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Contractions quasisimilar to an isometry

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1. Introduction. The bounded linear operators T_1 and T_2 on complex, separable Hilbert spaces $\mathfrak{H}_1$ and $\mathfrak{H}_2$ are *quasisimilar* ($T_1 \sim T_2$) if there are operators $X\colon \mathfrak{H}_1 \to \mathfrak{H}_2$ and $Y\colon \mathfrak{H}_2 \to \mathfrak{H}_1$ with trivial kernel and dense range such that $XT_1 = T_2X$ and $YT_2 = T_1Y$. This paper is concerned with the question when a contraction is quasisimilar to an isometry. This problem has been studied before: in [12] for contractions with finite defect indices, [5] for subnormal contractions and [15] for hyponormal contractions. Our main result in this paper (Theorem 2.7) generalizes all these previous ones. We show that a contraction T whose $C_{\cdot 0}$ part has finite multiplicity is quasisimilar to an isometry if and only if its $C_{\cdot 1}$ part is of class C_{11} and its $C_{\cdot 0}$ part is quasisimilar to a unilateral shift. These latter conditions can further be expressed in terms of the inner and outer factors of the characteristic function of T. In § 3, we show that in certain circumstances quasisimilarity to an isometry even implies unitary equivalence and partially verify a conjecture we proposed in [15].

Recall that a contraction T ($\|T\| \leq 1$) is of *class* $C_{\cdot 0}$ (resp. $C_{0 \cdot}$) if $T^{*n}x \to 0$ (resp. $T^n x \to 0$) for all x; T is of *class* $C_{\cdot 1}$ (resp. $C_{1 \cdot}$) if $T^{*n}x \not\to 0$ (resp. $T^n x \not\to 0$) for all $x \neq 0$. $C_{\alpha\beta} = C_{\alpha \cdot} \cap C_{\cdot \beta}$ for $\alpha, \beta = 0, 1$. Any contraction T can be uniquely triangulated as $T = \begin{bmatrix} T_1 & * \\ 0 & T_2 \end{bmatrix}$, where T_1 and T_2 are of classes $C_{\cdot 1}$ and $C_{\cdot 0}$, respectively (called the $C_{\cdot 1}$ and $C_{\cdot 0}$ *parts* of T). A contraction T can also be decomposed as $U \oplus T'$, where U is a unitary operator and T' is completely nonunitary (c.n.u.); U and T' are called the *unitary part* and *c.n.u. part* of T. T is said to be of *analytic type* if it has no singular unitary direct summand. For such T, the functional calculus, $\varphi(T)$ for $\varphi \in H^\infty$ is well-defined. For the details and other properties of contractions, readers are referred to Sz.-Nagy and Foiaş' book [7].

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Let T_1 and T_2 be operators on $\mathfrak{H}_1$ and $\mathfrak{H}_2$, respectively. We use $T_1 \overset{d}{\prec} T_2$ to denote that there is an operator X: $\mathfrak{H}_1 \to \mathfrak{H}_2$ with dense range and satisfying $XT_1 = T_2X$, and $T_1 \prec T_2$ if the intertwining X is both injective and with dense range (called a *quasiaffinity*). $T_1 \overset{d}{\sim} T_2$ if $T_1 \overset{d}{\prec} T_2$ and $T_2 \overset{d}{\prec} T_1$; $T_1 \sim T_2$ if $T_1 \prec T_2$ and $T_2 \prec T_1$. T_1 is *similar* to T_2 ($T_1 \approx T_2$) if the intertwining operator X is invertible; T_1 is *unitarily equivalent* to T_2 ($T_1 \cong T_2$) if X is unitary. The *multiplicity* μ_T of an operator on $\mathfrak{H}$ is the minimum cardinality of a set $\mathfrak{K} \subseteq \mathfrak{H}$ for which $\mathfrak{H} = \bigvee_{n=0}^{\infty} T^n \mathfrak{K}$. Note that $T_1 \overset{d}{\prec} T_2$ implies $\mu_{T_1} \geqq \mu_{T_2}$. In the following, we use S_n to denote the unilateral shift with multiplicity n acting on H_n^2.

