Bibliographie

Applications of Combinatorics and Graph Theory to the Biological and Social Sciences, Edited by Fred Roberts (The IMA Volumes in Mathematics and its Applications), IX+345 pages, X+156 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong, 1989.

One of the principal motive powers of the development of mathematics is the hard demand on more and more application which appear from the side of living sciences. For the people who are makers or users of pure and applied mathematics, a very interesting experience is to find the meeting point of mathematics and the living nature, biology and social sciences.

This volume contains fifteen exciting overviews concerning the above topics. The leading idea of the book is formulated in the first paper (by F. Roberts), drawing up seven fundamental concepts, as RNA chains as "words" in a 4 letter alphabet, Interval graphs, Competition graphs or niche overlap graphs, Qualitative stability, Balanced signed graphs, Social welfare functions, and Semiorders.

Diversity of human and biological sciences manifests itself in the interesting and multicoloured topics in the remaining forteen papers. The list of authors (in the order of papers) is: J-P. Barthelemy, M. B. Cozzens, N. V. R. Mahadev, J-C. Falmagne, P. C. Fishburn, B. Ganter, R. Wille, E. C. Johnsen, V. Klee, J. C. Lundgren, J. S. Maybee, J. K. Percus, P. H. Sellers, P. D. Straffin Jr.

The volume is mainly based on the proceeding of a workshop which was organized in course of an IMA program on Applied Combinatorics.

J. Kozma (Szeged)

Applied Mathematical Ecology, Edited by Simon A. Levin, Thomas G. Hallam and Louis J. Gross (Biomathematics, 18), XIV+491 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1989.

This book contains the subject-matter of the Second Autumn Course on Mathematical Ecology held at the International Centre for Theoretical Physics in Trieste, Italy in November and December of 1986. The contents and the structure of the book is introduced by the editors in the Preface as follows: "This book is structured primarily by application area. Part II provides an introduction to mathematical and statistical applications in resource management. Biological concepts are interwoven with economic constraints to attack problems of biological resource exploitation, conservation of our natural resources and agricultural ecology. Part III consists of articles on the fundamental aspects of epidemiology and case studies of the diseases rubella, influenza and AIDS, Part IV addresses some problems of ecotoxicology by modelling the fate and effects of chemicals in equatic systems. Part V is directed to several topics in demography, population biology and plant ecology, with emphasis on structured population models."

The list of authors shows that the Autumn Course was participated by the most outstanding experts in Mathematical Ecology from all over the world. Their book must be found on the bookshelf of every specialist wishing to follow the main directions of the development of the field.

L. Hatvani (Szeged)

E. Arbarello—C. Procesi—E. Strickland, Geometry today, Giornate di Geometria, Roma 1984 (Progress in Mathematics, 60), 329 pages, Birkhäuser, Boston—Basel—Stuttgart, 1985.

The meeting "Giornate di Geometria" was held at the "Dipartimento di Matematica, Istituto G. Castelnuovo" during the period 4—9 June 1984. There were many mathematicians on this conference from almost all of the area of geometry. At the same time some top specialists were also there such as S. Donaldson, W. Fulton, P. Griffiths, V. Kac, D. Kazdhan, D. Mumford for example only. The book contains almost all of the talks given at the meeting, hence the reader finds accounts on geometry ranging from algebraic curves to topology, from non linear equations to algebraic groups and number theory.

We recommend this book to all who are interested in the modern geometry.

Árpád Kurusa (Szeged)

P. L. Barz—Y. Hervier, Enumerative Geometry and Classical Algebraic Geometry (Progress in Mathematics, 24), X+252 pages, Birkhäuser, Boston—Basel—Stuttgart, 1982.

This book is based on the conference held at the University of Nice during the period 23—27 June 1981. The major areas of the activity were enumerative geometry, curves and cycles and multiplicities. We mention that half of the papers are written in french. The papers of the book are from L. Gruson, C. Peskine, R. Piene, F. Catanese, W. Fulton, R. Lazarsfeld, D. Laksov, A. Beauville, A. Hirschowitz, M. S. Narasimhan, P. Le Barz, I. Vainsencher and S. L. Kleiman.

We recommend this book to graduate students and resarchers as well.

Árpád Kurusa (Szeged)

P. Biler—A. Witkowski, Problems in Mathematical Analysis, (Pure and Applied Mathematics) v+227 pages, Marcel Dekker, Inc. New York—Basel, 1990.

This is a truly excellent collection of problems in mathematical analysis, although several problems from other mathematical disciplines are also included. The level of problems varies considerably, but most of them are above the level of standard textbook exercises. Most of them require some trick or strong theoretical background. Some of the problems are very hard and have the flavour of research results. The collection was selected from several sources, many of them were taken from the American Mathematical Monthly.

The book, which contains about 1200 problems, is divided into nine sections: real and complex numbers, sequences, series, functions of one variable, functional equation and functions of several variables, real analysis, analytic functions, Fourier series and functional analysis. Each of them gives a very thorough account of the given field through fascinating problems, although I have found the first chapter more entertaining than the other ones. It starts out with the easy exersice that for irrational a and b the power a^b can be rational. But the trick is nice: $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}=2$. However, the next problem, that for a $c>8/3$ asks for the existence of a ϑ such that $[\vartheta^{c^n}]$ is prime for every n, is a much more challenging one.

Unfortunately the hints given to the problems are very scarce and very often of little use since they rather give the reference to the source of the problem than lending help in the solution. This makes the use of the book rather cumbersome since no one wants to run to the library every time he gets stuck with a particular problem. Sometimes no hint or reference is given at all, which may be puzzling for many readers in case of problems like the one which asks if it is possible to divide a square into an odd number of triangles of the same area (try it!). I found it a pity that the authors do not give more detailed hints or full solutions which would have made the book even more outstanding.

