Acta Sci. Math., 54 (1990), 247—256

Non type-preserving automorphism groups of buildings and normalizing Tits systems

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0. Introduction

It is known that a very important class of groups, including real and complex reductive Lie groups and algebraic groups [1], [4], [7], [11], [12], [13], finite simple groups other than the alternating or sporadic groups [12], and also some infinite dimensional groups associated to Kac—Moody Lie algebras [15], [16] and some infinite dimensional transformation groups [9], give rise to a Tits system (or BN pair) [2], [12] and hence act on some simplicial complex which satisfies very striking geometric and combinatorial properties, axiomatized in the theory of buildings [2], [12].

A basic property of a building (for definitions, see Sec. 1) is that it admits "type mappings", the set of which is parametrized by the group of permutations of the vertices of a given chamber [12]. Furthermore, a group with Tits system acts on its building in a type preserving way [12]. By a Theorem of J. Tits ([12], 3.11, p. 44), there is a certain converse to this situation: any group of type preserving automorphisms of the building which acts transitively on the set of pairs (C, A), where C is a chamber of given type and A an apartment containing it, is a group with Tits system, with respect to B and N the stabilizers of C and A, respectively.

In this paper we consider groups of non-type preserving automorphisms of a building which satisfy the analogous condition for chambers and apartments whose type is not fixed. Non type-preserving elements of such groups (the "polarities") and their centralizers play an important role in the theory of simple groups (see for instance [14]). We show that such a group together with B and N as above, gives rise to a "normalizing Tits system", a notion which we have introduced in [9]. This means that G satisfies all the hypotheses of a Tits system except for the property of groups with Tits system that all elements of any generating set S of the Weyl

Received March 13, 1987 and in revised form May 17, 1989.

group fail to normalize B. Under our hypotheses, the subgroup of G preserving the type still acts transitively on the set of pairs of oriented chambers and apartments, as above and hence, by Tits' Theorem, gives rise to a Tits system. We use this fact in the proof that G has a *normalizing* Tits system. The subgroup of type preserving automorphisms is a normal subgroup of G. When the building is of finite rank, then it is of finite index in G. In [9] we had given these results in the special case where the building is a homogeneous tree.

1. Basic definitions

In this paragraph we establish some basic notation and briefly recall a few definitions and results concerning buildings, taken from [12].

Given any set X, we denote the group of all bijective maps of X by $S(X)$. By a simplicial complex I we shall here mean a set $M(I)$ together with a family of subsets $\mathrm{Sim}(I)$, the simplices of I, which is closed under taking subsets. Given a simplex S, we denote the set of points of $I(S)$ by $M(S)$. I is completely determined by the set of maximal elements of $\mathrm{Sim}(I)$, which we shall denote by $\mathrm{Ch}(I)$. We shall write the action of the automorphism group of I on $M(I)$ or $\mathrm{Sim}(I)$ on the right, $m \to m \cdot g$, except when we wish to name the homomorphism α: $\mathrm{Aut}(I) \to S(M(I))$ or $S(\mathrm{Sim}(I))$, in which case we shall write $\alpha(g)(m)$, etc.

Definition 1.1. A simplicial complex $A=(M(A), \mathrm{Sim}(A))$ is said to be *"thin"* iff the following conditions are verified:

(i) For any maximal simplices, C, C' there exists a chain of maximal simplices, γ: $C=C(0), \dots, C(n)=C'$ such that the successive intersections $C(i) \cap C(i+1)$ are of codimension 1 in both $C(i)$ and $C(i+1)$. A maximal simplex is called a *chamber*. Two chambers whose intersection is of codimension 1 are called *adjacent*, an their intersection is called their wall. γ is called a *gallery* between C and C'.

(ii) If (C, C') and (D, D') are two pairs of adjacent chambers, such that $C \cap C' = D \cap D'$ then the sets $\{C, C'\}$ and $\{D, D'\}$ coincide.

One of the basic results about thin complexes is the following result on the set of chambers: Given a set X, a bijection from some ordinal o onto X is called a *tuple* in X. By fixing a given tuple, we can identify the set of all tuples in X with $S(X)$.