2. Main results. We start with the following proposition.

Proposition 2.1. *Let T be a contraction on $\mathfrak{H}$ and $1 \leqq n < \infty$. Then $T \sim S_n$ if and only if $T \overset{d}{\sim} S_n$. Moreover, in this case, T is of class C_{10}, and there exist quasiaffinities X: $\mathfrak{H} \to H_n^2$ and Y: $H_n^2 \to \mathfrak{H}$ which intertwine T and S_n and such that $XY = \delta(S_n)$ and $YX = \delta(T)$ for some outer function δ in H^∞.*

Proof. Assume that $T \overset{d}{\sim} S_n$. We first show that T is of analytic type. Let $T = U_s \oplus T'$ on $\mathfrak{H} = \mathfrak{H}_1 \oplus \mathfrak{H}_2$, where U_s is a singular unitary operator and T' is a contraction of analytic type, and let $Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$: $H_n^2 \to \mathfrak{H} = \mathfrak{H}_1 \oplus \mathfrak{H}_2$ be an operator intertwining S_n and T and with dense range. Then Y_1 intertwines S_n and U_s and has dense range in $\mathfrak{H}_1$. It can be lifted to an operator $\tilde{Y}_1$ which intertwines the minimal unitary extension U of S_n and U_s (cf. [4, Corollary 5.1]). Since U is absolutely continuous and U_s is singular, $\tilde{Y}_1$ must be the zero operator (cf. [4, Theorem 3]). Hence $Y_1 = 0$ and it follows that $T = T'$ is of analytic type.

Let X: $\mathfrak{H} \to H_n^2$ be an operator intertwining T and S_n and with dense range. Then XY commutes with S_n and has dense range in H_n^2. We may assume that $\|XY\| \leqq 1$. Thus XY is the operator Φ_+ of multiplication by a contractive operator-valued analytic function Φ on H_n^2 which is even outer (cf. [7, Lemma V.3.2]). By [7, Proposition V.6.1], Φ has a scalar multiple $\delta \in H^\infty$: there exists another contractive analytic function Ω such that $\Omega(\lambda)\Phi(\lambda) = \delta(\lambda)I$ and $\Phi(\lambda)\Omega(\lambda) = \delta(\lambda)I$ ($|\lambda| < 1$). Since Φ is an outer function, we may take δ to be outer (cf. [7, Theorem V.6.2]). Let $Z = \Omega_+ X$. Then Z intertwines T and S_n and $ZY = (\Omega_+ X)Y = \Omega_+ \Phi_+ = \delta(S_n)$. Multiplying both sides by Y, we obtain $YZY = Y\delta(S_n) = \delta(T)Y$ (here we need the fact that T is of analytic type). Since Y has dense range, we infer that $YZ = \delta(T)$. Note that δ is outer implies that $\delta(S_n)$ and $\delta(T)$ are quasiaffinities (cf. [7, Proposition III.3.1]). It follows easily that X, Y and Z are all quasiaffinities. This shows that $T \sim S_n$. That T is of class C_{10} can be easily deduced.

Corollary 2.2. *Let T be a contraction of analytic type and $1 \leqq n < \infty$. Then $T \sim S_n$ if and only if $\mu_T = n$ and $T \overset{d}{\prec} S_n$.*

Proof. The assertion follows from Proposition 2.1 and the fact that $\mu_T = n$ implies that $S_n \overset{d}{\prec} T$ (cf. [15, Lemma 2.3]).

When T is subnormal, the preceding corollary was essentially proved by HASTINGS [5, Proposition 4.1]. For another set of conditions in order that $T \sim S_n$, compare [1, Theorem 2.8].

Corollary 2.3. *Let $T = \begin{bmatrix} T_1 & * \\ 0 & T_2 \end{bmatrix}$ on $\mathfrak{H} = \mathfrak{H}_1 \oplus \mathfrak{H}_2$ be a contraction of analytic type. If T_1 is not missing and $T_2 \overset{d}{\prec} S_n$, then $\mu_T \geqq n+1$.*

Proof. Since $\mu_T \geqq \mu_{T_2} \geqq n$, we may assume that $n < \infty$. Let $X\colon \mathfrak{H}_2 \to H_n^2$ be an operator intertwining T_2 and S_n and with dense range. Let $Y = [0\ X]\colon \mathfrak{H} = \mathfrak{H}_1 \oplus \oplus \mathfrak{H}_2 \to H_n^2$. Then Y intertwines T and S_n and has dense range. If $\mu_T = n$, then $T \sim S_n$ by Corollary 2.2 and so by the proof of Proposition 2.1 Y is injective, which implies that $\mathfrak{H}_1 = \{0\}$, a contradiction. Hence we have $\mu_T \geqq n+1$.

The next theorem characterizes those contractions which are quasisimilar to a unilateral shift with finite multiplicity in terms of their characteristic functions. It generalizes [12, Lemma 1] for contractions with finite defect indices. For any contraction T, let Θ_T denote its characteristic function (consult [7] for its definition and properties).

