiable on an open set in the configuration space $\mathbf{R}^n$. However, it may often happen that a single equation defined on an open set describes the motions only locally. For example, in the case of the double plain pendulum the configuration space is a two dimensional torus, which cannot be mapped by any single one-to-one function onto an open set in $\mathbf{R}^2$. But we can find an "atlas" for the entire torus with "charts" giving coordinates only for some parts of the torus. In the other words, wanting to study the motions globally one needs the differentiable manifold technique. But the text-books based upon this approach (e.g. R. Abraham's and J. E. Marsden's or V. I. Arnold's books) demands essentially more than the standard undergraduate advanced calculus texts give. This gap has been bridged by the present excellent lecture notes.

The first part is an easy mathematical introduction, in which the reader can get acquainted with such concepts as differentiable manifold, tangent space, tangent boundle, double tangent, etc. In the second part the authors show how the theory can be used for the development of the theory of Lagrangian mechanics directly from Newton's law, and give some applications (the spherical pendulum, rigid bodies).

This well-written book is highly recommended to students, applied mathematicians and theoretical physicists as well as to mathematicians interested in applications of the modern mathematics.

László Hatvani (Szeged)

Hüseyin Kocak, Differential and Difference Equations through Computer Experiments (With Disketts Containing PHASER: An Animator/Simulator for Dynamical Systems for IBM Personal Computers), XV+224 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1986.

Nowadays the "strange attractor" is a key word of both theoretical and applied dynamical systems. It is an attracting set of the phase space that is more complicated than an equilibrium point or a limit cycle studied by classics. And this kind of attractors has been discovered by using numerical integration of a "simple" polynomial differential equation. E. Lorenz, a mathematician-meteorologist was investigating the motion of a layer heated from below. Using a routine numerical algorithm he got a strange attractor and noticed that the solutions behaved at almost random. Despite the strong efforts of many mathematicians, most of the properties noticed have not been proved yet theoretically. So it can be understood that computer experiment is becoming a very important tool in the theory of dynamical systems, in other words, the computer is becoming the mathematician's laboratory. Kocak's book makes this tool available also for those scholars not having any programming knowledge.

The first part gives a synopsis of the facts from the theory of differential equations, difference equations and numerical methods that are prerequisite for the book. The second part is a handbook of PHASER. It should be noted here that the program is a masterpiece. Let us cite the author to describe how it works and what it does: "It is an extremely versatile and easy-to-use program, incorporating state-of-the-art software technology (menus, windows, etc.) in its user interface. The user first creats, with the help of a menu, a suitable window configuration for displaying a combination of views-phase portraits, text of equations, Poincaré sections, etc. Next, the user can specify, from another menu, various choices in preparation for numerical computations. He or she can choose, for instance, to study from a library of many dozen equations, and then compute solutions of these equations with different initial conditions or step sizes, while interactively changing parameters in the equations. From yet another menu, these solutions can be manipulated graphically. For example, the user can rotate the images, take sections, etc. During simulations, the solutions can be saved in various ways: as a hardcopy image of the screen, as a printed list, or in a form that can be reloaded into PHASER at a later time for demonstrations for further work."

The third part briefly describes the over sixty differential and difference equations stored in the permanent library of PHASER, among them the Lorenz equation, van der Pol's oscillator, Lotka—Volterra equation, Mathieu's equation, the restricted problem of three bodies on the plane. One can meet with different kinds of bifurcations, strange attractors, homoclinic orbits etc. Moreover, phaser provides a menu entry for adding new equations to the library without any programming knowledge so that each user can easily enlarge the library according to personal needs.

Summing up, this unusual book with the diskettes gives an invaluable help for using computers in teaching, research and application of differential equations.

