

Bibliographie

G. M. Adelson-Velsky—V. L. Arlazarov—M. V. Donskoy, Algorithms for Games, X+197 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1988.

The original Russian edition was published in 1978. Though the progress in the development of computer chess programs has been rapid, present translation is still interesting since it deals with the basic ideas of the research problems rather than with specific programs and programming techniques.

The book consists of four chapters. Chapter 1 is devoted to a description of a two-person game with complete information. It contains the definition of the game tree and the position score furthermore brand- and- bound method of searching for the best move in a position. Some simple theorems on the decomposition of a game tree are proved as well as theorems on valuation of positions for finding the best moves.

Chapter 2 is devoted to heuristic methods for choosing a move in a contemplated position. A probabilistic method is used for justifying a heuristic algorithm. Shannon's model with the concept of evaluation function and depth of the search are introduced.

Chapter 3 (entitled The Method of Analogy) is devoted to the definition and study of moves which are independent of positions (analogous positions) having thus analogous consequences in different positions. In Chapter 4 (Algorithms for Games and Probability Theory) constructions of probabilistic models for two-person games and calculation of model scores on a probabilistic basis are investigated.

The authors of the book had written the program of KAISSA which won the First International Championship for Chess Programs in 1974.

In the Appendix the reader finds a list of chess programs which took part in 1974—1977 competitions, and an interesting historical survey of game programming of computer age up to 1978.

Zoltán Blázsik (Szeged)

Brian A. Barsky, Computer Graphics and Geometric Modeling Using Beta-splines (Computer Science Workbench) IX+156 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988.

Specialist of β -splines B. A. Barsky gives the following conception of this book in the Introduction: "The underlying concept of this work is the synthesis of two useful concepts: the application of tension to a shape; and the study of a parametrically defined shape as fundamental geometric measures." The whole method is based on considering the continuity of these two differential geometric notions, which are of basic importance in the investigation of their geometric shape.

Even the reader who is unfamiliar with the theory of curves and surfaces can easily catch ideas of the considerations concerning continuity of the unit tangent and curvature vectors of a

curve given by a piece-wise representation. The visualization problems of these concepts by computer graphic methods are introduced in a clear way, as well.

As to the technical applications, the most important feature of the β -spline curves and surfaces is that the base points render local control possibility. The shape of the curve and the surface can be modified locally, furthermore the so called shape parameters determine the "tightness" and "looseness" of the curves and surfaces as they fit close the control polygon or surface.

Independent chapters are devoted to the cases of uniform, respectively continuously varying shape parameters. In the both cases a method is explained in detail for determination of the β -spline curve (surface). For this purpose the author shows REDUCE computer algebra system developed at the Department of Computer Science at the University of Utah. This is a perspicuous program for the evaluation of the unknown coefficient functions of β -spline curves and surfaces. Boundary conditions (respectively, end conditions) are analyzed in original chapters, including the problem of classification.

One of the greatest merits of the book is the excellent collection of figures and pictures. They help the reader to understand the concepts above more exhaustively, and demonstrate the effectiveness of β -spline representation. Especially remarkable are the figures analyzing the relations between control polygons (surfaces) and shape parameters, by the side of which the reader can see the synthetic image appearing on the monitor. Wide possibilities of β -splines are illustrated by nice colour pictures in the 19th chapter.

The book is recommended to readers interested in ability of β -spline technique. However, it should be a pleasure first of all for those mathematicians and computer scientists who want to deal with computer graphic and design problems. In the latter case, the summary in the 20th chapter with an outlook on further research directions; the enclosed Reduce programs in the Appendix, and the Bibliography on Curves and Surfaces including about 400 references are very useful.

József Kozma (Szeged)

M. Berger—B. Gostiaux, *Differential Geometry: Manifolds, Curves and Surfaces* (Graduate Texts in Mathematics, 115), XII+474 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988.

First of all we must sound out that this is an extremely good book. The observant readers must certainly find great pleasure in reading this book, because of its clear style and very nice setting up. Although one has to read this book to know its taste we try to say some words about it.

This book can be regarded as an enlarged and revised version of M. Berger's book "*Géométrie Différentielle*" (1972). In order to know something about the building up of this book it is worth to quote Berger's words about his aims in lectures he read in Paris in 1969—71 served as the basis of this beautiful book:

"First, to avoid making the statement and proof of Stokes' formula the climax of the course and running out of time before any of its applications could be discussed. Second, to illustrate each new notion with nontrivial examples, as soon as possible after its introduction. And finally, to familiarize geometry-oriented students with analysis and analysis-oriented students with geometry, at least in what concerns manifolds."

While the first nine chapters are based on the above mentioned book absolutely, the last two chapters are an attempt to remedy the notorious absence in the original book of any treatment of surfaces in three-space, an omission all the more unforgivable in that surfaces are some of the most common geometrical objects, not only in mathematics but in many branches of physics".

Although we have not detailed the book's contents we suppose these words above can take a fancy to reading of this book, hence we call again attention of everybody interested in differential geometry on graduate level or reading lectures about it to this well illustrated nice book.

Árpád Kurusa (Szeged)

K. H. Borgwardt, The Simplex Method. A Probabilistic Analysis, XI+268 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

Since the Simplex-Method was discovered by George B. Dantzig it has been in the central of interests of researches. One of the most interesting problem in connection with the Method is the discrepancy of its worst-case behaviour and the good practical behaviour of it.

This book — giving a comprehensive probabilistic analysis of the so-called Two-Phase Simplex Method — attempts to resolve this discrepancy.

After an extensive introduction in which the author overviews the known results of the probabilistic analysis of the Symplex Method including some theorems from the field of stochastic geometry, the papers of Smale and Haimowich and of course their own earlier results as well.

Because of the analysis is based on the shadow vertex algorithm, the first chapter reviews this algorithm. The next two sections deal with giving an upper bound for the average number of pivot step of the algorithm. The research culminates in the Theorem 6 of the Chapter 3 in which the author postulates that the average number of pivot steps ($E_{m,n}$) for the complete Simplex Method is polynomial, namely if we have m inequalities with n variables then

$$E_{m,n} = O(m^{1/(n-1)}n^4).$$

Chapter 4 studies the asymptotic average behaviour of the Simplex Method. (The author uses the term "asymptotic" in the sense that $m \rightarrow \infty$, and n is fixed.)

Upper bounds have been given in integral form, for certain classes of distributions including the uniform and the Gaussian distributions as well.