Theorem 2.4. *Let T be a contraction and $1 \leqq n < \infty$. Then $T \sim S_n$ if and only if T is of class C_{10}, $\mu_T = n$ and there exists a bounded analytic function Ω such that $\Omega \Theta_T = \delta I$ for some outer function δ in H^∞.*

Proof. Assume that $T \sim S_n$. It is easily seen that T is of class C_{10} whence c.n.u. We may consider its functional model, that is, consider T acting on $\mathfrak{H} = H^2_{\mathfrak{D}_*} \ominus \ominus \Theta_T H^2_{\mathfrak{D}}$ by $Tf = P(e^{it} f)$ for $f \in \mathfrak{H}$, where $\mathfrak{D} = \overline{\operatorname{ran}(I - T^*T)^{1/2}}$, $\mathfrak{D}_* = \overline{\operatorname{ran}(I - TT^*)^{1/2}}$ and P denotes the orthogonal projection onto $\mathfrak{H}$ (cf. [7, Proposition VI.2.1]). By Proposition 2.1, there exist quasiaffinities $X\colon \mathfrak{H} \to H_n^2$ and $Y\colon H_n^2 \to \mathfrak{H}$ which intertwine T and S_n and satisfy $XY = \delta(S_n)$ and $YX = \delta(T)$ for some outer function δ in H^∞. Note that $Xf = \Phi f$ for $f \in \mathfrak{H}$ and $Yg = P(\Psi g)$ for $g \in H_n^2$, where Φ and Ψ are bounded analytic functions satisfying $\Phi \Theta_T = 0$ (cf. [7, Theorem VI.3.6]). From $XY = \delta(S_n)$ and $YX = \delta(T)$, we deduce that $\Phi \Psi = \delta$ and $\Psi \Phi - \delta = -\Theta_T \Omega$ for some bounded analytic function Ω. Since Θ_T is an inner function (cf. [7, Proposition VI.3.5]), we have

$$\Omega \Theta_T - \delta = \Theta_T^* \Theta_T (\Omega \Theta_T - \delta) = \Theta_T^* (\Theta_T \Omega - \delta) \Theta_T = \Theta_T^* (-\Psi \Phi) \Theta_T = 0.$$

Therefore $\Omega\Theta_T=\delta I$ as required. The reverse implication follows as in the proof of [12, Lemma 1].

Using Proposition 2.1, Theorem 2.4 and [14, Theorem 2.1], we can obtain the following interesting result.

Theorem 2.5. *Let* $T=\begin{bmatrix} T_1 & * \\ 0 & T_2 \end{bmatrix}$ *be a contraction. If* $T_2 \overset{d}{\sim} S_n$ *for some* $1 \leq n < \infty$, *then* $T \sim T_1 \oplus T_2$.

Proof. If T is c.n.u., then the conclusion follows from the results cited above. For general T, let $T=U\oplus T'$ on $\mathfrak{H}=\mathfrak{K}\oplus\mathfrak{L}$, where U is unitary and T' is c.n.u. Assume that $T=\begin{bmatrix} T_1 & * \\ 0 & T_2 \end{bmatrix}$ is acting on $\mathfrak{H}=\mathfrak{H}_1\oplus\mathfrak{H}_2$. We first check that $\mathfrak{H}_2\subseteq\mathfrak{L}$. Since $T_2 \overset{d}{\sim} S_n$ implies that T_2 is of class C_{10} by Proposition 2.1, for any $x\in\mathfrak{H}_2$, we have $T^{*m}x=T_2^{*m}x\to 0$ as $m\to\infty$. If $x=x_1\oplus x_2$, where $x_1\in\mathfrak{K}$ and $x_2\in\mathfrak{L}$, then $U^{*m}x_1\to 0$. Since U is unitary, this implies that $x_1=0$ and thus $x=x_2\in\mathfrak{L}$. This proves $\mathfrak{H}_2\subseteq\mathfrak{L}$ which is equivalent to $\mathfrak{K}\subseteq\mathfrak{H}_1$. It is easily seen that

$$T=\begin{bmatrix} U & 0 & 0 \\ 0 & T_1' & * \\ 0 & 0 & T_2 \end{bmatrix} \quad \text{on} \quad \mathfrak{H}=\mathfrak{K}\oplus(\mathfrak{H}_1\ominus\mathfrak{K})\oplus\mathfrak{H}_2.$$

Since $\begin{bmatrix} T_1' & * \\ 0 & T_2 \end{bmatrix}$ is c.n.u., from above we have $\begin{bmatrix} T_1' & * \\ 0 & T_2 \end{bmatrix}\sim T_1'\oplus T_2$ and therefore $T\sim$ $\sim U\oplus T_1'\oplus T_2=T_1\oplus T_2$.