László Hatvani—János Karsai (Szeged)

J. L. Koszul, Lectures on Fibre Bundles and Differential Geometry, (Tata Institute of Fundamental Research, Lectures on Mathematics and Physics, 20) IV+127 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

The first edition in 1960 of these Lectures was one of the first explanations of the general connection theory making a significant influence on the further development of both differential geometry and the applications in mathematical physics. In the present time the wide-ranging interest of fibre bundle technique and of the notion of connections on principal and vector bundles has increased considerably and the present "classical" treatment of this modern theory can serve as a very good introduction to the differential geometric methods used in the mathematical manifold and Lie group theory and in their applications in Yang—Mills theory and in the related fields. The first two Chapters are devoted to the coordinate free differential calculus on manifolds and to the notion of differentiable bundles. In Chapters III and IV there is given the explanation of the notion of connections on principal bundles and holonomy groups. In Chapters V and VI the attention is focused on derivation laws on the associated vector bundles determined by the connection on principal bundle and to the applications in holomorphic connection theory.

Péter T. Nagy (Szeged)

J. P. LaSalle, The Stability and Control of Discrete Processes (Applied Mathematical Sciences, 62), V+150 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1986.

The book is concerned with systems whose development in time can be described by difference equations. The system is observed at any integer point of time, and it is assumed that the state of the system at time $n+1$ is completely determined by its state at time n. This means, that $x(n+1)=T(x(n))$, where $x\in\mathbf{R}^m$ is the state variable and the function $T\colon \mathbf{R}^m\to\mathbf{R}^m$ is given. Therefore, if the initial state $x(0)$ is known, then the future of the system can be computed. However, not only computing problems arise. For example, if $\bar{x}$ is a periodic point or equilibrium (i.e. $T(\bar{x})=\bar{x}$) then it is important to know whether or not it is stable. This means that $x(n)$ remains arbitrarily close to $\bar{x}$ for all n if $x(0)$ is sufficiently close to $\bar{x}$. As is known, the stability theory for the continuous processes (for the differential equations) has been developed by A. M. Lyapunov. LaSalle has established the corresponding theory for difference equations. During this extension a great number of deep questions were to be solved, and the new theory is interesting and useful not only for those dealing with discrete processes but also for mathematicians interested in differential equations.

The second part of the book is devoted to the control system $(x(n+1)=Ax(n)+f(n))$, where the matrix A is given, f is the control function. This model often appears in controlling vehicles, economy, illnesses, epidemics, populations, floods, crime, manufacturing processes, etc. The book is concluded by the stabilization by feedback.

The book was published posthumously with the assistance of Kenneth Meyer, one of the students of LaSalle.

Anyway, this monograph also has the characteristic feature of every LaSalle's book and paper: it gives a very clear and plastic presentation of a sophisticated theory, which is enjoyable and useful equally for students, users of mathematics and mathematicians.

László Hatvani (Szeged)

Tamás Matolcsi, A Concept of Mathematical Physics (Models in Mechanics), 335 pages, Akadémiai Kiadó, Budapest, 1986.

This is a continuation of the author's monograph "A Concept of Mathematical Physics, Models for Space-Time" published in 1984. The notations and results of that monograph are used and referred to throughout this volume.

The author sets forward his program in the introduction: "The modelling of some sort of physical phenomena means a construction of a *category*. The objects of the category are the models and we require that there be no morphisms between the models of different physical phenomena, there be morphisms between models of similar phenomena and two models be isomorphic if and only if the modelled phenomena are physically identical."

In this book he presents mathematical models of mechanical phenomena. The models of classical and quantum mechanics (nonrelativistic and special relativistic) presented here are based on a consistent application of the basic principles of *covariance* and *relativity*. The construction of mechanical models takes up the first half of the book, the second half is devoted to mathematical tools. Among the topics touched upon in the second part of the book are the following: probability theory on subset lattices and Hilbert lattices, star algebras, elements from functional analysis and from the theory of group representations, representations of space-time groups, basic notions concerning symplectic manifolds and Poisson brackets.