Definition 1.2. Given a simplicial complex I, and a simplex X of I, we shall denote the set of ordered tuples in $M(X)$ by $\langle X \rangle$; we can identify $\langle X \rangle$ with $S(M(X))$. We denote by $\langle \mathrm{Ch}(I) \rangle$ the set $\mathrm{Ch}(I) \times \mathrm{Typ}(I)$ (see Definitions 1.3. and 1.4.).

Definition 1.3. A simplicial complex $I=(M(I), \mathrm{Sim}(I))$ together with a family of subcomplexes Ap, called *apartments*, is called a *building* iff the following conditions are satisfied:

i) Any subcomplex A in Ap is thin.

ii) For any two chambers C, C' of I, there exists an apartment A which contains both C and C'.

iii) If the intersection of two apartments A, A' contains two chambers C and D, then there exists an isomorphism $q\colon A \to A'$ which fixes C and D, as well as all their faces.

For a chamber C in a building I, we shall let $\mathrm{Sim}(C)$ and $M(C)$ denote the simplices and points of I contained in C, respectively.

Definition 1.4. A *type mapping* (relative to a fixed chamber C) for a building is a mapping $t\colon M(I) \to M(C)$ which restricts to a bijection on $M(C')$ for each chamber C'. We shall denote the set of type mappings by $\mathrm{Typ}(I)$.

The basic property of type mappings of a building is the following result ([12]' Ch. 3):

Theorem 1.1. *Any type mapping t is uniquely determined by the permutation $\sigma\colon M(C) \to M(C)$ which t induces by restriction to $M(C)$.*

One has an obvious action of $S(M(C))$ on the set of type mappings of I, given by the formula

$$\sigma(t)(m) = \sigma(t(m)). \tag{1}$$

By Theorem 1.1, this action is regular (i.e. free and transitive), and hence we can identify a type mapping with a permutation of $M(C)$. Indeed, we shall identify the permutation σ with the unique type mapping inducing σ on $M(C)$. Given a point m of I and a type map t, $t(m)$ is called the type of m, and given a wall S between two chambers, the cotype of S is the unique element of $M(C)$ which is not in the image $t(S)$.

The notion of a Tits system (or BN pair) was first defined in [11]. For some of its implications see also [1], [2], [3], [7], [12], [13], as well as the papers referred to in [12].

Definition 1.5. Let G be a group, N and B two subgroups, S a set. We say that a quadruple (G, B, N, S) is a Tits system if and only if

(T. 1) the subgroups B and N generate G.

(T. 2) $B \cap N$ is normal in N.

Furthermore the group $W = N/B \cap N$ has a set S of generators of order 2 such that

(T. 3) for each w in W, s in S, the inclusion $B \cdot w \cdot B \cdot s \subseteqq B \cdot w \cdot s \cdot B \cup B \cdot w \cdot B$ holds

(T. 4) for each s in S, $s \cdot B \cdot s \neq B$.

Clearly, in (T.3) one can also write $B \cdot w \cdot B \cdot s \cdot B$ instead of $B \cdot w \cdot B \cdot s$. The principal result of this paper is a generalization of the following result of J. Tits [12, 3.11, p. 46].

Theorem 1.2. *Let G be a group of type-preserving automorphisms of a building acting transitively on the set of pairs* (C, A), *where C is a chamber (of fixed type) and A an apartment containing it. Let B and N be the stabilizers in G of C and of A, respectively. Then* $B \cap N$ *is normal in N, and there exists a set of generators S in* $W = N/N \cap B$ *such that the quadruple* (G, B, N, S) *forms a Tits system.*

2. Non-type preserving automorphisms

We shall fix a building I. We consider the action of $G = \mathrm{Aut}(I)$ and certain subgroups of it (which we shall define later) on $\langle \mathrm{CH}(I) \rangle$ (Definition 1.2). G will always denote a subgroup of $\mathrm{Aut}(I)$. We consider throughout this paper a fixed chamber C; and t the unique type map t, defined with respect to C which induces the identity on $M(C)$. By means of it, we can identify $\langle \mathrm{CH}(I) \rangle$ with $\mathrm{Ch}(I) \times S(M(C))$. By this identification a typed chamber $\langle D \rangle$ for which $t(\langle D \rangle) = \mathrm{id}$ is identified with the pair (D, id). The reader is warned that this contrasts with the usual terminology of buildings ([12] for instance), where the letters C, D, etc. refer to chambers with fixed type under a given type mapping, whereas here we use C, D to denote the "abstract" chambers. Similar remarks apply to apartments and simplices of a building. When we wish to emphasize this point, we shall speak of abstract chambers, apartments, etc.