We remark that it is unknown whether the preceding theorem is still valid under $n=\infty$, that is, when $T_2 \overset{d}{\sim} S_\infty$ or $T_2\sim S_\infty$. In a very special case, this is indeed true.

Theorem 2.6. *Let* $T=\begin{bmatrix} T_1 & * \\ 0 & T_2 \end{bmatrix}$ *be a contraction. If* T_2 *is similar to an isometry, then* T *is similar to* $T_1\oplus T_2$.

Proof. If T is c.n.u., this follows from [8, Theorem 2.4] and [14, Theorem 2.1]. For the general case, assume that T_2 is similar to the isometry $V=W\oplus S_n$, where W is unitary and S_n is some unilateral shift. It is easily seen that T_2 can be triangulated as $\begin{bmatrix} T_3 & * \\ 0 & T_4 \end{bmatrix}$ with $T_3\approx W$ and $T_4\approx S_n$. Letting $T_5=\begin{bmatrix} T_1 & * \\ 0 & T_3 \end{bmatrix}$, we have $T=\begin{bmatrix} T_5 & * \\ 0 & T_4 \end{bmatrix}$. Since $T_4\approx S_n$, proceeding as in the proof of Theorem 2.5 we obtain $T\approx T_5\oplus T_4$. On the other hand, $T_3\approx W$ implies that $T_5\approx T_1\oplus T_3$ and $T_2\approx T_3\oplus T_4$ (cf. [9, Theorem 2.14]). Thus $T\approx T_1\oplus T_3\oplus T_4\approx T_1\oplus T_2$ as claimed.

Now we are ready for our main result.

Theorem 2.7. *Let* T *be a contraction and* $T=\begin{bmatrix} T_1 & * \\ 0 & T_2 \end{bmatrix}$ *be its triangulation of type* $\begin{bmatrix} C_{\cdot 1} & * \\ 0 & C_{\cdot 0} \end{bmatrix}$. *Assume that* $\mu_{T_2}<\infty$. *Then the following statements are equivalent:*

(1) *T is quasisimilar to an isometry;*

(2) *T_1 is of class C_{11} and T_2 is quasisimilar to a unilateral shift;*

(3) *Θ_e (the outer factor of Θ_T) is outer from both sides, Θ_i (the inner factor of Θ_T) is inner and $*$-outer, and there exists a bounded analytic function Ω such that $\Omega\Theta_i=\delta I$ for some outer function δ in H^∞.*

Moreover, if T is of analytic type and is quasisimilar to the isometry V, then there are quasiaffinities X and Y intertwining T and V such that $XY=\delta(V)$ and $YX=\delta(T)$.

Proof. (1)⇒(2): Assume that $T\sim V=U\oplus S_n$, where U is unitary. Since T_1 and U are of class $C_{\cdot 1}$ and T_2 and S_n are of class $C_{\cdot 0}$, we can easily deduce that $T_2\overset{d}{\sim}S_n$. This, together with $\mu_{T_2}<\infty$, implies that $T_2\sim S_n$ by Proposition 2.1. On the other hand, $T\prec V$ implies that T is of class $C_{1\cdot}$ whence T_1 is of class C_{11}.

(2)⇒(1): This follows from Theorem 2.5.

(2)⇔(3): Since Θ_e and Θ_i correspond to the characteristic functions of T_1 and T_2, respectively, this follows from Theorem 2.4 and [7, Proposition VI.3.5].

The assertion concerning the intertwining quasiaffinities can be deduced easily from [14, Theorem 2.1].

As we remarked in § 1, the preceding theorem generalizes [11, Theorem 3] for contractions with finite defect indices, [5, Corollary to Theorem 4.5] for subnormal contractions and [14, Corollary 3.11] for hyponormal contractions. An example of Hastings [5] shows that (1) may not imply (2) without the assumption $\mu_{T_2}<\infty$. It is interesting to contrast this theorem it with [10, Theorem 2] where "quasisimilarity" is replaced by "similarity" in which case $\mu_{T_2}<\infty$ won't be needed.

3. Some consequences. In this section, we will derive two results for which an operator quasisimilar to an isometry is even unitarily equivalent to it. More precisely, we show that if $V=U\oplus S_n$ is an isometry, where U is unitary and $n<\infty$, and T is a quasinormal operator or $T\in\operatorname{Alg} V$ (the weakly closed algebra generated by T and 1), then $T\overset{d}{\sim}V$ implies $T\cong V$. For the first one, we prove the following more general result.