In the Chapter 5 the author introduces a modified version of the Two-Phase Simplex Method solving the so-called rotation invariant model with n additional nonnegativity constraints. It has been proved that the expected number of pivot steps of this algorithm is not greater than

$$m^{1/(n-1)}(n+1)^4 \frac{2}{5} \pi \left(1 + \frac{e\pi}{2}\right).$$

An Appendix including definitions and proofs for Gammafunction and Betafunction closes the book. The book is well-organized readable (in mathematical sence), but I have to mention that the lack of some definitions causes that the book is not absolutely "self-contained".

G. Galambos (Szeged)

CAAP '88, 13th Colloquium on Trees in Algebra and Programming, Proceedings, Nancy 1988. Edited by M. Dauchet and M. Nivat (Lecture Notes in Computer Science, 299), VIII+304 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1988.

This volume contains the proceedings of the 13th Colloquium on Trees in Algebra and Programming, held on March 21—24, 1988, in Nancy.

CAAP '88, following the tradition, is devoted to trees, which are a basic structure for Computer Science, and which are explicitly or implicitly studied in a lot of papers in this volume. But

CAAP '88 covers also a wider range of topics in Theoretical Computer Science: Algorithms and complexity on trees and other structures; Abstract data types and term rewriting; Logic, parallelism and concurrency.

We warmly recommend this book to everybody, working in Theoretical Computer Science.

Sándor Vágvölgyi (Szeged)

G. S. Campbell, *An Introduction to Environmental Biophysics*, XV + 159 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1986.

Environmental biophysics is a specialized branch at the borderline of physics and biophysics and mainly concentrates on energy exchange processes taking place in our environment exposed to solar radiation and variations in air humidity. The principal emphasis of the book is to present the differential equation formalism for mass and heat transfer and it gives an introduction into the mathematical physics of rate equations. Special attention has been paid to a quantitative analysis of energy balance using the continuity equation. Then it goes on to apply the general principles to selected examples and in the second part of the book the energy and mass transfer models are applied to exchange processes between organisms and their microenvironment. Throughout the book the basis principles are illustrated by several examples which are rather useful in gaining an understanding the subject. The illustrations are superb and rather useful additions to the text. This book is addressed to the physics and biophysics undergraduate student of conventional course background. The author does not review mathematical physics, but it is a useful supplement for those who have met the concepts in other courses. It can be used as a textbook of environmental biophysics or a supplementary reference source of classical mechanics for first year undergraduate courses. At the end of each chapter further problems are presented which can be very useful additions to conventional physics courses.

L. I. Horváth (Szeged),

Classic Papers in Combinatorics, Edited by Ira Gessel and Gian-Carlo Rota, X + 489 pages, Birkhäuser, Boston—Basel—Stuttgart, 1987.

Excellent papers from different fields of the combinatorics are presented in this collection. Without giving a complete enumeration on the contents of the book we give some significant results. From the Ramsey theory we meet the basic paper of Ramsey from 1930, the classical paper of Erdős and Szekeres (1935), the Erdős—Rado theorem on the partition calculus. The new results are represented by the Graham's, Leeb's, Rotschild's papers on the categorical underpinning of Ramsey theorem.

Withney's paper (1932) presents the first paper on the theory of matroids. Tutte's paper included in this book roots in the matroid theory. Classical papers are presented from the graph coloring (Brooks, Lovász). The matching theory represent 8 papers among them the opening papers of Hall (1935), Halmos (1958) and Dilworth. Here we can find the lot-cited papers of Ford and Fulkerson, Tutte's paper on factors of graphs and Edmonds's efficient matching algorithms.

One of the editors (Rota) has used Pólya's paper on picturewriting to establish the theory of Möbius functions. His work was extended by Crapo.

On the field of the extremal set theory the first paper is due to Katona. Clements's, Kruskal's, Kleitman's and Erdős's results are cited in this part.

Again an Erdős's paper on the probabilistic method is in the collection. This paper is very important since it helps to prove a lot of existence theorems in the graph theory. Lovász's contribution to the Ulam reconstruction problem is an ingenious use of the inclusion-exclusion principle.

It was a great pleasure of the referee that the Hungarian mathematicians who played significant role in this field of mathematics are present in this collection with a weight.

G. Galambos (Szeged)

Underwood Dudley, A Budget of Trisections, XV + 169 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1987.

From time to time every mathematical institute receives letters in which the authors "solve,, some famous problems. They prove the Fermat Conjecture, the Goldbach Conjecture, they duplicate the cube with compass and straightedge and so on. Numerous amateurs try the trisection of the angle. (This is impossible with straightedge and compass as was proved by P. L. Wantzel in 1837.) Archimedes trisected the angle using a compass and a straightedge with two scratches on it. This is a non-euclidean construction and you will find some more examples of this kind in the first chapter. The second and third chapters (Characteristics of Trisectors, Three Trisectors) enlighten the personalities of these amateurs. The fourth chapter contains the collection of trisections.

This book is a curious, extraordinary work. I have never seen anything similar to this.

Everyone can read it with minimal mathematical background. The author writes in the Introduction: "What follows, then, is something which has never been done before: it is an effort to do something which may be as impossible as trisecting the angle: namely to put an end to trisections and trisectors".

L. Pintér (Szeged)

Dynamics of Infinite Dimensional Systems, Edited by Shui-Nee Chow and Jack K. Hale (NATO ASI Series, Series F: Computer and Systems Sciences, 37), IX + 514 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

This is the Proceedings of the NATO Advanced Study Institute on Dynamics of Infinite Dimensional Systems, held in Lisbon, Portugal, May 19—24, 1986.

In recent years it has become a general method in the researches into partial differential equations (PDE's) and functional differential equations (FDE's) to consider these equations as dynamical systems on functional spaces. The purpose of this workshop was to bring together research workers from the various areas coming with several different backgrounds and interests. The papers investigate asymptotic behaviour of solutions (e.g. stability properties oscillation, bifurcation) for such equations as semilinear and nonlinear parabolic and elliptic PDE's integrodifferential equations dissipative systems, FDE's with finite and infinite delay, infectious disease model, wave equation and reaction diffusion system.

L. Hatvani (Szeged)

Foundations of Logic and Functional Programming, Proceedings, Trento 1986. Edited by M. Boscarol, L. Carlucci Aiello and G. Levi (Lecture Notes in Computer Science 306), IV + 218 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1988.

This volume contains ten papers presented at the workshop on "Foundations of Logic and Functional Programming" held in Trento, Italy, December 15—19, 1986.

The titles of invited contributions are: 1. C. Talcott: Rum. An intensional theory of function and control abstractions. 2. L. Cardelli: Typechecking dependent types and subtypes. 3. C. Böhm: Reducing recursion to iteration by means of pairs and N -tuples. 4. J.-L. Lassez, M. J. Maher and K. Marriott: Unification revisited. 5. C. Zaniolo and D. Sacca: Rule rewriting methods for efficient implementations of Horn logic.