G acts on $\mathrm{Ch}(I)$ in the obvious way. We shall denote this action by $p\colon G \to S(\mathrm{Ch}(I))$ and shall sometimes abbreviate $p(g)(D)$ by $D \cdot g$. G acts also on $\mathrm{Typ}(I)$:

$$q(g)(a)(m) = a(m \cdot g), \tag{2}$$

for any type map a and any m in $M(I)$.

As an immediate consequence of Theorem 1.1, one has

Lemma 2.1. *The stabilizer $G(a)$ in G of any type mapping a is normal in G. $G/G(a)$ acts freely on* $\mathrm{Typ}(I)$. *We denote the G-orbit of t by* $\mathrm{Typ}(G)$. *It forms a subgroup of* $S(M(C))$.

Definition 2.1. An automorphisms g is called *type preserving* if and only if it lies in $G(a)$, for some (and hence by the Lemma for all) type mappings. We denote by $G(0)$ the subgroup of type preserving automorphisms in G.

Finally, there is an action of G on $\langle \mathrm{Ch}(I)\rangle = \mathrm{Ch}(I) \times S(M(C))$, which is simply the product action of G on $\mathrm{Ch}(I)$ and on $\mathrm{Typ}(I)$.

$$\langle D\rangle \cdot g = (D, \sigma)\cdot g = \big(p(g)(D), q(g)(\sigma)\big).$$

The action of G on $\mathrm{Typ}(I)$ defines a homomorphism of G into the set of bijections of $\mathrm{Typ}(I)$:

(3) $$1 \to G(0) \to G \xrightarrow{q} H \to 1,$$

with H contained in $S(\mathrm{Typ}(I))$.

Furthermore, we had seen in Ch. 1 that one can identify the set $\mathrm{Typ}(I)$ with $S(M(C))$ by virtue of the regular action of that latter group, which we can hence identify with the left regular action of $S(M(C))$ on itself. Moreover one sees immediately that the actions of G and of $S(M(C))$ on $\mathrm{Typ}(I)$ commute. It is well known that the commutant of right regular action of a group on itself is left regular action of the same group. Hence we can regard the exact sequence (3) as follows-

(3′) $$1 \to G(0) \to G \xrightarrow{q} \mathrm{Typ}(G) \to 1$$

and we can identify $\mathrm{Typ}(G)$ with a subgroup of $S(M(C))$.

We consider the G-equivariant projections

$$P\colon \langle \mathrm{Ch}\rangle \to \mathrm{Ch}, \quad Q\colon \langle \mathrm{Ch}\rangle \to \mathrm{Typ}(I)$$

defined by the decomposition of $\langle \mathrm{Ch}\rangle = \mathrm{Ch} \times \mathrm{Typ}(I)$ on the two factors. In fact, they are "fibred extensions" in the sense of [8], in (3′), G acts on the fibres for the projection Q, and $G(0)$ is the set of elements in G for which, for all chambers D' $Q(g\cdot(\langle D\rangle)) = D = Q(\langle D\rangle)$, and similarly for the pair (P, p).

Proposition 2.1. *G acts faithfully on* $\mathrm{Ch}(I)$.

We leave proof to the reader.

Definition 2.2. Let A be an apartment of I, and H a subgroup of $S(M(C))$ (thought of as a set of type mappings of the building). Then let $\langle \mathrm{Ch}(A; H)\rangle = \{\langle C\rangle | C$ is in A and $t(\langle C\rangle)$ is in $H\}$. The pair $(M(A), \langle \mathrm{Ch}(A; H)\rangle)$, will be called the *typed apartment* $\langle A; H\rangle$, H its group of types, and A its underlying abstract apartment. Set-theoretically, $\langle \mathrm{CH}(A; H)\rangle$ is the product $\mathrm{Ch}(A) \times H$.

We shall now fix a subgroup H of $\mathrm{Typ}(I)$. From now on we shall make the following assumption about our group G, which is the direct analogue of the hypotheses of Theorem 1.2 for typed apartments and non type-preserving groups.

(S) Standard hypothesis. G acts transitively on the set of pairs $(\langle C\rangle, \langle A;\ \mathrm{Typ}(G)\rangle)$, such that $\langle C\rangle$ is a typed chamber of type h contained in *a* typed apartment $\langle A;\ \mathrm{Typ}(G)\rangle$ with h in $\mathrm{Typ}(G)$.