I would very strongly recommend the book to both students and teachers, but everyone who likes problem solving, which, according to many of us, is the heart of mathematics, will find hours, and hours of fun and enjoyment in the problems.

Vilmos Totik (Szeged)

A. Bohm—M. Gadella, Dirac Kets, Gamow Vectors and Gel'fand Triplets. The Rigged Hilbert Space Formulation of Quantum Mechanics, (Lecture Notes in Physics 348), VIII+119 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1989.

This book presents the Rigged Hilbert space formulation of Dirac's bracket formalism of quantum mechanism, preferred by most physicists for its elegance and practicality in actual calculations. It is an extension of the first author's well-known lecture notes (LNP 78) on the subject.

Dirac's formalism of bras and kets has been considered as mathematical nonsense by von Neumann, whose Hilbert space formulation became the standard, mathematically rigorous model of quantum mechanics. The right mathematics for describing Dirac's formalism appeared by the invention of the theory of distributions in the fifties, and the concept of Gel'fand triplets and the nuclear spectral theorem in the sixties, which make sense of the complete system of eigenvectors of self-adjoint operators with continuous spectrum. The discovery of these beautiful mathematical theories was inspired by Dirac's heuristic ideas.

This book not only gives a clear exposition of the mathematics of the Rigged Hilbert space formulation of Dirac's approach to quantum mechanics, in a languagage accessible to physicists, but also presents interesting physical applications concerning decaying states and resonances, by using the concept of Gamow vectors.

The reviewer recommends this volume to everybody interested in quantum mechanics, especially to graduate students studying physics or functional analysis, and university instructors lecturing on quantum mechanics.

László Fehér (Szeged)

David M. Burton, Elementary Number Theory (second edition), XVII+450 pages, Wm. C. Brown Publisher, Dubuque, Iowa, 1989.

The theory of numbers has occupied a unique position in the world of mathematics. This position is due to several facts, e.g., it has an unquestioned historical importance, it has several easily formulated but hardly solvable problems (this is the reason why it arises the interest of many amateurs), it is one of the best subjects for early mathematical instruction. We share Gauss' opinion, ,'Mathematics is the Queen of science, and number theory the Queen of mathematics".

The elementary number theory is an integral part of almost all undergraduate mathematical curriculum, therefore several textbooks are available on this topic. Nevertheless, Burton's very readable book is unique in some sense among them.

In each chapter we can read a historical introduction or/and they end with a historical outline. Evoking a nice old practice, we can find a pearl of quotation from mathematicians, philosophers or writers at the top of every chapter. There are problems at the ends of the chapters (the total amount is about 600) ranging in difficulty from the purely mechanical to challenging theoretical questions. They form an integral part of the text, to require the reader's active participation.

This second edition is an enlarged version of the first one (Allyn and Bacon, 1980). The substantial changes are an entirely new section on cryptography, the enlargement of the section on Fermat numbers, introduction of a variety of new topics, e.g., Merten's conjecture, absolute semiprimes, amicable number pairs, and primes in arithmetical progression. Some 150 additional problems are also included.

The first nine chapters (Some Preliminary Considerations, Divisibility Theory in the Integers, Primes and Their Distribution, The Theory of Congruences, Fermat's Theorem, Number-Theoretic Functions, Euler's Generalization of Fermat's Theorem, Primitive Roots and Indices, The Quadratic Reciprocity Law) can be used as a basic material of a one semester course. The additional four chapters (Perfect Numbers, The Fermat's Conjecture, Representation of Integers as Sums of Squares, Fibonacci Numbers and Continued Fractions) are independent of each other. They may be taken up at pleasure. Despite of the material is mostly classical, there are several hints to modern results, too (only in the second edition). Among the five appendices there are an outline of the prime number theorem and answers to selected problems.

This well-organized textbook is warmly recommended to any undergraduate number theory course.

Lajos Klukovits (Szeged)

Categorical Methods in Computer Science, Edited by H. Ehrig, H. Herrlich, H.-J. Kreowski and G. Preuss (Lectures Notes in Computer Science, 393), VI + 350 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1989.

This volume contains the papers presented at the International Workshop on Categorical Methods in Computer Science with Aspects from Topology held in Berlin in September 1988. The material is organized into three parts. The following quotation is from the Preface. "In part 1 we have collected papers concerning categorical foundations and fundamental concepts from category theory in computer science. Applications of categorical methods to algebraic specification languages and techniques, data types, data bases, programming, and process specifications are presented in part 2. The papers on categorical aspects from topology in part 3 mainly concentrate on special adjoint situations like cartesian closedness, Galois connections, reflections, and coreflections, which are of growing interest in categorical topology and computer science."

The volume can be recommended to those interested in categorical methods in computer science.

Z. Ésik (Szeged)

Collected papers of Paul Turán, Edited by P. Erdős. 3 Volumes, XXXVIII + 2665 pages. Akadémiai Kiadó, Budapest, 1990.

Paul Turán was born in 1910 in Budapest, Hungary. He died in 1976. He achieved a remarkably prolific career with publishing two books and 246 papers. The three volumes of his collected works contain the collection of most of his papers (some of them written to very specific audience were omitted). Many of the earlier papers are in German bacause the works were reproduced photocopically, a process that amplifies the immense variety of Turán's work. The exceptions to this are only the papers written in Hungarian that were translated to English. Naturally, his books about the power sum method invented by him in 1938 are not included in the list, but there are many research papers dealing with the method and its applications.

The main areas in which Turán worked are as follows: power sum method and its applications (some 70 papers), analytic number theory (cc. 60 p.), elementary number theory (15 p.), function theory (22 p.), approximation theory and interpolation (34 p.), Fourier series (8 p.), differential equations (11 p.), statistical group theory (18 p.), combinatorics (16 p.), numerical solutions of equations (10 p.), polynomials (8 p.).