In this monograph the material is treated from a uniform viewpoint of principle. This book is not an easy reading but it is well worth studying for everybody interested in its subject.

L. Gy. Fehér (Szeged)

Kazuo Murota, Systems Analysis by Graphs and Matroids, Structural Solvability and Controllability, (Algorithms and Combinatorics, Volume 3), 281 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

This monograph is devoted to the study of the structural analysis of a system of linear/nonlinear equations and the structural controllability of a linear time-invariant dynamical system. The outline of the contents of this book is as follows:

In the first Chapter mathematical preliminaries are given. Basic results in graph theory and matroid theory are mentioned and some useful relevant theorems as the Dulmage—Mendelsohn decomposition of bipartite graphs are shown. This chapter presents some results on the submodular functions as well.

Chapter two is devoted to a graph-theoretic method for the structural analysis of a system of equations. First the structural solvability of a system of equations is formulated. The L-decomposition and the M-decomposition of graphs are introduced in connection with Menger-type linkings, together with their applications to the hierarchial decomposition of a system of equations into smaller subsystems.

Chapter 3 presents graph-theoretic conditions to the structural controllability of a linear dynamical system expressed in the descriptor: $F \cdot dx/dt = Ax + Bu$. Some known results on controllability condition of a descriptor system are mentioned, too. Various descriptions of a dynamical system are compared from the viewpoint of structural analysis.

Physical observations are made for providing the physical basis for the more elaborate and faithful mathematical models adapted in the second half of the book. It is explained in Chapter 4 that two different kinds are to be distinguished among the nonvanishing numbers characterizing real-word systems. Algebraic implications motivate the introduction of "mixed matrix" and "physical matrix".

In Chapter 5 a matroid-theoretic method is developed for the structural analysis of a system of equations. The rank of a mixed matrix is characterized, and an efficient algorithm for computing is described. Matroidal conditions are given to the structural solvability under the refined formulation.

In the last Chapter a structural controllability of a dynamical system is investigated. The dynamical degree is characterized in connection with the independent-flow problem. Relations to other works are mentioned.

G. Galambos (Szeged)

Stefan Pokorski, Gauge Field Theories (Cambridge Monographs on Mathematical Physics), XIV+394 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1987.

This new volume in the authorative series Cambridge Monographs on Mathematical Physics deals with physical and technical aspects of gauge theories.

The author first presents an overview of the standard $SU(3) \times SU(2) \times U(1)$ model, then he gives a short introduction to (path integral formulation of) perturbative quantum field theory and Feynman rules for Yang—Mills theories. In the following there is a careful discussion of the renormalization program. Separate chapters are devoted to quantum electrodynamics, renormalization group techniques and quantum chromodynamics. The book contains a detailed examination of global and gauge symmetries and their breaking schemes. The important topics of chiral symmetry, its breaking and chiral anomalies are also treated in detail. A fair amount of space is given to questions concerning scale invariance and low energy effective Lagrangians. The last chapter contains a discussion of basic elements of supersymmetric field theory.

The author presented here an extraordinarily wealthy material on theoretical methods and computational techniques of gauge field theories underlying our present understanding of elementary particle phenomena. The book is clearly written and practically self-contained, the reader is only assumed to have some familiarity with standard quantum field theory in its canonical formulation. Consequently, this book is warmly recommended to every research worker and graduate student interested in modern developments of gauge theories.

L. Gy. Fehér (Szeged)

George Pólya, The Pólya Picture Album: Encounters of a Mathematician. Edited by G. L. Alexanderson, 160 pages, Birkhäuser, Boston—Basel, 1987.