The titles of submitted contributions are: 1. P. Miglioli, U. Moscato and M. Ornaghi: PAP: A logic programming system based on a constructive logic. 2. E. Giovannetti and C. Moiso: A completeness result for E -unification algorithms based on conditional narrowing. 3. N. Guarino: Representing domain structure of many-sorted Prolog knowledge bases. 4. A. D'Angelo: Horn: An inference engine prototype to implement intelligent systems. 5. E. G. Omodeo: Hints for the design of a set calculus oriented to Automated Deduction.

This book is recommended to everybody working in the theory of Logic and Functional Programming.

Sándor Vágvölgyi (Szeged)

M. Goresky—R. MacPherson, Stratified Morse Theory (Ergebnisse der Mathematik und ihrer Grenzgebiete), XIV + 272 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988.

This book consists of three parts and a nice introduction. This introduction makes absolutely clear basis for the three distinct subjects of three parts. The parts contain: a systematic exploration of the natural extension of Morse theory to include singular spaces; a large collection of theorems on the topology of complex analytic varieties; the calculation of the homology of the complement of a collection of flat subspaces of Euclidean space.

The only common thing in these parts is the application of the Morse theory, but we think the appearance of these subjects in one book was a very good and natural idea.

To end our review we establish that this book is very nice in its form, contents and also its getting-up. We are sure that it will become a fundamental book of its subject.

Árpád Kurusa (Szeged)

Martin Grötschel—László Lovász—Alexander Schrijver, Geometric Algorithms and Combinatorial Optimization, XII + 362 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988.

In spite of the fact that many of the most frequently used combinatorial algorithms were based on the discrete structure of the problems in the last several years geometric methods have played more significant role in combinatorial optimization.

In the focus of this book states the investigation of two geometrical algorithms: the ellipsoid method and the basis reduction. The first one has been developed by L. G. Khachiyan for linear programs and the authors examined it deeply in their earlier papers as well. The roots of the second method go back to Hermite and Minkowski, and it has been used for the polynomial time solvability of integer linear programming in fixed dimension by Tardos and H. W. Lenstra.

The first two sections of the book contain preliminaries. A list of the main problems (The Weak Optimization Problem, the Weak Violation Problem, the Weak Validity Problem, the Weak Separation Problem, the Weak Membership Problem) are introduced in Chapter 2. The next section contains the description of the ellipsoid method. Applications and specializations of the method

are collected in Chapter 4. The algorithms concerning the different characteristics of a convex set are approximations (because of the nature of the method).

The next two sections contain the basis reduction algorithm for lattices and its applications. Different combinations of the ellipsoid method with basis reduction are given for the programming in fixed dimension. The last four chapters contain further applications: Chapter 7 gives some basic examples, in Chapter 8 there is given a deep survey of the basis reductions. Specific fields of the application of the ellipsoid method are discussed in the finishing sections.

The book has a clear style. It may be a useful piece of reading not only for experts but for students as well.

G. Galambos

John L. Kelley—T. P. Srinivasan, Measure and Integral, Volume 1 (Graduate Texts in Mathematics, 116), X+150 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1988.

The measure and integral have been two basic notions of analysis and probability theory since their beginnings. Nowadays they play an important role also in other branches of pure and applied mathematics. This book is a systematic exposition of the theory of measure and integration emphasizing the part of the theory most commonly used in functional analysis.

The book consists of two kinds of text. The body of the text, requiring only a first course in analysis as a background, is a study of abstract measures and integrals. It establishes Borel measures and integration for R . The chapters are followed by supplements, which are more informal and present such parts of the theory as Borel measures and integration for R^n , integration for locally compact Hausdorff spaces, invariant measures for groups, Stieltjes integration, Haar measure, the Bochner integral.

The method of presentation differs from the standard one, namely, at first integrals are constructed, then measures are derived from them. The integral is extended to some R^* valued functions, and measures with R^* values; signed measures and indefinite integrals are also treated.

The well-written book can be highly recommended to mathematicians especially those dealing with functional analysis.

L. Harvani (Szeged)

Neal Koblitz, A Course in Number Theory and Cryptography (Graduate Texts in Mathematics), IV+208 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1987.

Since the fifties number theory has changed in an extremely rapid manner. Only a few decades ago this theory had no practical use. Gauss called it the "Queen of Mathematics". The results are interesting and sometimes surprising, the methods can be delightful, and nowadays there appear more and more new applications. A course is interesting and the publication of a book is justified if it has got some original distinguishing features. In my opinion the reader will enjoy this book. One of its characteristic features is the algorithmic approach, emphasizing estimates of the efficiency of the techniques. Cryptography is in the centre of the discussions. The inclusion of some very recent applications of the theory of elliptic curves seems to be originally new.

The first two chapters — Some topics in elementary number theory and Finite fields and quadratic residues — give a general background. Some of the proofs are omitted (one finds them in introductory textbooks). A characteristic (unusual) topic is the estimation of the number of bit operations needed to perform different tasks by computer.

The following four chapters—Cryptography; Public key; Primality and factoring; Elliptic curves — are similar to a fascinating novel. Especially the chapter on public key supplies astonishing novelties for the readers who are inexperienced in this theme. Let us cite the last sentences of this chapter from which consequences may be drawn on the discussion and further we can see that the book holds the benefits of a lecture: "At the present time there is no known polynomial time algorithm for solving the iterated knapsack problem, i.e., the public key cryptosystem described in the last paragraph. However, there are some promising approaches to generalizing Shamir's algorithm. It is not unlikely that intensive research on this problem would before long produce an efficient algorithm for breaking the iterated knapsack cryptosystem. In any case, most experts, traumatized by Shamir's unexpected breakthrough, do not have much confidence in the security of any public key cryptosystem of this type."

Several various exercises increase the interest of the work, answers and in more difficult cases solutions are given.

Although we can read on the cover: "No background in algebra or number theory is assumed", however, in my opinion the reader needs some experience in the theory and in this case she/he can find great enjoyment in this text and very much of it indeed.

L. Pintér (Szeged)

Max Koecher, *Klassische elementare Analysis*, 211 pages, Birkhäuser Verlag, Basel—Boston, 1987.