Also included are some most interesting writings discussing the lifelong achievements of L. Fejér (his master), P. Erdős (his lifelong friend and coauthor), K. Rényi, A. Rényi, A. Baker (for Fields Medal), S. Knapowski (his student and coauthor), S. Ramanujan and young Hungarian mathematicians that were the victims of fascism. He himself was in a nazi labour camp during second world war, where he initiated extremal graph theory. One can read about this in two affectionate obituaries contained in volume I by P. Erdős and G. Halász.

Turán's works have initiated several new directions and stimulated the research of an enormous number of mathematicians, so it is natural that in many areas there have been new developments since the publications of his results. Therefore it is most appropriate that the collected works contain many mathematical notes written by L. Alpár, P. Erdős, G. Halász, J. Pintz, M. Simonovits, J. Szabados, M. Szalay and P. Vértesi on the progress in the subjects of the different papers. Unfortunately no list of these notes is included although it would have made easier for the reader to keep trac of these developments.

Paul Turán's collected papers are not the type of books that one would read from the beginning to the end, although I found it impossible to quickly paging through the volumes because quite often a title, a formula or a problem caught my eyes that forced me to read further. I am certain that browsing among Turán's papers will be truly enjoyable for every mathematician even if his or her field is completely different. No library can afford to miss these volumes, and for many of us it will be very pleasant to have them on our bookshelf.

Vilmos Totik (Szeged)

R. Courant—F. John, Introduction to Calculus and Analysis, Vol. I, XXIII+661 pages; Vol. II, XXIII+954 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong, 1989.

Although the first volume of this book was originally published in 1969 and the second one in 1974 it remained one of the best textbooks introducing several generations of mathematicians to higher mathematics. This book leads the students to the heart of the mathematical analysis preparing them for an active application of their knowledge. The main goal of this book is to exhibit the interaction between mathematical analysis and its various applications emphasizing the role of intuition furthermore the importance of the union of intuitive imagination and deductive reasoning. Numerous examples and problems are given at the end of the chapters. Some are challenging, some even difficult; most of them supplement the material in the text. The book is adressed to students on various level, to mathematicians and engineers. Volume I contains among others the following chapters: Integral and Differential Calculus; The Techniques of Calculus; Applications in Physics and Geometry; Taylor's Expansion; Infinite Sums and Products; Trigonometric Series; Differential Equations.

The most characteristic chapters of Volume II are: Functions of Several Variables and Their Derivatives; Vectors, Matrices, Linear Transformations; Applications; Multiple Integrals; Relations Between Surface and Volume Integrals; Differential Equations; Functions of Complex Variable.

This excellent book is highly recommended both to instructors and students.

J. Németh (Szeged)

CSL '88, Edited by E. Börger, H. Kleine Büning and M. M. Richter (2nd Workshop on Computer Science Logic, Duisburg, FRG, October 1988), Proceedings. (Lectures Notes in Computer Science, 385), VI+399 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1989.

This volume is a collection of 24 papers presented at the workshop "Computer Science Logic" held in Duisburg, from October 3 to 7, 1988. The papers cover a broad class of topics ranging from logical aspects of computational complexity to the acceptance of ω-regular languages under various fairness constrains. Below we briefly discuss three contributions of particular interest to the reviewer.

In the paper "Characterizing complexity classes by general recursive definitions in higher types" by A. Goerdth, it is proved that recursive definitions of rank $n+1$ correspond to the complexity class $\bigcup(\mathrm{DTIME}\ (\exp_n(p(x))): p(x)$ *a polynomial*). Consequently, due to a hierarchy theorem of complexity classes, rank n recursive definitions form a proper hierarchy.

Star-free regular languages have attracted a lot of interest in theoretical computer science. By McNaughton's theorem, star-free regular sets are exactly those definable by some first-order sentence in a suitably chosen language. The paper "Interval temporal logic and star-free expressions" by D. Lippert relates star-free languages and a generalisation thereof to interval temporal logic, a kind of logic introduced for the specification of digital circuits.

Automata and tree automata have continued to play an important role in establishing decidability of certain logics. In the paper "On the emptiness problem of tree automata and completeness of modal logics of programs" by H. Wagner, it is proved that the non-emptiness problem of alternating tree automata is P-complete. This result is then used to show that the satisfiability problem of Propositional Dynamic Logic with a repeat construct is EXPTIME-complete.

The volume can be recommended to those interested in recent research in logical aspects of theoretical computer science.

Z. Ésik (Szeged)

Gerald A. Edgar, Measure, Topology, and Fractal Geometry, (Undergraduate Texts in Mathematics), +230 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong, 1990.

Nowdays the fractals are in the center of the scientists' interest. Since Benoit Mandelbrot established the notion and phylosophy of fractals, quite a lot of books were published on this subject.

Now here is a mathematics book about fractals. The authors' main aim was to give a systematic discussion of the topological and measure theoretical background and to present the most important ideas of fractal geometry.

In Chapter 1 the most basic examples of fractal sets are introduced, such as the Cantor set, the Sierpinski Gasket, the Koch curve, to motivate the "whole story". Chapter 2 is a very good introduction to the topology of metric spaces and Chapter 3 contains the basics of topological dimension theory (small and large inductive dimensions). Chapter 4 is devoted to the complete and detailed discussion of the self-similarity and the more general "graph self-similarity". Here can be found the description of iterated function systems which is an efficient way of generating fractal sets, discovered is recent years by Michael Barnsley. In Chapter 5 the Lebesgue measure and the general methods of generating outer measures and measures are discussed. In Chapter 6 the Hausdorff measure and the Hausdorff dimension are introduced and various fractal dimensions are compared to each other. Finally in Chapter 7 some additional topics are discussed.

Each section contains several exercises for practicing the use of the notions and theorems. The book is written in a nice style illustrated by a lot of figures.

This text is recommended to students as a first course on fractal geometry but it may be useful to anybody who is interested in the rigorous mathematical background of fractals.