Proposition 3.1. *If T is a contraction whose c.n.u. part is of class $C_{\cdot 0}$ and $V=U\oplus S_n$ is an isometry with $n<\infty$, then $T\overset{d}{\sim}V$ implies $T\sim V$.*

Proof. Let $T=U'\oplus T'$, where U' is unitary and T' is c.n.u. Since U' and U are of class $C_{\cdot 1}$ and T' and S_n are of class $C_{\cdot 0}$, we deduce from $T\overset{d}{\sim}V$ that $T'\overset{d}{\sim}S_n$. Thus $T'\sim S_n$ by Proposition 2.1 and therefore $T\sim U'\oplus S_n\overset{d}{\sim}V$. [15, Lemma 3.4] yields that $U'\oplus S_n\cong V$. Thus $T\sim V$ as asserted.

Corollary 3.2. *If T is a hyponormal operator and $V=U\oplus S_n$ is an isometry with $n<\infty$, then $T\overset{d}{\sim}V$ implies $T\sim V$.*

Proof. $T \overset{d}{\sim} V$ implies that their spectra are equal [2, Theorem 2], so are their spectral radii: $r(T)=r(V)$. Hence $\|T\|=r(T)=r(V)=1$ showing that T is a contraction. Now the assertion follows from Proposition 3.1 and the fact that c.n.u. hyponormal contractions are of class $C_{\cdot 0}$ [6].

Corollary 3.3. *If T is a quasinormal operator and $V=U\oplus S_n$ is an isometry with $n<\infty$, then $T\overset{d}{\sim} V$ implies $T\cong V$.*

Proof. By Corollary 3.2, we have $T\sim V$. For quasinormal T, this implies $T\cong V$ (cf. [15, Proposition 4.2]).

Now for our final result. In [15], we asked whether for isometry V, $T\in \operatorname{Alg} V$ and $T\sim V$ imply $T\cong V$, and showed that this is indeed the case if $T\approx V$ [15, Proposition 4.6]. We will now verify its validity when $V=U\oplus S_n$ with $n<\infty$. We start with the following. For any operator T, $T^{(n)}$ denotes the direct sum of n copies of T.

Lemma 3.4. *Let T be a contraction. If $T^{(n)} \overset{d}{\sim} S_n$ for some $1\leqq n<\infty$, then $T\sim S_1$.*

Proof. Since $\Theta_{T^{(n)}}=\Theta_T^{(n)}$, Proposition 2.1 and Theorem 2.4 imply that $T^{(n)}$ is of class C_{10} and there exists a bounded analytic function Ω such that $\Omega\Theta_T^{(n)}=\delta I$ for some outer function δ in H^∞. If Φ denotes the (1, 1)-entry of Ω, then $\Phi\Theta_T=\delta I$. Thus, by Theorem 2.4 again, $T\sim S_k$ for some $1\leqq k<\infty$. Since $S_n\sim T^{(n)}\sim S_{kn}$, we conclude that $k=1$ and $T\sim S_1$.

Proposition 3.5. *Let $V=U\oplus S_n$ be an isometry with $n<\infty$. If $T\in\operatorname{Alg} V$ and $T\overset{d}{\sim} V$, then $T\cong V$.*

Proof. Let $U=U_s\oplus U_a$, where U_s and U_a are singular and absolutely continuous unitary operators, respectively. In view of [15, Lemma 4.3], we may assume that V is not unitary. Hence $T\in\operatorname{Alg} V$ implies that $T=W\oplus\varphi(U_a\oplus S_n)$, where $W\in\operatorname{Alg} U_s$ and $\varphi\in H^\infty$ (cf. [13, Lemma 1.3] and [11, Lemma 3.11]). This shows that T is hyponormal and therefore $T\overset{d}{\sim} V$ implies, by Corollary 3.2, that $T\sim V$. If φ is a constant function, then T is normal whence $T\sim V$ implies that V is unitary, a contradiction. Thus φ is nonconstant and therefore $\varphi(S_n)$ is completely nonnormal (cf. [15, Lemmas 4.4 and 4.5]). Hence $T\sim V$ implies that $W\oplus\varphi(U_a)\cong U$ and $\varphi(S_n)\overset{d}{\sim} S_n$ by [5, Proposition 3.5]. We apply Lemma 3.4 to obtain that $\varphi(S_1)\sim S_1$. It follows from [3, Theorem 1] that $\varphi(S_1)\cong S_1$ whence $\varphi(S_n)\cong S_n$ and $T\cong V$ follows.

Added in proof. Takahashi [16] showed that for isometry V, $T\in\operatorname{Alg} V$ and $T\sim V$ imply $T\cong V$ which answered the question asked in [15].

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