The text is divided into six parts. Chapter 1 is a preparatory part the main idea of which is the investigation of the connection between the classical golden section problem, the Fibonacci numbers and continued fractions. An algebraic application of golden section is also given. Chapter 2 introduces the notions of convergence of sequences and series of real numbers. In Chapter 3 the Riemann integral is defined, the integration methods are acquainted and at the end of the chapter the logarithm function as an integral and its inverse, the exponential function are introduced. Chapter 4 is devoted to algebraic and number theoretic applications. Chapter 5 deals with convergence of function sequences and series, the power series of elementary functions are deduced, the partial fraction decomposition of cotangent function and by using the power series representation of the arctangent function a series of $\frac{\pi}{4}$ are considered. Chapter 6 discusses famous classical problems of elementary analysis. Here Bernoulli polynomials, Euler series, Euler and Poisson summations and the Gamma-Function are investigated.

The book is a pearl of the mathematical literature. It is highly recommended to students for learning analysis in the first two semesters.

L. Gehér (Szeged)

Paul Koosis, *The logarithmic integral. I* (Cambridge Studies in Advanced Mathematics 12), XI+606 pages, Cambridge University Press, Cambridge—New York—New Rochelle—Melbourne—Sydney, 1988.

The frequent appearance of $\int_{-\infty}^{\infty} M(t)/(1+t^2) dt$ (or its transformed form) in more or less different branches of mathematical analysis as well as in their applications naturally raises the question: What is the role of this integral in the analysis? One, who is interested in this theme,

should consult with the present book. Moreover, having even a look at its "Contents", the prospective reader will surely find something of his particular interest.

The first two Chapters are devoted to Jensen's formula, the celebrated Szegő's theorem and the familiar Poisson integral. Chapter III, called for more frequently in the subsequent ones, deals with "Entire functions of exponential type", i.e.: entire functions satisfying $|f(z)| \leq Ce^{A|z|}$. The rest of the Chapters are entitled as follows: "IV. Quasianalycity", "V. The moment problem on the real line", "VI. Weighted approximation on the real line", "VII. How small can the Fourier transform of a rapidly decreasing non-zero function be?", "Persistence of the form $dx/(1+x^2)$ ". An "Addendum" improves the content of Chapter VII by discussing some recent results.

The author pays attention to show how things grow up from simple ideas. The reader, familiar with an introduction to the theory of real and complex functions, and a bit of functional analysis, will find only a few cases, when he needs to look for supplementary material. Exact references help the readers to find way in such situations. "Bibliography for volume I" lists approximately 80 items, including a number of books.

The argumentations are detailed to such an extent that one can follow them easily. However, a large area for the reader's activity is provided by giving "Problems" accompanied with hints (if necessary). By solving these problems (mostly of own interest) one can deeply understand, how to use the methods of the discussed theme, and thus possibly feels to be stimulated to do research work in analysis.

Reading this book, everybody will certainly be caught by the author's enthusiasm: "It is a beautiful material. May the reader learn to love it as I do." Thus, it mustn't escape the reader's attention that this book is completed by "Contents of volume II."

Endre Durszt (Szeged)

Hary Krishna, Computational Complexity of Bilinear Forms (Lecture Notes in Control and Information Sciences, 94), XVI + 166 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1987.

The book contains two parts: the first one has four chapters and is a deep study of the relationship between the computation of bilinear forms and the linear error-correcting codes. The two chapters of the second part describe an application of the class of linear codes showed in Part I.

In details, Chapter 2 discusses the multiplicative complexity of certain noncommutative algorithms that are usable to compute a system of k bilinear forms and establishes a connection between linear (n, k, d) codes and algorithms. Using the property of duality it is shown that the multiplicative complexity of the bilinear forms is the same as the multiplicative complexity of an aperiodic convolution algorithm with length $(k+d+1)$.

In Chapter 3 efficient algorithms are developed for aperiodic convolutions. In Chapter 4 bilinear algorithms — basing on two approaches developed in the previous Chapter — are presented for aperiodic convolution of sequences defined over $GF(2)$ and $GF(3)$.

Chapter 5 shows the decoding procedure for the class of codes obtained from the aperiodic convolution algorithms, moreover it is established that the length- and the errorcorrecting capability of these codes can be varied easily. As a consequence it has been proved that the encoder/decoder can be designed to incorporate a large number of these codes into the same configuration.

The next two chapters deal with the basic automatic repeat request schemes, with their protocols and their generalization.

G. Galambos (Szeged)

Fred Kröger, Temporal Logic of Programs (EATCS Monographs on Theoretical Computer Science, 8), VIII + 148 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

Temporal logic is a branch of modal logic. Its basic idea is that the truth of an assertion may depend on a discrete time scale. As a logic of this kind, it can be used to describe properties of programs in a natural way where the execution sequence of a program plays the role of the time scale.

Besides the Introduction, the book consists of seven chapters. Chapter I provides a detailed discussion of syntax and semantics of propositional temporal logic. In addition to the usual modal operators, the nexttime operator and the atnext operator are taken as primitive. If A and B are formulas, then A *atnext* B expresses that A will hold at the next time point than B holds. Other temporal operators are introduced as derived ones. Soundness and completeness of an axiomatization of propositional temporal logic is established in Chapter II. Some induction principles are also included. Chapter III is devoted to first order temporal logic. No completeness theorem is stated.

A basic (parallel) programming language is the subject of Chapter IV. Program properties are formalized and classified as safety (or invariance), liveness (or eventuality) and precedence properties. The rest of the book is devoted to program verification using temporal logic. Invariance and precedence properties are discussed in Chapter V and eventuality properties in Chapter VI. Hoare's calculus is embedded in temporal logic in Chapter VII.

It is shown in each case how program verification rules can be derived within the system, these are however the only theorems incorporated. Several examples are discussed.

The volume can be recommended to graduate students with interest in program verification.

Z. Ésik (Szeged—Munich)

Yurii T. Lyubich, Introduction to the Theory of Banach Representation of Groups, VI + 223 pages, Birkhäuser Verlag, Basel—Boston—Berlin, 1988.

This is a translation of the original Russian edition. The book consists of five chapters. The first three chapters are devoted to give the mathematical background needed in the last two chapters. Chapter one deals with the basic properties of bounded linear operators in Banach spaces and with commutative Banach algebras. Chapter 2 introduces the notions of topological groups and topological semigroups, a brief reference to invariant measures and means is also given. Chapter 3 gives a glimpse into the elements of general representation theory. Chapter 4 presents the representation theory of compact groups and semigroups in the space of bounded operators of a Banach space. In the final chapter the representation theory of locally compact Abelian groups can be found. In the text a rich collection of exercises and examples is given, serving as the illustration of the ideas.

L. Gehér (Szeged)

Erkki Mäkinen, On context-free derivations (Acta Universitatis Tamperensis, ser. A, vol. 198), 94 pages, Tampere, 1985.