J. Kincses (Szeged)

Bernard d'Espagnat, Reality and the Physicist. Knowledge, duration and the quantum world, 280 pages, Cambridge University Press, Cambridge—New York—New Rochelle—Melbourne—Sydney, 1989.

Since the very beginning of this science, quantum mechanics has always been a source and a field of philosophical debates. The founders of this discipline were fully aware of the fact, that the results of quantum theory are in sharp contradiction with the concepts of classical mechanics. They revealed, that microscopic objects are strongly influenced by the measuring apparatus, and the term of physical phenomenon makes sense only, if we take into account the whole apparatus that produces a result of an observation. This process is a kind of collapse, in which the original state of the system changes radically and irreversibly. According to the orthodox view, this final step takes place in the mind of the observer, which is in contradiction with realism, with the principle of the existence of independent reality. For a long time physicists gave up the idea of digging more deeply into such questions, regarding them to belong to the field of philosophy. They rather made use of the calculation rules of quantum physics, which proved to be a very succesful theory.

The debate has been showing a revival in recent years, because it turned out, that the consequences of most simple and logical assertions about a physical system can be put into the form of inequalities (the Bell inequalities), the validity of which can be tested by (much less simple) experiments. And the experiments show, that these most plausible inequalities are violated, while the predictions of quantum mechanics are confirmed. The novelty of these experiments lies in the fact, that the collapse manifests itself on a macroscopic scale, when the "parts" of a single quantum system are several meters apart.

The book, whose author is a well-known theoretical physicist and philosopher, is intended to clear up the situation, stating as precisely as possible the different views on this problem. It is out of question, that d'Espagnat enriches the concept of independent reality and its relation to physical observations. He places his own views somewhere between those of the positivists and the materialists. The book concentrates on the philosophical aspects of the issue, and almost totally avoids the mathematical technicalities, as well as the description of physical experiments. This is certainly a merit, because the book can be recommended to everybody, interested in natural philosophy and the fundamental problems of the material world. Nevertheless, I would propose that the reader should get acquainted with the article by the same author in the Scientific American (vol. 241, p. 158, 1979), where some part of the background is explained in simple terms. This paper, as well as a short synopsis of more recent experimental work, might have been added as an appendix to the text. But anyway, there are a plenty of deep and interesting thoughts in this book, and it enforces us to think over: how absurd independent reality can be on the quantum level.

M. G. Benedict (Szeged)

H. Gross, Quadratic Forms in Infinite Dimensional Vector Spaces (Progress in Mathematics, 1), XXII+419 pages, Birkhäuser, Boston—Basel—Stuttgart, 1979.

Well, one can say this book is old (ten years have gone since its publication) but I think no one can say it is absolete. Although many new results have born in the last decade, for example H. A. Keller's non classical Hilbert space (Math. Z. 172, 41—49), most of the more important results until 1979 are collected in this book together with the directions of the present researches. Its clear style and carefully considered built up makes it still the best book in the subject in my opinion.

The contents of the book are gathered around some important notions and theorems. These are the sesquilinear forms, the diagonalization of $\aleph_0$-forms, the Witt decomposition for Hermitean $\aleph_0$-forms, the quadratic forms and the theorems of Witt an Arf. Every sections are written almost just like a paper closed with specific reference list and some of them with appendix. This helps the reader very well.

I think this book should be on the shelf of every mathematician who makes research on this subject.

Árpád Kurusa (Szeged)

Günther Hämmerlin—Karl-Heinz Hoffmann, Numerische Mathematik, XII+448 pages, Springer-Verlag, Berlin—Heidelberg—New York (Grundwissen Mathematik, B. 7), 1989.

This book offers all the material of the customary one-year introductory courses and a lot of extras.

Its main merit is the clear and intelligent mathematical treatment of the problems. There are numerous consize proofs, illuminating examples and fascinating historical remarks throughout the text.

Even the first Chapter on numerical calculations and algorithms contains supplements on backward error analysis branch-and-bound algorithms and complexity issues.

In Chapters 2 and 3 Numerical Linear Algebra, i.e., Systems of Linear Equations and the Eigenvalue Problem are treated.

The main body of the book follows: Chapter 4: Approximation, Chapter 5: Interpolation and Chapter 6: Splines. In this part the standard topics are investigated with deep mathematical insight. Moreover, the questions of the (two and) finite dimensional interpolation and approximation are touched on.

Chapter 7: Integration starts with the elementary interpolation quadrature rules, extrapolation methods and departs to the special issues of optimal quadrature rules and Monte Carlo methods.

Chapter 8: Iteration gives the basic material on iteration methods for systems of linear and nonlinear equations.

The final Chapter 9: Linear Programming traces out the theoretical background the different variants of the Simplex Method and ends with the polynomial algorithms of Karmarkar and Khachyan.

A Guide on General Literature, an Index and 270 not-only-routine exercises complete this valuable book. It can be recommended to anybody who wants to get a general overview of the spirit and the methods of the Numerical Analysis.

J. Virágh (Szeged)

Micha Hofri, Probabilistic Analysis of Algorithms (Text and Monographs in Computer Science), XV+240 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1987.

To analyse of an algorithm there are two different basic methods. One of them has the objective to find the running time of the algorithm operation in the worst-case in the term of some specified function.

In the other one the operations of algorithms are shown to be associated with probabilistic concepts and processes. In this sense there are two subclasses: On one hand there are explicitly introduced operations in the algorithm and they are choosed on the basis of random elements (pseudorandom numbers, simulated coin flipping etc.). On the other hand we have the operations of a deterministic algorithm and we consider the input data over which some probability measure can be stipulated.

Among the algorithms for which the book provides detailed analyses, the reader finds examples of both varieties. Chapter 1 shows that the second type brings up methodological and conceptual problems that the first case need not entail. Since the probabilistic analysis of algorithms, as a discipline, draws on a fair number of mathematics Chapter 2 is dealing with some of them as introduction to asymptotics, generating functions, integral transforms, combinatorial calculus, asymptotics from generating functions and some selected results from probability theory.