Imagine Albert Einstein, "young and good looking, not the Einstein we usually see", and young Lisi Hurwitz, whom you don't know, playing the violin as a duet and Adolf Hurwitz whom of course you know playfully conducting with a drumstick. This is the cover photo of this most enjoyable selected personal picture album. Then picture yourself to be conducted and guided through his album by Uncle George Pólya himself, at his best humour, describing the people or the occasion you see, relating the pictures to each other, and telling stories and anecdotes most charmingly, with intelligence and wit, and with obvious fondness towards all these people even if the story has a mild edge. This is exactly what you get in this book, a guided tour through the Pólya album by the late Professor Pólya, a nice afternoon in Palo Alto, California. His words were taped and transcribed. Therefore, the advanture is very intimate. Most, but not all, of the stories from Pólya's famous lecture "Some mathematicians I have known" [*Amer. Math. Monthly* 76(1969), 746—753] are told again, some of them almost verbatim (he must have told them many times), but there are quite a number of new ones, new at least to the reviewer, like the one about the absent-mindedness of Paul Lévy, or Pringsheim's remark that "Rosenthal was just a special case of Blumenthal".

Of necessity, the book is rather Magyar. The editor's care in using proper Hungarian first names and especially in accenting without an error deserves special mention. All the more so that such a care, an elementary courtesy, seems to have died out with the generations of the Pólyas.

The nicest things are of course the pictures themselves. One notices that quite a few of the photos in the illustrated history of the International Mathematical Congresses by D. A. Albers, G. L. Alexanderson and C. Reid [Springer-Verlag, New York, 1987; a review of which is in these *Acta* 51 (1987), p. 503] were in fact taken from Pólya's album. The present book has an introduction (pp. 7—8), a really intelligent biographical sketch of Pólya by the editor (pp. 9—22), and the photos with Pólya's accompanying remarks take the pages 23—155. A useful index of names completes the album. Any decent mathematics library will want to have a copy of it. It would still be better just to leave a copy in the coffee lounge or mail room of the Department of Mathematics.

Slips of the memory make narratives more authentic. The following nice little contradiction (or is it really a contradiction?) was left without remark by the editor. Probably this was intentional, and if so, then rightly so. On page 78 Pólya says: "And here are the Nevanlinnas and myself. These pictures were taken in Switzerland the year Nevanlinna came to take Weyl's place at the ETH, when Weyl went to Princeton to the Institute." On the other hand, on page 131 he says: "Ernst Völlm, myself, and Heinz Hopf in Switzerland, 1949. Hopf had replaced Weyl at the ETH when Weyl went to the Institute at Princeton." Was Nevanlinna declined in Zürich? Did he just go there to take the place but did not like it? Or, who took Weyl's place?

Sándor Csörgő (Szeged)

Lothar Sachs, A Guide to Statistical Methods and to the Pertinent Literature. Literatur zur Angewandten Statistik, XI+212 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1986.

About 5500 statistical key words and phrases are arranged in alphabetical order, a smaller portion of which is in German. To each entry reference numbers are assigned which represent 1449 papers and books from the statistical literature listed also in alphabetical order. The orientation is very much toward applications. Although the book cannot compete with recent encyclopedic works, it may prove to be useful to practicing applied statisticians and to research workers from many fields who use statistical methods as a quick and handy guide.

Sándor Csörgő (Szeged)

Robert I. Soare, Recursive Enumerable Sets and Degrees (Perspectives in Mathematical Logic), XVIII+437 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

The study of computable functions and computably generated (or recursively enumerable) sets of numbers goes back to the 1930's when Gödel proved his Incompleteness Theorem and Church, Gödel, Kleene, Post and Turing formulated several versions of computability. Since then recursion theory has become one of the basic parts of mathematical logic.

A classical topic, initiated by Post, deals with the classification of sets of integers into "degrees" on the basis of how difficult it is to compute them. Two sets are said to belong to the same class called degree or degree of unsolvability if they are "equally difficult to compute" and degrees are partially ordered by the relation "is more difficult to compute than". Degree theory studies this structure.

This book mainly deals with the degree theory of r.e. degrees, i.e. degrees that contain an r.e. set (and is complemented by M. Lerman: Degrees of Unsolvability (1983), also in the Omega Series).