Given a context-free grammar, its Szilard language contains one word for each terminating derivation. Szilard languages also arise with restricted types of derivations such as leftmost derivations, depth-first derivations and breadth-first derivations. The book provides a good survey of

results on Szilard languages: basic properties and relation to the Chomsky hierarchy, decision problems, recognition of Szilard languages, etc. Unrestricted Szilard languages are related to m -counter automata and left Szilard languages to simple pushdown automata. The importance of depth-first derivations lies in the fact that depth-first Szilard languages are context-free yet they are more general than leftmost derivations. The last chapter is devoted to the relation of Szilard languages to grammatical similarity.

The book can be recommended to graduate students and computer scientists with interest in formal languages and compiler construction.

Z. Ésik (Szeged—Munich)

Mathematical Foundations of Programming Language Semantics, Edited by M. Main, A. Melton, M. Mislove and D. Schmidt (Lecture Notes in Computer Science, 298), VIII + 637 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988.

This volume is the proceedings of the Third Workshop on Mathematical Foundations of Programming Language Semantics held at Tulane University, New Orleans, Louisiana, in April, 1987. The 32 contributions (4 invited and 28 selected) are organized into six chapters. The subject matter covers a wide range from category theory and λ -calculus to domain theory and implementation issues.

The invited addresses are the following: J. W. Gray: A Categorical Treatment of Polymorphic Operations; The main thesis is that 2-categories provide the right framework for studying polymorphic operations, i.e., operations that behave the same everywhere. J. D. Lawson: The Versatile Continuous Order; A survey of basic properties of continuously ordered sets including two natural topologies. S. D. Brookes: Semantically Based Axiomatics, A discussion on the basic ideas of Hoare's calculus. N. D. Jones et al.: MIX: A Self-Applicable Partial Evaluator for Experiments in Compiler Generation (Extended Abstract). The volume does not contain the text of the invited talks given by G. Plotkin and D. Scott.

The volume can be recommended to researchers and graduate students with interest in semantic issues.

Z. Ésik (Szeged—Munich)

Particle Physics, A Los Alamos Primer, Edited by N. G. Cooper and G. B. West, XI + 199 pages, Cambridge University Press, Cambridge—New York—New Rochelle—Melbourne—Sidney, 1988.

Particle physics is one of the most challenging fields for the human thought, and likewise for the budget of those few countries and organizations that can afford to finance the enormous costs of experimental particle physics. This book, which is a collection of articles written by a group of particle physicists at Los Alamos, is divided into two main parts. The first one is a theoretical framework. The authors explain here what are meant by the fundamental physical particles as quarks-leptons, gauge bosons, and how the related mathematical ideas: gauge fields, spontaneous symmetry breaking, quantum chromodynamics, etc., have emerged in the last 20 years. The subject is treated on a variety of technical levels and will certainly be enjoyed by anyone who is interested in the modern developments of natural sciences. Physicists working in other fields than particle physics will like this book too because everything is explained on the level of ordinary four dimensional electrodynamics and quantum mechanics. I think also the professional particle physicist may obtain much

help reading this book because it demonstrates how to expose this most difficult subject in a simple manner.

The second part of the book acquaints us with the grandiose experiments have been done so far, or planned in the future by particle physicists. We can learn about "underground science", as the enormous detectors, that are to be detecting proton decay, or neutrino oscillations — predicted by some theories — are located in deep mines. Not less interesting are the details of huge accelerators with diameters of tens of kilometers etc.

Each article is illustrated with several figures that help very much to understand the physical ideas. In the end of the book the authors express their personal viewpoint in a lively discussion about their profession, and also about social psychology of the particle physics community.

Physical theories of the first half of our century yielded us, among others, the theoretical basis of atomic power plants, but at present one can only hope that some time in the future high energy physics will also provide us with practical comfort. And though physicists are convinced that this will come true one day, the situation is more idealistic at present. We can only state that the gigantic and very expensive experiments serve merely to prove that deep mathematical ideas, such as Lie groups, supersymmetry, gauge invariance etc. have their origins in reality. Nevertheless these theories have strong predictive power and will allow mankind to control reality in an ever increasing manner.

M. G. Benedict (Szeged)

S. J. Patterson, *An Introduction to the Theory of the Riemann Zeta-Function* (Cambridge Studies in Advanced Mathematics, 14), 156 pages, Cambridge University Press, Cambridge—New York—New Rochelle—Melbourne—Sidney, 1988.

One of the most famous problems of mathematics is the so called Riemann Hypothesis. This states that all the zeros of the zeta-function lie on the "critical line" $\left\{z: \operatorname{Re} z = \frac{1}{2}\right\}$. (This is one of the several forms of the conjecture.) This was formulated in 1859 by B. Riemann, and it occurs in the eighth problem of the famous 23 unsolved problems presented by D. Hilbert before the International Congress of Mathematicians in 1900.

The following little story told by G. Pólya in a speech characterizes the importance of the problem. Somebody allegedly asked Hilbert, "If you would revive, like Barbarossa, after five hundred years, what would you do?" "I would ask" said Hilbert, "Has somebody proved the Riemann Hypothesis?"

The problem had resisted for over 100 years the efforts of mathematicians.

Several examples prove that the most fruitful and exciting task is to build a bridge over mathematical branches which are seemingly far off. The zeta-function is a meromorphic function, it can be investigated by the techniques of complex analysis and at the same time it yields important and characteristic results concerning the integers. Through the history of the zeta-function a long series of the world's greatest mathematicians (the enumeration is almost impossible) obtained determinant results: Two widely known classical summaries were written by E. Landau and E. C. Titchmarsh.

This book grew out of a lecture course about the Riemann Hypothesis and Weil's point of view concerning it. In determining the direction of the investigations the Riemann Hypothesis plays a central role. Chapter headings are: Historical introduction; The Poisson summation formula and the functional equation; The Hadamard product formula and explicit formulae of prime number theory; The zeros of the zeta-function and the prime number theorem; The Riemann Hypothesis and the Lindelöf Hypothesis; The approximate functional equation; Appendices.

An interested reader having a good background in analysis and number theory should be able to read the main part of the book. For the reviewer one of the most attractive features of this work is the concise but clear style of the treatment. The appendices make the reading of the texts easier. Various exercises in an unusually large number constitute an essential part of the book. (Some hints would be useful for the reader concerning the more difficult examples.) The thorough examination of this book offers the reader a good possibility to study special problems and to do some research. Last but not least the work consists of only 156 pages.

L. Pintér (Szeged)

Efim M. Polishchuk, Continual Means and Boundary Value Problems in Function Spaces, 159 pages, Birkhäuser Verlag, Basel—Boston—Berlin, 1988.