The remining part of the book gives applications. Chapter 3 presents algorithms over permutations (locating the largest term in a permutation, representations of permutations, analysis of sorting algorithms). Chapter 4 contains algorithms for communications network, and Chapter 5 is dealing with bin packing problems.

This is a good book which is recommended to all people who are working in the given fields.

G. Galambos (Szeged)

Irregularities of Partitions, Edited by G. Halász and V. T. Sós (Algorithms and Combinatorics, 8), VIII+168 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1989.

The problem of uniform distribution of sequences has now become an important part of number theory, and this is also true for Ramsey theory in relation to combinatorics. This volume is the homogeneous account of a workshop held at Fertőd in Hungary. Participants discussed the recent emergence of close links between Ramsey theory in combinatorics and the theory of uniform distribution in number theory.

The titles and authors of papers are: J. Beck and W. W. L. Chen: Irregularities of Point Distributions Relative to Convex Polygons; J. Beck and J. Spencer: Balancing Matrices with Line Shifts II; M. Cochand and P. Duchet: A Few Remarks on Orientation of Graphs and Ramsey Theory; P. Erdős, A. Sárközy and V. T. Sós: On a Conjecture of Roth and Some Related Problems I; Ph. Flajolet, P. Kirschenhofer and R. F. Tichy; Discrepancy of Sequences in Discrete Spaces; P. Frankl, R. L. Graham and V. Rödl: On the Distribution of Monochromatic Configurations; A. Gyárfás: Covering Complete Graphs by Monochromatic Paths; H. Lefmann: Canonical Partition Behavior of Cantor Spaces; L. Lovász and K. Vesztergombi: Extremal Problems for Discrepancy; J. H. Loxton: Spectral Studies of Automata; M. Mendes France: A Diophantine Problem; J. Nesetril and P. Pudlák: A Note on Boolean Dimension of Posets; Zs. Tuza: Intersection Properties and Extremal Problems for Set Systems; G. Wagner: On an Imbalance Problem in the Theory of Point Distribution.

Z. Blázsik (Szeged)

W. Klingenberg, Lineare Algebra und Geometrie, zweite verbesserte Auflage, XIII+293 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1990.

This book consists of ten chapters. The material of the first five chapters has an algebraic character. In accordance with the didactical motivation, the text starts with an exposition of the classical algebraic structures: groups, rings and fields. Then moduls and vector spaces, basis systems, dimension of vector spaces and dual spaces are defined. Matrices are first formally defined and then the connection between matrices and linear operators is showed. Solution of linear equation systems and the notion of determinants are also given. At the end of the algebraic part eigenvalues and normal forms of linear operators are discussed and as an application linear differential equation systems are investigated. With the sixth chapter starts the geometric part. First normed vector spaces are introduced. Affin and projectiv spaces are considered over general finite dimensional vector spaces. If the finite dimensional vector space is endowed with an euclidean norm, then the affin space over this one supplies the euclidean, the projectiv space supplies the elliptic geometry. If the vector space is endowed with a Lorenz metric, then the affine space over this supplies the hyperbolic geometry. The main theorem of the affin and projectiv spaces with which the general collineations are characterized is completed with Staudt theorem concerning bijections of a projectiv line.

L. Gehér (Szeged)

R. Kress, Linear Integral Equations (Applied Mathematical Sciences, 82), XI+299 pages. Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1989.

Sometimes the classical theory of integral equations serves as an introduction to the abstract theory of compact operators, on the other hand the theory of integral equations is derived as an important application of the operator theory. Mostly the numerical methods are treated separately.

The aim of the author of this book is to attach the same value on the theory, the application and on the numerical methods. This is a considerable task from scientific and pedagogical aspects as well. Integral equations are useful for engineers, too. Therefore it is desirable that the work should be readable for them. So presenting a modern introduction one cannot begin by saying "you must have solid backgrounds in differential and integral calculus, in differential equations, in complex function theory, in functional analysis, in numerical methods" and so on. (Something like this is often presumed implicitly.) The author of this book relies on bases which — I think — are expectable from trained readers. Some useful and necessary topics are briefly presented.

Roughly speaking the work consists of four main fields: the Riesz—Fredholm theory for integral equations of the second kind; the classical applications (Laplace and heat equation, singular integral equations); introduction to the numerical solution of the equations and finally, ill-posed integral equations of the first kind. These are done in 18 chapters.

The proofs are clear, detailed in a suitable manner. In several cases the considerations are more elementary and straightforward than as customary.

As an example let us quote here the outline of the third chapter. This consists of three points: Riesz theory for compact operators (in my opinion remarks which may be found for example here such as "The main importance of the results of the Riesz theory for compact operators lies in the fact that we can conclude existence from uniqueness as in the case of finite dimensional linear equations." are valuable for the readers; Spectral theory for compact operators (the former results in terms of spectral analysis); Volterra integral equations (the result is formulated in the classical way and also in terms of spectral theory). Finally, as at the end of the other chapters as well we find interesting problems without solutions but with hints in some cases.

I am sure that after reading this book everyone will like integral equations a bit better which was indeed the author's aim.

L. Pintér (Szeged

Y. A. Kubyshin—J. M. Mourão—G. Rudolph—I. P. Volobujev, Dimensional Reduction of Gauge Theories, Spontaneous Compactification and Model Building, (Lecture Notes in Physics, 349), X+80 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1989.

At present there is a general agreement among physicists that the experimental facts of particle physics are correctly reproduced by the so called standard model. The most challenging problem of theoretical particle physics is to produce a theory unifying the Weinberg-Salam-Glashow model of the electroweak force with the quantum chromodynamics describing the strong interaction, and, if possible, describing gravity as well. The dimensional reduction approach to this problem presented in this monograph is a modern version of the ideas put forward by T. Kaluza and O. Klein in the twenties.