We shall fix $\langle C \rangle$ contained in $\langle \mathrm{Ch}\,(A)\rangle$, and denote by B and N the stabilizers of $\langle C \rangle$ and $\langle A; H\rangle$, respectively. We shall always denote $\mathrm{Typ}\,(G)$ by H. Given a pair of subgroups B, N, with $B \cap N \lhd N$, $W = N/N \cap B$, we shall write right, left and double cosets $n \cdot B$, $B \cdot n$, $B \cdot n \cdot B$ unambiguously as $w \cdot B$, $B \cdot w$, etc. where w is the image of n in W under the natural projection.

The formula (B) of the following Proposition 2.2 is an obvious generalization of the Bruhat decomposition (see [3.0] for the Bruhat decomposition in its original setting, and also the references on Tits systems).

Proposition 2.2.
(i) $B \cap N \lhd N$.
(ii) *G is the disjoint union of all double cosets $B \cdot w \cdot B$, w in W:*

$$\text{(B)} \quad G = B \cdot W \cdot B.$$

Proof. We first prove that $B \cap N \lhd N$. As above, we let t be the unique type mapping defined with respect to C which is the identity on $M(C)$. Let T be the stabilizer of $\langle C \rangle$ in N. Then, by Lemma 2.1, T is contained in $\ker(q)$. We shall denote by o the homomorphism defining the action of N on $\mathrm{Ch}\,(A)$, i.e. the restriction of the homomorphism p to N, restricted to the invariant subset $\mathrm{Ch}\,(A)$. T is in the kernel of that homomorphism. On the other hand, $N \cap \ker(q) \cap \ker(o)$ is obviously contained in T. Hence T is the intersection of two normal subgroups of N, hence is normal.

We let $W = N/N \cap B$. To show (B), it suffices to show that for any $\langle D \rangle$ in $\langle \mathrm{Ch}\,(I)\rangle$, there exists b in B and n in N such that $\langle D \rangle = \langle C \rangle \cdot n \cdot b$. By Lemma 2.1, there exists a typed apartment $\langle A' \rangle$ containing $\langle C \rangle$ and $\langle D \rangle$. By hypothesis there exists an element b' in G which maps the pair $(\langle C \rangle, \langle A' \rangle)$ into the pair $(\langle C \rangle, \langle A \rangle)$. By definition of B, b' is in B. Applying the hypothesis again, we see that there exists an element n in G which maps the pair $(\langle C \rangle, \langle A \rangle)$ to $(\langle D \rangle \cdot b', \langle A \rangle)$. By definition of N, n is in N. Putting $b = (b')^{-1}$, $\langle D \rangle = \langle C \rangle \cdot n \cdot b$.

Using the G-equivariant projections P and Q onto $\mathrm{Ch}\,(A)$ and $\mathrm{Typ}\,(G)$ respectively, the disjointness of the decomposition (B) follows immediately from Bruhat decomposition of $G(0)$, which follows from Tits' Theorem 1.2, and the fact that $G/G(0)$ acts freely on $\mathrm{Typ}\,(G)$. This proves Bruhat decomposition.

For any subgroup P of G, $P(0)$ will denote the intersection of P with $G(0)$. We note that T and B are both contained in $G(0)$ by Lemma 2.1. Hence the extension

$$1 \to T \to N \to W \to 1$$

restricts to a subextension

$$1 \to T \to N(0) \to W(0) \to 1$$

with $W(0)$ contained in W.

Lemma 2.4. *W acts regularly on* $\langle \mathrm{CH}(A, H)\rangle$.

The proof is routine and is left to the reader.

Lemma 2.5. *Assuming* (S), *the homomorphism q restricted to N and to* $\langle \mathrm{Ch}(A, H)\rangle$ *defines an extension*

$$1 \to W(0) \to W \to H \to 1. \tag{4}$$

$W = W(0) \rtimes H$. *There exists a splitting from H into W which maps H onto the stabilizer in W of C.*

Proof. By the Standard hypothesis (S), q restricts to a surjection from N onto H. Since we have seen that T is contained in the kernel of q, q defines a surjection, still denoted by q, from W onto H, with kernel $W(0)$. We now restrict q to the stabilizer in W of C. Since, by the previous Lemma, W acts regularly on $\langle \mathrm{Ch}(A)\rangle$, this restriction is still surjective. It is clearly injective. Inverting that restriction from H onto the stabilizer, we obtain the splitting.