The main purpose of this book is to develop the theory of integration of infinite dimensional spaces and to give applications to boundary value problems for function domains. The text is divided into four parts. In the first part the definitions of uniform and normal functional domains are introduced. The notion of the main value of a functional over a domain is given and explicit formulae for its calculation are deduced. The procedure of functional averaging is shown to result in a Dirac measure, which is a generalized function. At the end of this part several definitions of the functional Laplace operator are presented. The second part is devoted to study the weak Dirichlet problem for normal domains with boundary values from the Gâteaux class, furthermore the Poisson equation and the solution of an exterior Dirichlet problem in a function space are considered. In the third part a completely different approach to the functional boundary value problems is proposed. Also boundary value problems for uniform domains are investigated. The final part deals with the extension of some of the previous results to boundary value problems with a general elliptic functional operator using the theory of diffusion processes and the compact extension of a function domain.

The material is as self-contained as it is possible. The book is recommended to research workers who are familiar with measure theory and functional analysis.

L. Gehér (Szeged)

Recent Developments in Mathematical Physics, Edited by H. Mitter and L. Pittnes, XI+323 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

The demand for mathematical rigor appears in theoretical physics mostly when "rude" physics itself shows that "something is wrong". This is the point where it is worth to try more exact methods; and it turns out very often that behind the new mathematics there is something new in physics as well. Mathematical rigor has the advantage that the physical model, its assumptions and restrictions, can be formulated in a most compact way. The 34 articles, contained in this book are written in this spirit. They are the texts of the lectures given at a meeting in Schladming, Austria, in 1987, in honour of Professor W. Thirring. Both classical and quantum mechanical problems are considered as well as problems in statistical physics and quantum field theory. The book will be interesting for mathematicians and also for physicists who like the mathematical style in theoretical physics. It will be useful for anyone who wants to see at least a part of fields of present day mathematical physics.

M. G. Benedict (Szeged)

Recent Topics in Theoretical Physics (Proceedings in Physics, 24). Edited by H. Takayama, IX+129 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988.

The volume consists of 10 lectures on recent developments in theoretical physics, held at the Yukawa Memorial Symposium, in 1986, in Nishinomiya. The topics of the lectures can be divided into two classes. The first one is high energy physics and cosmology. The titles contain: superstrings, lattice quantum chromodynamics, the quark structure of the nucleus, solar neutrinos, the very early universe, and gravitational collapse. The rest of the volume is devoted to some most recent developments of solid state theory, such as the quantum Hall effect, diffusion of heavy particles in metals, spin glasses, and pattern formation. The lectures are written on a high level, mostly by well known Japanese specialists. Nevertheless, the style is introductory and the text is aimed at any physicist independently from his special field of research. Concepts unfamiliar to the nonexpert are explained in simple terms. From the book one can learn what is in the centre of interest of theoretical physics now. It can be recommended to mathematicians and experimental physicists as well.

M. G. Benedict (Szeged)

Rewriting Techniques and Applications, Edited by J. P. Jouannaud, 216 pages, Academic Press, London—Orlando—San Diego—New York—Austin—Montreal—Sydney—Tokyo—Toronto, 1987.

This volume contains a selection of papers presented at the first international conference on Rewriting Techniques and Applications held in May 1985, in Dijon, France. The material is reprinted from the Journal of Symbolic Computation, Volume 3; Numbers 182, 1987. The 8 selected papers are: B. Buchberger: History and Basic Features of the Critical-pair/completion procedure; R. V. Book: Thue Systems as Rewriting Systems; N. Dershowitz: Termination of Rewriting; M. Rusinowitch: Path of Subterms Ordering and Recursive Decomposition Ordering Revisited; J. Hsiang: Rewrite Method for Theorem Proving in First Order Theory with Equality; K. A. Yelick: Unification in Combinations of Collapse-free Regular Theories; E. Tiden and S. Arnborg: Unification Problems with One-sided Distributivity; D. Benenav, D. Kapur and P. Narendran: Complexity of Matching Problems.

The first 3 papers are invited and provide good surveys on 3 different topics of symbolic computation. The following is a quotation from the Editorial by Jean-Pierre Jouannaud.

"The paper by Bruno Buchberger relates the history of the most important discovery in term rewriting theory: the notion of a critical pair, and its natural consequence, the completion algorithm. The reader will find his bibliography very helpful.

The paper by Ronald Book synthesizes at least ten years of research on Thue Systems, with a particular emphasis on the role of Church—Rosser properties in deciding important questions related to Thue Systems.

The paper by Nachum Dershowitz is a beautiful presentation of the current state of knowledge of termination. Moreover, he gives a new coding of Turing machines by rewrite rules, which leaves open the uniform termination problem of the one rule case only."

The book can be recommended both to researchers and graduate students interested in the field.

(Accepted 15.11.1988)

Z. Ésik (Szeged—Munich)

Paulo Ribenboim, *The Book of Prime Number Records*, XXII+476 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1988.

Having read this book the opinion of A. Rényi came to my mind (he was a Hungarian mathematician (1921—1970), mentioned in this book in the Index of Names, too). He wrote somewhere that the really beautiful, interesting, very significant and genuine things will never become ordinary or boring. This is true for the field of mathematics as well. One will never be tired of Euclid's proof stating that there exists infinitely many prime numbers. This proof is astonishing and it is a delightful experience every time just as when climbing up a peak the scenery opens up in front of us.

To tell the truth I didn't quite understand why Rényi mentioned this very example. After reading this book it became obvious for me.

Consider the above mentioned theorem. You find it in Chapter 1 having the title: How many prime numbers are there? Giving Euclid's classical proof you find some records. Denote by $p\#$ the products of all $q \leq p$, where q and p are primes. The largest known prime of the form $p\# + 1$ is $13649\# + 1$ and it was discovered by H. Dubner in 1987. (This number has 5862 digits.) Then several other proofs are given for the infinity of prime numbers: Kummer's proof; Pólya's proof (this uses the idea: it is enough to find an infinite sequence of natural numbers $1 < a_1 < a_2 < \dots$ that are pairwise relative prime); Euler's proof investigating the product of $1/(1 - 1/p_i)$ which leads to important developments; Thue's proof (this applies the fundamental theorem of unique factorization of natural numbers as product of prime numbers); Perrot's proof requiring the convergence of $\sum (1/n^2)$; Auric's proof; Métrod's proof; Washington's proof done via commutative algebra (this comes from 1980); and the last is Fürstenberg's proof that appeared in 1955 and is based on topological ideas. I think that there are only a few mathematicians, who don't find something new for themselves in this first chapter concerning a well-known theorem.

The further questions (at the same time chapter headings) are: How to recognize whether a natural number is a prime? Are there functions defining prime numbers? How are the prime numbers distributed? Which special kinds of primes have been considered? Heuristic and probabilistic results about prime numbers.