In the first part of the book the authors discuss the dimensional reduction of pure Yang-Mills theories. In particular, they present a general method for calculating the scalar potential. The second part is devoted to the dimensional reduction of gravity and to spontaneous compactification. In the final part matter fiels and some aspects of model building are considered.

Throughout the book, the authors make extensive use of homogeneous spaces of Lie groups and connections on fibre bundles. They exhibit the global aspects of the dimensional reduction method and give all the important formulae in local terms as well.

It seems that till now nobody succeded in constructing a model describing the fundamental interactions in a unified scheme, which is satisfactory in all respects. The dimensional reduction approach to constructing such a model deserves further investigation. This book is primarily intended for researchers and graduate students working on this program. It is also recommended to physicists and mathematicians interested in unified field theories and in applications of differential geometry.

László Fehér (Szeged)

Serge Lang, Undergraduate Algebra (Undergraduate Text in Mathematics.), IX+256 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1987.

The book is a second part of an algebra program which is addressed to undergraduates. The theme of Chapter 1 is the set of the real numbers. After some basic properties such important definitions are introduced as the greatest common divisor, the unique prime factorization and the equivalence relations and congruences. The next two Chapters are dealing with the groups and rings with general definitions on mappings, the homomorphisms and automorphisms. Among the groups the permutation groups, the cyclic groups and the finite Abelian groups are studied in details. In the Chapter on rings there are mentioned some basic theorems on their homomorphisms. In Chapter 4 the polynomials are considered. The Euclidean Algorithm, the greatest common divisor, the unique factorization and the partial fractions are introduced. The Chapter is closed by examinations on polynomials over the integers, the principial rings and the factorial rings. Vector spaces and modules are considered in Chapter 5. After some basic definitions (vector space, subspace, generators, basis, homomorphism, kernel) some theorems are presented on the dimension of a vector space. Subsections are dealing with the linear maps, the modules, the factor modules and the free Abelian groups.

For familiar readers have been suggested the next two Chapters which are dealing with some linear groups as the general linear group $(GL_n(K))$. Theorems are introduced for the structure of $GL_2(F)$ and $SL_2(F)$. The Chapter 7 considers the elements of Field Theory: embeddings, splitting fields are mentioned. Basic theorems on the Galois Theory are given. In Chapter 8 the finite fields are considered. Chapter 9 introduces some theorems on the real and complex numbers, and the book is closed with the examinations on the sets. In this section such well-known theorems are considered as the Zorn-Lemma and the Schroeder—Bernstein Theorem.

This book is an elementary text in the Algebra and so a lot of examples are introduced together with the development of the abstractions. The author intended to write a self contained book. The aim has been obtained.

G. Galambos (Szeged)

W. Y. Lick, Difference Equations from Differential Equations (Lecture Notes in Engineering), X+282 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1989.

In mathematical physics and in other branches of the practical problems the first task is to construct a correct model. (Of course every model has some imperfection.) In mechanics one generally obtains a differential equation. The investigation of the obtained equation is twofolded. By using qualitative methods one gets general properties of the solutions and on the other hand we try to present the solution or an approximation of the solution in an explicit form. In general this is done by translation of the differential equation into accurate, stable and physically realistic difference equation and the investigation of this task is the aim of the author.

There are several methods to form difference equations from differential equations. A brief and clear survey of these methods can be found in the Preface. The advantages and disadvantages of every single method are enumerated. In the author's opinion the volume integral method seems to be superior to other methods, therefore this is the primary method used in this book. The application of this single procedure makes the work easier to understand and at the same time it gives more possibility for deriving new and improved difference equations.

The book consists of five chapters. In the first four ones the volume integral method is applied for ordinary and partial differential equations. (Parabolic, hyperbolic and elliptic partial differential equations are treated separately.) Chapter 5 contains the applications of the ideas and algorithms treated formerly for special problems. Let us list them: currents in aquatic systems; the transport of fine-grained sediments in aquatic systems; chemical vapor deposition; free-surface flows around submerged or floating bodies.

The text is clearly written and well-organized. The emphasis on the important role of the basic physical problem is a characteristic feature of the investigations.

In my opinion this book is valuable not only for phisicists, engineers and computer scientists but for mathematicians who are interested in the qualitative theory of differential equations, as well.

L. Pintér (Szeged)

D. Lüst—S. Theisen, Lectures on String Theory, (Lecture Notes in Physics 346), VIII+346 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1989.

In the past few years string theory has been one of the most active areas of theoretical, mathematical physics. Although its relevance for explaining the mysteries of Nature still has not been proven, there can not be any doubt at all that it greatly contributed to the interaction of mathematics and physics. For example, string theory played an important role in the development of conformal field theory, which involves the fascinating mathematics of the Kac-Moody and the Virasoro algebras.

This introduction to string theory is an expanded version of the lectures given by the authors at the Max-Planck-Institut für Physik und Astrophysik in Munich in fall and winter 1987/1988. The authors present a standard introduction to the bosonic and fermionic strings in the critical dimensions, and give a detailed description of the covariant lattice construction of four-dimensional heterotic strings. They give a clear introduction to conformal field theory, including the supersymmetric version, and emphasize its role in constructing four-dimensional strings.

This book will prove useful for graduate students and researchers interested in string theory and is warmly recommended.

László Fehér (Szeged)

Mathematical Logic and Applications, Edited by J. Shinoda, T. A. Slaman and T. Tugué (Lecture Notes in Mathematics, 1388), 222 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong, 1989.

These are the proceedings of the '87th Meeting on Mathematical Logic and its Applications held at the Research Institute of Mathematical Sciences of Kyoto University during August 3—6, 1987. The authors are C. T. Chong, Y. Kakuda, H. Katsutani, S. Kobayashi, M. Shimoda, J. Shinoda and T. A. Slaman, T. A. Slaman and W. H. Woodin, T. Yamakami and M. Yasugi.