Part A contains an introduction to recursion theory (computable functions, r.e. sets, reducibilities, complete and creative sets, the recursion theorem, the jump operator, the arithmetical hierarchy, etc.).

The latter parts contain more advanced results.

Part B describes Post's initial problem (are there more than two r.e. degrees?), the initial results of Post and Kleene—Post, simple, hypersimple, hyperhypersimple sets and the solution of Friedberg and Muchnik to Post's problem explaining the fundamental finite injury priority method.

Part C explains the infinite injury priority method and gives deeper results about the upper semi-lattice of r.e. degrees such as the Density Theorem of Sacks, the theorem of Lachlan and Yates about the existence of minimal pairs, a theorem of Lachlan about nonbranching degrees and many others. It also discusses another important structure on r.e. sets: the lattice of r.e. sets formed under inclusion, proving e.g. splitting theorems and the existence of maximal sets. The relationship between the two structures is also considered (e.g. the connections between high degrees and maximal sets). The final chapter deals with index sets, e.g. the Index Set Theorem of Yates.

Part D contains more recent results which already lead toward current research. The topics include promptly simple degrees, priority arguments even more powerful (and more complicated) than the previous ones (leading to a proof of Zachlan's Nonbounding Theorem) and Soare's theorem about the automorphisms of the lattice of r.e. sets. The last chapter contains most recent work such as the unsolvability results of Herrmann, Harrington and Shelah (without proofs) and a valuable collection of open problems.

The book, written by one of the main researchers of the field, gives a complete account of the theory of r.e. degrees. Without requiring any preliminaries, the author set up and realized the aim to "bring the reader to the frontiers of current research" which is even more to be appreciated considering the high stage of development of the field. The definitions, results and proofs are always clearly motivated and explained before the formal presentation; the proofs are described with remarkable clarity and conciseness.

The book is highly recommended to everyone interested in logic. It also provides a useful background to computer scientists, in particular to theoretical computer scientists. Reading the book, one can agree with the author who points out similarities between the beauty of this field of mathematics and the art of the Renaissance. It can be added that his book reflects this beauty.

Zoltán Fülöp—György Turán (Szeged)

Richard J. Trudeau, The Non-Euclidean Revolution, XIII+269 pages with 257 Illustrations, Birkhäuser, Boston—Basel—Stuttgart, 1987.

There is a more than 2000-year-old controversy whether the Euclidean geometry is the true description of the physical world. This philosophical and mathematical debate climaxed in the first half of the last century with the invention of the non-Euclidean geometry. As a result of this "new" geometry, from the second half of the 19th century mathematicians and scientists changed the way they viewed their subject. This was a real scientific revolution. R. J. Trudeau considers it as significant as the Copernican revolution in astronomy, the Newtonian revolution in physics or as the Darwinian revolution in biology.

According to the author's aim this book proceeds on three levels. On the first this is a book on plane geometry (both Euclidean and hyperbolic) with extra material on history and philosophy. On the second this is a book on a scientific revolution, and on the third level this book is about the possibility of significant, absolute certain knowledge about the world.

To read this very interesting and enjoyable book only a sound knowledge of high school (secondary school) geometry is needed. In the first chapter we can read on the origin of the deductive geometry, and on introduction to the axiomatic method. The second chapter deals with Euclidean geometry. The short Chapter 3, entitled Geometry and the Diamond Theory of Truth contains philosophical material. In Chapter 4 we can read about the attempts to prove or disprove Postulate 5 of Euclid. The next two chapters deal with the possibility of the non-Euclidean geometry and the hyperbolic geometry. In Chapter 7 we can read about consistency questions. The last chapter deals with the question of truth. Almost every chapter ends with exercises and notes.