Let us mention only a few of the records: the largest known prime of the form $k \times 2^n + 1$ with $n \geq 2$ is $7 \times 2^{54486} + 1$ having 16402 digits (J. Young (1987)); the largest known prime of the form $n^2 + 1$ is $17^2 \times 2^{18802} + 1$ (Keller (1984)).

Let us consider another record concerning the famous Waring's problem: for every $k \geq 2$ there exists a number $r \geq 1$ such that every natural number is the sum of at most r k th powers. If such a number r exists denote by $g(k)$ the smallest possible one. While these phenomena tend to become more regular for sufficiently large numbers another characteristic number is introduced: denote by $C(k)$ the minimal value of r such that every sufficiently large integer is the sum of r k th powers, obviously $C(k) \leq g(k)$. Waring's problem (the existence of $g(k) < \infty$ for arbitrary k) was first solved by Hilbert in 1909. Here you have the records on $g(3)$ and $C(3)$. (In the book the reader finds much more in detail.) J. A. Euler (L. Euler's son) $g(3) \leq 9$. E. Maillet (1985) $g(3) \leq 21$, $C(3) \leq 4$, $g(3)$ exists; A. Fleck (1906) $g(3) \leq 13$; A. Wieferich (1906) $g(3) \leq 9$, $g(3)$ best possible; E. Landau (1909) $C(3) \leq 8$; Yu. V. Linnik $C(3) \leq 7$. Present status: $4 \leq C(3) \leq 7$; recent computations of Bohman and Fröberg as well as of Romani (1982) point to the likelihood that $C(3) = 4$.

We could enumerate several interesting problems from this work but we have no space. (One of my favourites is the discussion of Dirichlet's famous result on arithmetic progressions, and a related question established by Sierpinski in 1959: Let $a_1, a_2, \dots, a_m, b_1, b_2, \dots, b_n$ be any digits ($0 \leq a_i, b_j \leq 9$), satisfying $b_n = 1, 3, 7$ or 9 . Then there exist infinitely many prime numbers p which

are written in base 10, with the a_i as initial digits and the b_j as final digits: $p = a_1, a_2, \dots, a_m, b_1, b_2, \dots, b_n$.)

I liked this book. (For the reviewer this is the book of prime numbers with records.) It is well written in a conversational style, and with evident enthusiasm. The Bibliography which is compiled carefully is extremely useful. Reading on prime numbers is similar to playing tennis: it is marvellous in your youth and in your old age, too.

L. Pintér (Szeged)

J. L. C. Sanz—E. B. Hinkle—A. K. Jain, Radon and Projection Transform—Based Computer Vision (Algorithms, A Pipeline Architecture, and Industrial Applications), VIII+123 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988,

This book provides a description of the applicability of Radon and projection transforms to computer vision and processing. Particularly it deals with novel machine vision architecture ideas that make real-time projection-based algorithms a reality.

The authors concern themselves with several image analysis algorithms for computing (for example the projections of gray-level images along linear patterns, i.e. the Radon transform). They provide fast methods to transform images into projection space representations and to backtrace projection space information to the image domain which are suitable for implementation in a pipeline architecture.

We recommend this book to the beginners and also to the specialists, since it includes a survey of the architecture trends and some novel algorithms in computer vision.

Árpád Kurusa (Szeged)

Jaroslav Smital, On Functions and Functional Equations, VII+155 pages, Adam Hilger, Bristol and Philadelphia, 1988.

The text consists of five chapters. The introductory one summarizes the elementary ideas concerning functions. The second chapter studies functional equations of several variables, and solves the Cauchy functional equation starting different initial assumptions. The third chapter dealing with iterations is the most important part of the text playing a central role in the book. Chapter 4 gives the application of the iteration method for the study of population growth model. The final chapter investigates linear functional equations, the Abel and the Schröder equations.

Only elementary mathematical knowledge of the reader is supposed.

L. Gehér (Szeged)

Song Jian—Yu Jingyuan, Population System Control, XI+286 pages, China Academic Publishers, Beijing and Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988.

During the last century the world's population has increased enormously. Today more than five billion people live on Earth and the population has been increasing further. Will the resources of energy and food be enough for the mankind? Philosophers have always shown great concern about this problem throughout history. The earlier works on population studies, however, used figurative and literary language and methods and were not of a scientific nature. The modern natural

sciences are required to have theories that can be quantifiably tested and verified. Recently population studies has been included among these sciences thanks to the use of statistical and qualitative research methodologies. One of the most significant steps in this direction was that researchers have begun to regard the population evolution of a community as a dynamic process. The book gives an excellent account on the latest results of Chinese systems analysts achieved by the investigation of this mathematical model.

Chapter 1 (Introduction) gives a survey on the history and basic ideas of population studies and formulates tasks of population cybernetics. In Chapter 2 the continuous, discrete and stochastic population equations are derived. In the continuous model the state function is the agedistribution density function $p(a, t)$. (Roughly speaking, if $\Delta a > 0$ is small, then the total number of people of age between a and $a + \Delta a$ at time t is $p(a, t) \cdot \Delta a$.) The state function in the discrete model is a vector: $x(t) = (x_0(t), x_1(t), \dots, x_m(t))$, where $x_i(t)$ is the total number of persons in year t whose full age is within the age interval $[i, i+1]$. The model is a first order partial differential equation and difference equation, respectively. In Chapter 3 demographic indices (average lifetime and life expectancy, net population reproduction rate, average female fertility rate) are expressed by the state functions $p(a, t)$ and $x(t)$. Chapter 4 contains the dynamic analysis of population systems based upon the evolution equations. The most interesting (in reviewer's opinion) Chapter 5 is concerned with stability problems for population systems. The authors prove that the necessary and sufficient condition of stability in Liapunov's sense for a population system is that the total fertility rate should not exceed a critical value. Chapters 6 and 7 are devoted to population projections and policies, and description of the population structure in an ideal society. The concluding Chapter 8 presents an optimization theory of birth control policy and its applications.

This book will be very useful for mathematicians as well as social scientists dealing with population dynamics and population policy.

L. Hatvani (Szeged)

STACS 88, Edited by R. Cori and M. Wirsing (Lecture Notes in Computer Science, 294), IX+404 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988,

The fifth Symposium on Theoretical Aspects of Computer Science was held in Berdeaux in February 1988. The volume contains the text of the invited talk "Geometry of Numbers and Integer Programming" by C. P. Schnorr as well as 34 selected contributions that cover a wide range of theoretical computer science: Algorithms, Complexity, Formal Languages, Rewriting Systems and Abstract Data Types, Graph Grammars, Distributed Algorithms, Geometrical Algorithms, Trace Languages, Semantics of Parallelism. In addition to the technical contributions, eight software systems presented at the symposium are reviewed.