Vilmos Totik (Szeged)

Meyberg-Vachenauer, Höhere Mathematik 1. Differential — und Integralrechnung Vektor — and Matrizenrechnung, XIV+517 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1990.

The text is divided into eight chapters. The first chapter is of introductory character. Here the real and complex numbers, vectors, lines and planes are introduced. In the second chapter the limit of number sequence, the limit value and continuity of functions of one variable are defined. The third chapter is devoted to developing the differentiation theory of functions and its applications- At the end of this chapter the exponential and logarithm functions are introduced and discussed. Chapter 4 deals with the integration theory of functions and applies the theory for determining the length of a curve, the area of a rotation surface and the volume of a rotation body. Numerical in. tegration is also shortly discussed. Chapter 5 introduces the concept of convergence of number series and function series. The power series and especially the Taylor series are examined in detail. Chapter 6 is a glimpse into linear algebra, where the usual notions and theorems are given. Chapter 7 investigates functions of several variables, defines the differentiation of such functions and finally develops the differentiation theory of functions with vector values. Chapter 8 treats the theory of integration of functions of two and three variables and the theory of line and surface integration.

The book is recommended to students in the first two semesters.

L. Gehér (Szeged)

Angelo B. Mingarelli—S. Gotskalk Halvorsen, Non-Oscillation Domains of Differential Equations with Two Parameters (Lecture Notes in Mathematics 1338), XI+109 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988.

Nowadays the literature of the qualitative theory of the linear second order ordinary differential equations fills almost a whole library. To find some new and interesting result is not an easy task. In this book the authors present an important problem, new results and open questions. The starting point is the investigations of R. A. Moore on the equation $y''+(-\alpha+\beta\cdot B(x))y=0$ (1), where α, β are real parameters, the function B is continuous, periodic of period one and has mean value equals to zero. Several well-known equations (Hill, Mathieu etc.) are of this form with various B. Equation (1) will be called disconjugate on R if and only if every nontrivial solution has at most one zero in R. (1) will be called non-oscillatory on R if and only if every nontrivial solution has at most a finite number of zeros in R. The pairs $(\alpha, \beta)\in R^2$ for which (1) is disconjugate resp. non-oscilatory constitute the disconjugacy domain resp. nonoscillation domain of (1) and these sets are denoted by D resp. N. Moore proved that $D=N$ and N is a closed, convex unbounded set. The main problem of this book is the investigation of sets D and N of the equation $y''+(-\alpha\cdot A(x)+$ $+\beta\cdot B(x))y=0$, where x is nonnegative, the functions A, B are Lebesgue integrable on every compact subset of the nonnegative reals. In cases treated in this work D is colsed, convex, bounded or unbounded set, $D\subset N$, N is convex, but N is not always closed. Naturally more interesting questions arise on D and N and their connections. Chapter headings are: Introduction, Scalar linear ordinary differential equations; Linear vector ordinary differential equations; Scalar Volterra—Stieltjes integral equations.

From the style of this work it seems to me that the authors do not take the topic as their own hunting-field and they would not mind if somebody else should solve some of their problems.

L. Pintér (Szeged)

J. D. Murray, Mathematical Biology (Biomathematics, 19), XIV+767 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1989.

The most intensive development of sciences can, to my mind, be waited in the biology. One of the good omens of this is that more and more biological models are constructed and investigated by mathematical methods. This is the way that creates a good possibility of the interaction of mathematical and biological researches and the established involvement would be useful not only for the development of biology, but the mathematics itself should benefit from this connection.

Murray's new book takes an inspiring influence on the involvement of these two sciences. It contains a lot of models from several branches of the biology, for example from the population ecology, reaction kinetics, biological oscillators, the developmental biology, the evolution, the epidemology and so on. The most important biological laws of studied phenomena can be found in it; therefore, the reader will attain a great practice in modelling of biology.

To understand and follow this book, no serious preliminary biological knowledges are needed. The reader has to be familiar only with the basic calculus and differential equations. The authors have involved only deterministic models described by ordinary differential equations, delay equations, integro-differential equations, partial differential equations and their discrete analogies. The used mathematical tools, as such as the catalogue of singularities in the plane, Poincaré-Bendixson theorem, Routh—Hurwitz conditions, Juri conditions, Hopf bifurcation theorem, the properties of Laplacian operators in bounded domains are collected in an appendix that is a great help to the reader.

The book contains simpler and more particular models, too. So, on the one hand this book is an excellent handbook for investigators working in the field of biomathematical modelling. The reason is not only that it provides a good survey on deterministic models of the biology, but its style is suitable for giving inspirations for further researches. On the other hand, the simpler models in the book assure possibility to use it as an introduction for beginer scientific workers in this branch and also in the teaching differential equations.

The book is easy-to-read. The clearness is assured by numerous figures, diagrams. At the same time, the style is deeply interesting since the results obtained by theoretical methods are compared with experimental dates.

The book is recommended to specialists in biomathematics, differential equations, to biologists interested in mathematics, and to graduated students in mathematics and biology.

J. Terjéki (Szeged)

New Integrals, Proceedings. Coleraine 1988. Edited by P. S. Bullen, P. Y. Lee, J. L. Mawlin, P. Muldowney and W. F. Pfeffer (Lecture Notes in Mathematics, 1419), 202 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1990.

In recognition of the pioneering work done by Ralph Henstock in the field of post-Lebesgue integration theory, the 1988 Summer Symposium on Real Analysis was held in Coleraine. The papers in this volume cover current research in generalised Riemann, Denjoy and Perron integration. The 15 papers contained in this volume are written by R. Henstock, P. S. Bullen, T. S. Chew, S. F. L. de Foglio, C. Pierson-Gorez, J. Kurzweil and J. Jarnik, S. Leader, P. Y. Lee, P. Maritz, P. Muldowney, P. Mikusinski and K. Ostaszewski, W. F. Pfeffer, V. A. Skvortsov, J. D. Stegeman.