We note that the Standard hypothesis implies immediately that the hypotheses of Theorem 1.2 of [12] are satisfied, and hence that the quadruple of $(G(0), B(0), W(0), S)$, for some suitable S, constitute a Tits system, and hence that the following corollary is true. However, it also follows immediately from Proposition 2.2:

Proposition 2.3. $G(0) = B \cdot W(0) \cdot B$.

Proof. Writing an element in $G(0)$, as $g = b \cdot n \cdot b'$, and keeping in mind the fact that $B = B(0)$ is contained in $G(0) = \ker(q)$, we see that n also lies in $\ker(q)$, hence in $N(0)$. The result follows.

The following notion was introduced (in slightly different form) in [9].

Definition 2.3. Let G be a group, N and B two subgroups. We say that a triple (G, B, N) *gives rise to a normalizing Tits system* if and only if there exists a set S for which the axioms (T. 1), (T. 2), and (T. 3) of Definition 1.5 are verified but no such S for which (T. 4) is also verified. If S is such a set, one calls the quadruple (G, B, N, S) a normalizing Tits system.

Theorem 2.1. *Let G satisfy the hypothesis* (S), *and let B, N, W, $\langle C\rangle$, $\langle A; H\rangle$ be as above. Let $W = W(0) \rtimes H$ be the splitting of Lemma* 2.5. *Then there exists a set of generators S of $W(0)$, such that, for any set of generators R of H,* $(G, B, N, R \cup S)$ *form a normalizing Tits system. More precisely, all elements of R normalize B. There exists no set U of generators of W for which* (T. 4) *is also verified.*

Proof. As we have remarked, by [12], Theorem 1.2, there exists a set of generators of order 2 in $W(0)$, S for which $(G(0), B(0), N(0), S)$ forms a Tits system. We have seen that $B(0)=B$.

We choose this set S of generators of $W(0)$ and any set R of generators of H. We already know Bruhat decomposition. Hence we conclude immediately that the axioms (T. 1) and (T. 2) are satisfied. We need to verify (T. 3), i.e. we must show for each element in the generating set of W, $R\cup S$, that the condition $B\cdot w\cdot B\cdot t\cdot B= =B\cdot w\cdot t\cdot B\cup B\cdot w\cdot B$ is satisfied. In fact we shall prove that for all w in W,

(T. 3. H) $B\cdot w\cdot B\cdot h\cdot B = B\cdot w\cdot h\cdot B$ for any h in H

(T. 3. S) $B\cdot w\cdot B\cdot s\cdot B\subset B\cdot w\cdot s\cdot B\cup B\cdot w\cdot B$ for any s in S.

We consider the splitting of the extension

$$1\to W(0)\to W\to H\to 1$$

defined in Lemma 2.5: H acts regularly on the set of typed chambers (C is our fixed chamber) $\{\langle C\rangle=(C, h)|h \text{ in } H\}$. It follows from Lemma 2.1 that B is the intersection of the stabilizers in G of each of these typed chambers. Hence H normalizes B. Hence $B\cdot h=h\cdot B$, and $B\cdot W\cdot B\cdot h=B\cdot W\cdot h\cdot B$, which proves (T. 3. H).

Let $w=hw'$, then

$$B\cdot w\cdot B\cdot s\cdot B = B\cdot h\cdot w'\cdot B\cdot s\cdot B = h\cdot B\cdot w'\cdot B\cdot s\cdot B=h\cdot B\cdot w'\cdot s\cdot B\cup h\cdot B\cdot w'\cdot B=$$

$$= B\cdot w\cdot s\cdot B\cup B\cdot w\cdot B.$$

This proves (T. 3. S).

It remains to show that there exists no set U of generators of W for which (G, B, N, U) is a Tits system. This follows from the fact that B is not self-normalizing, violating a well-known property of groups with Tits system.