The wide range and high quality ensure that every computer scientist will find at least one paper of his own interest.

Z. Ésik (Szeged—Munich)

Topics in Operator Theory, Constantin Apostol Memorial Issue, Edited by Gohberg, 274 pages, Birkhäuser Verlag, Basel—Boston—Berlin, 1988.

The text starts with a short glimpse of the life and mathematical results of Constantin Apostol. List of his publications is also given. Eleven papers can be found in the book. Their subjects are: operator theory and operator algebras. The first paper contains a result of Constantin Apostol

(On the Spectral Equivalence of operators). The other papers are written by different, in general, well known authors, all of them are dedicated in memory of Constantin Apostol.

The book is highly recommended to research workers interested in the modern functional analysis.

L. Gehér (Szeged)

Stephen Wiggins, Global Bifurcations and Chaos. Analytical methods (Applied Mathematical Sciences, 73), XIV+494 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1988.

The first chaotic phenomena arose in deterministic nonlinear dynamical systems fifteen years ago. As it can be followed also in our Review Section, since that time an unusually great number of texts and monographs have been published devoted to the theoretical and applied problems of these phenomena. This book is concerned with the following three fundamental questions exciting both mathematicians and applied scientists: What is meant by the term "chaos"? What mechanism does chaos result? How can one predict when chaos will occur in a specific dynamical system?

It is pointed out in the book that the global bifurcation can often be the mechanism for producing deterministic chaos (the final answers are far from known). The global bifurcation means a qualitative change in the orbit structure of an extended region of phase space.

The first chapter contains the background for ordinary differential equations and dynamical systems (including such notions as conjugacies, invariant manifolds, structural stability, genericity, bifurcations, Poincaré maps) which are derived for a dynamical system to exhibit chaotic behaviour. The reader can find here a clear and exact, easily readable description of the Smale horseshoe which is the prototypical map possessing a chaotic invariant set, and which is absolutely essential for understanding what is meant by the term "chaos". The chapter includes also a good introduction to symbolic dynamics. Chapter 3 is concerned with homoclinic and heteroclinic motions, which typically result global bifurcation and chaotic behaviour in deterministic systems. A homoclinic orbit connects an unstable equilibrium to itself, a heteroclinic one connects two unstable equilibria. In the fourth chapter a variety of perturbation techniques are developed which allow the scientists to detect homoclinic and heteroclinic orbits. These are such generalizations and improvements of the Melnikov—Arnold method which are applicable to arbitrary finite dimensional systems and allow for slowly varying parameters and quasiperiodic excitation.

The book is written in an excellent style. It is selfcontained, requiring only the knowledge of calculus. During the exposition of the complicated notions the author first gives some examples of specific physical systems so that the reader may develop some intuition. After this he gives the exact mathematical definition.

This excellent book can be highly recommended to every mathematician, user of mathematics or student interested in qualitative theory of dynamical systems and its applications.

L. Hatvani (Szeged)

Eberhard Zeidler, Nonlinear Functional Analysis and its Applications IV: Applications to Mathematical Physics, XXIII+975 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1988.

This book is the fourth of a five-volume survey on the main principles and methods of nonlinear functional analysis and its applications. The main goal of the book is to give an exact clear exposition

of the field which is self-contained and accessible to the nonspecialists, which combines the classical and modern ideas, and which builds a bridge between the language and thoughts of physicists and mathematicians. The specific nature and importance of the problems of mathematical physics are well expressed by M. Atiyah's opinion, which is cited in the book: "The more I have learned about physics, the more convinced I am that physics provided, in a sense, the deepest applications of mathematics. The mathematical problems that have been solved, or techniques that have arisen out of physics in the past, have been the lifeblood of mathematics. The really deep questions are still in the physical sciences. For the health of mathematics at its research level, I think it is very important to maintain that link as much as possible". However, reading physical literature mathematicians often complain that the presentation is not rigorous and exact enough, and vice versa, physicists find the mathematical methods too abstract and useless for their purposes covering the physical thoughts. The present book helps to solve this difficulty. Mathematicians will feel it comfortable because it uses precise mathematical language, at the same time the reader can learn a lot about the physical interpretation. On the other hand, the physicists can recognize the familiar physical ideas and can get acquainted with their justification.

Similarly to the previous volumes, the chapters are grouped into blocks according to applications:

Applications in Mechanics: Ch. 58. Basic Equations of Point Mechanics; Ch. 59. Dualism Between Wave and Particle, Preview of Quantum Theory, and Elementary Particles.

Applications in Elasticity Theory: Ch. 60. Elastoplastic Wire; Ch. 61. Basic Equations of Nonlinear Elasticity Theory; Ch. 62. Monotone Potential Operators and a Class of Models with Nonlinear Hooke's Law, Duality and Plasticity, and Polyconvexity; Ch. 63. Variational Inequalities and Signorini Problem for Nonlinear Material; Ch. 64. Bifurcation for Variational Inequalities; Ch. 65. Pseudomonotone Operators, Bifurcations, and the von Kármán Plate Equations; Ch. 66. Convex Analysis, Maximal Monotone Operators and Elasto-Viscoplastic Material with Linear Hardening and Hysteresis.

Applications in Thermodynamics: Ch. 67. Phenomenological Thermodynamics of Quasi-Equilibrium and Equilibrium States; Ch. 68. Statistical Physics; Ch. 69. Continuation with respect to a Parameter and a Radiation Problem of Carleman.

Applications in Hydrodynamics: Ch. 70. Basic Equations of Hydrodynamics; Ch. 71. Bifurcation and Permanent Gravitational Waves; Ch. 72. Viscous Fluids and the Navier-Stokes Equations.

Manifolds and their Applications: Ch. 73. Banach Manifolds; Ch. 74. Classical Surface Theory, the Theorema Egregium of Gauss, and Differential Geometry on Manifolds; Ch. 75. Special Theory of Relativity; Ch. 76. General Theory of Relativity; Ch. 77. Simplicial Methods, Fixed Point Theory, and Mathematical Economics; Ch. 78. Homotopy Methods and One Dimensional Manifolds; Ch. 79. Dynamical Stability and Bifurcation in $B-S$ -spaces.

The chapters are followed by interesting problems supplying the body of the text and encouraging the reader's individual thinking.

Apparently, the book covers the whole spectrum of the significant applications of the nonlinear functional analysis. It will be very useful and inevitably important for mathematicians, physicists and students interested in applications of mathematical methods in physics.

L. Hatvani (Szeged)