This well organized material is warmly recommended to the wide mathematical community, especially to the teachers of mathematics. We share Felix Klein's view on the non-Euclidean geometry (it can be read in the Introduction written by H. S. M. Coxeter), who described it as "one of the few parts of mathematics which is talked about in wide circles, so that any teacher may be asked about it at any moment."

Lajos Klukovits (Szeged)

J. Wloka, Partial Differential Equations, XI+518 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1987.

This is the English translation of the successful textbook in German on the abstract theory of partial differential equations. A modern approach to this theory needs many sophisticated concepts and methods, so it is the cardinal problem of writing a self-contained text on it to find a good proportion of the cited and detailed prerequisites from the functional analysis. The book establishes a good balance in this respect. The reader should be familiar with the language and basic theorems of functional analysis relevant for analysis, but the less familiar material, such as the theory of Fredholm operators, Gelfand triples, abstract Green solution operators, the Schauder fixed point theorem and Bochner integral are thoroughly considered in separate sections.

The first chapter is an excellent introduction to the theory of distribution and Sobolev spaces working with the Fourier transformation. The second and third chapters give the principal part of the book. In the second chapter the Lopatinskiĭ—Šapiro condition and theorems on the index of elliptic boundary value problems are treated. It is a good choice that the L. Š. condition is formulated as an initial value problem for ordinary differential equations and not algebraically as a "covering condition". The third chapter is devoted to the strongly elliptic differential operators and the method of variations. In the fourth and fifth chapter those parabolic and hyperbolic equations are considered, respectively, for which the right-hand side, i.e. the derivatives with respect to the spatial variables is an elliptic differential operator. The sixth chapter gives a brief account on the difference method for the numerical solution of the elliptic equations and the wave equation.

This well-written book is recommended to graduate students, physicists and mathematicians interested in differential equations and mathematical physics.

László Hatvani (Szeged)

H. P. Yap, Some Topics in Graph Theory (London Mathematical Society Lecture Note Series, 108) 230 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1986.

The author of this book gave an optional course on Graph Theory to Fourth Year Honours students of the Department of Mathematics, National University of Singapore in the academic year 1982/83. This book has grown out from these lectures. It is not only suitable for using as a supplement to a course text at advanced undergraduate or postgraduate level but very useful to researchers in Graph Theory, too. The book consists of five chapters.

Each main part gives an up-to-date account of a particular topic in Graph Theory which is very active in current research. After the introduction and basic terminology the four main topics are Edge-colourings of Graphs (Chapter 2), Symmetries in Graphs (Chapter 3), Packing of Graphs (Chapter 4) and Computational Complexity of Graph Properties (Chapter 5).

In Chapter 2 after a few basic and important theorems for chromatic index Dr. Yap gives several properties of "so-called critical graphs". The author produces several methods for constructing critical graphs and counterexamples to the Critical Graph Conjecture. The main results of this chapter have been proved by Vizing, Fiorini, Yap, Gol'dberg and others.

"The investigation of symmetries of a given mathematical structure has always yield the most powerful results" wrote E. Artin. Chapter 3 studies various general properties of vertex — or edge — transitiv graphs and their automorphism groups. The author write Weiss' elegant proof of Tutte's famous theorem on S-transitiv cubic graphs. There are several theorems for Cayley graphs, and the author discusses some progress made towards the resolution of Lovász' question which asks whether or not every connected Cayley graph is Hamiltonian.

Packing of graphs is a NP-hard problem for arbitrary graphs, but for trees there exist polynomial time algorithms. The author presents several results for trees and small size graphs. The proof or disproof of Tree Packing Conjecture, Ringel's, Erdős and Sós', Bollobás and Eldridge's Conjecture wait for research workers.

A graph property is "elusive" if it cannot be found without all information of a hypothetical graph.

The connectedness and planarity are elusive properties. The main object of last chapter is to introduce a Two Person Game to tackle the problem whether or not a graph property is elusive.

Each chapter contains numerous examples, exercises and open problems for the reader.

Zoltán Blázsik (Szeged)