Example 1. Let I be projective flag variety over any field of dimension n. It is immediate to verify that the conditions of our result hold, with $\text{Typ}(\text{Aut}(I))= =Z/2\cdot Z$. Elements of the non-trivial coset of $G(0)$ in $\text{Aut}(I)$ are classically known as "polarities". They lie at the basis of the duality theorem of projective geometry. (Their analogues for buildings of type $D(4)$, where $\text{Typ}(\text{Aut}(I))$ is the the symmetric group on 3 letters, have been extensively studied in [14].)

A splitting of the extension of $\text{Aut}(I)\to Z/2\cdot Z$, is given by the map associating to every subspace its orthogonal complement, with respect to some inner product in the underlying space. Typed apartments are defined by unordered orthogonal bases, and consist of all ascending or descending flags formed from this basis. Given a chamber $\langle C\rangle$ in such an apartment, for instance the increasing flag defined by an ordering of that orthonormal basis in the underlying linear space, $e(1), \ldots, e(n+1)$, the splitting of the Weyl group of Lemma 2.5, would be given by the composition $t \circ s$, where t is the orthogonal complement map and s the permutation $i \to n+1-i$. It has the property that all terms of the flag are incident to their images under $t \circ s$. In particular, the "polarity" $t \circ s$ has the points corresponding to the linear subvariety generated by $\{e(i)\}, i=1, \ldots, [(n+1)/2]$, for self-adjoint points. (For an extensive discussion of self-adjoint points for groups of type $D(4)$, see [14].) The Theorem of this paper then says that the group $\mathrm{Aut}(I)=B \cdot W \cdot B$, with B the stabilizer of the flag $\langle e(0)\rangle, \ldots, (e(0), \ldots, e(i)\rangle, \ldots)$, and W the semi-direct product of the symmetric group on the basis $e(i)$ and $t \circ s$.

Example 2. I is a homogeneous tree. Then the automorphism group of I satisfies our conditions, and we find the result of [9].

Acknowledgement. The author gratefully acknowledges several very useful comments on an earlier version of this paper on the part of the referee. In particular, part of the proof given here of the proof of Theorem 2.1 is due to him.

References

[1] A. Borel, J. Tits, Groupes réductifs, *IHES Publ. Math.*, **27** (1965), 55—153.

[2] N. Bourbaki, *Elements de mathématiques. Groupes et algèbres de Lie*, Ch. 4—6, Hermann (Paris, 1968).

[3] F. Bruhat, Sur les représentations induites des groupes de Lie, *Bull. Soc. Math. France*, **84** (1956), 97—205.

[4] F. Bruhat, J. Tits, Groupes réductifs sur un corps local, *IHES Publ. Math., No.* **41** (1972), 5—251.

[5] C. Chevalley, Sur quelques groupes simples, *Tohoku Math. J.* (2) **7** (1955), 14—66.

[6] S. Eilenberg, S. McLane, Cohomology theory in abstract groups. II, *Annals of Math.*, **48** (1947), 51—78.

[7] N. Iwahori, H. Matsumoto, On some Bruhat decomposition and the structure of Hecke rings of p-adic Chevalley groups, *IHES Publ. Math., No.* **25** (1965), 5—48.

[8] B. Seifert, Fibred group extensions and the normalizer of a transformation group, in: *Operator algebras and group representations, Vol. 2*, Pitman (Boston, Mass.—London, 1984), pp. 140—151.

[9] B. Seifert, Highly transitive group actions on trees and normalizing Tits systems, *Pacific J. Math.*, **125** (1986), 447—458.

[10] B. Seifert, Transitivity properties, Bruhat decomposition and Tits systems for transformation groups, *J. London Math. Soc.*, (2), **29** (1984), 441—452.

[11] J. Tits, *Structures et groupes de Weyl*, Sem. Bourbaki, Exposé, No. 288 (1965).

[12] J. Tits, *Buildings of spherical type and (B, N) pairs*, Lecture Notes in Math., Vol. 386, Springer (Berlin—New York, 1974).

[13] J. Tits, Théorème de Bruhat et sous-groupes paraboliques, *C. R. Acad. Sci. Paris*, **254** (1962), 2910—2912.

[14] J. TITS, Sur la trialité et certains groupes qui s'en déduisent, *IHES, Publ. Math. No. 2* (1959), 13—60.
[15] J. TITS, *Résumé de cours du Collège de France,* Théorie des groupes (Paris, 1980—81).
[16] H. GARLAND, The arithmetic of loop groups, *IHES Publ. Math., No. 52* (1980), 5—136.

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