The wide range ensures that everybody interested in integral theory will find at least one paper of his own interest.

J. Németh (Szeged)

Number Theory and Dynamical Systems (London Mathematical Society Lecture Note Series, 134), Edited by M. M. Dodson and J. A. G. Vickers, 172 pages, Cambridge University Press, Cambridge—New York—Port Chester—Melbourne—Sydney, 1989.

Fifty years ago the title of this book would have been a great surprise. Nowadays number theory appears in various branches of practical applications. Therefore the connection of number theory and dynamical systems is not so astonishing, but at the same time it invariably holds that the combination of various branches produces significant results.

In connection with number theory and dynamical systems let us mention only two facts. One is the Kolmogorov—Arnold—Moser theorem concerning the question of stability of the solar system. The other is Furstenberg's proof of Szemerédi's theorem on arbitrarily long arithmetic progressions in infinite integer sequences. But we could cite several other examples, too.

This book consists of contributions from a Conference on Number Theory and Dynamical Systems held at the University of York in 1987. Perhaps a little characterizing are the addresses of the contributions: H. Rüssmann: Non-degeneracy in the perturbation theory of integrable dynamical systems; J. A. G. Vickers: Infinite dimensional inverse function theorems and small divisors; S. J. Patterson: Metric Diophantine approximation of quadratic forms; Caroline Series: Symbolic dynamics and Diophantine equations; S. G. Dani: On badly approximable numbers, Schmidt games and bounded orbits of flows; S. Raghavan and R. Weissauer: Estimates for Fourier coefficients of cusp forms; K. J. Falconer: The integral geometry of fractals; J. Harrison: Geometry of algebraic continued fractals; Michel Mendes Frances: Chaos implies confusion; J. V. Armitage: The Riemann hypothesis and the Hamiltonian of a quantum mechanical system.

Not only researchers in dynamical systems or in number theory can find interesting ideas in this volume but every curious mathematician, too.

L. Pintér (Szeged)

Numerical Methods for Ordinary Differential Equations, Proceedings of the Workshop held in L'Aquila, 1987. Edited by A. Bellen, C. W. Gear and E. Russo (Lecture Notes in Mathematics, 1368), VII+136 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1989.

This slim volume contains the following 8 invited lectures of the Workshop. C. Baiocchi: Stability in Linear Abstract Differential Equations. — A. Bellen: Parallelism Across the Steps for Difference and Differential Equations. — D. Di Lena, D. Trigiante: On the Spectrum of Families of Matrices with Applications to Stability Problems. — C. W. Gear: DAEs; ODEs with Constraints and Invarianst. — P. J. van der Houwen, B. P. Sommeijer, G. Pontrelli: A Comparative Study of Chebyshev Acceleration and Residue Smoothing in the Solution of Nonlinear Elliptic Difference Equations. — O. Nevanlinna: A Note on Picard—Lindelöf Iteration. — S. P. Norsett, H. H. Simonsen: Aspects of Parallel Runge—Kutta Methods. — L. F. Shampine: Tolerance Proportionality in ODE Codes.

In these research and survey papers the connections between the classical background of numerical initial value ODE methods and new reserarch aras such as differential-algebraic equations, effective stepsize control and parallel ODE solver algorithms for small — and large — scale parallel architectures are investigated.

This volume may be of interest to researchers and graduate students in the ODE field.

J. Virágh (Szeged)

G. Nürnberg, Approximation by Spline Functions, XI+243 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1989.

Splines play an important role in applied mathematics since they possess high flexibility to approximate efficiently, even nonsmooth functions which are given explicitly or only implicitly, e.g. by differential equations.

The aim of this book is to deal with basic theoretical and numerical aspects of interpolation and best approximation by polynomial splines in one variable.

In Chapter I basicly the unique solvability of interpolation problems for Chebyshev spaces is invesitgated, furthermore the construction of interpolating polynomials is given, and the best approximation by functions from Chebyshev spaces in the uniform norm, L_1-norm, L_2-norm is detailed.

Chapter II is devoted to the following main topics: Weak Chebyshev Spaces; B-Splines; Interpolation by Splines (for example Lagrange and Hermite Interpolation by Splines); Best Uniform Approximation by Splines (Algorithms with fixed knots and free knots are detailed); Best L_1-Approximation by Weak Chebyshev spaces; Best One-Sided L_1-Approximation by Weak Chebyshev Spaces and Quadrature Formulas; Approximation of Linear Functionals and Splines. From Appendix the section on Splines in Two Variables should be mentioned. The fact that a large number of new results presented in this book cannot be found in earlier books on spline makes it really valuable one.

This excellent book can be very useful for graduate courses on splines or approximation theory. Only basic knowledge of analysis and linear algebra is supposed.

The book is warmly recommended to everybody interested in approximation theory.

J. Németh (Szeged)

Ortogonal Polynomials, Theory and Practice, Edited by P. Nevai with the assistance of M. E. H. Ismail, (NATO ASI Series C., 294) xi+472 pages, Kluwer Academic Publishers, Dodrecht, 1990.

A NATO Advanced Study Conference was organized by P. Nevai during May 22, 1989 and June 3, 1989 in Columbus, Ohio on "Orthogonal Polynomials and Their Applications". The volume under review contains the proceedings of this conference. Most of the leading researchers of the theory of orthogonal polynomials and related subjects that live east or west of the USSR attended the conference, so its proceedings provide up to date insight of current research, the available methods and applications.

Two main parts can be distinguished in the book: there are papers the primary aim of which is to introduce the readers to applications of orthogonal polynomials, while others are dealing mainly with the extension of the theory. A few papers can be considered to belong to both parts. The theoretical papers can further be classified as those dealing with the algebraic aspects of the theory and the relation of orthogonal polynomials to special functions and combinatorics, while others discuss the analytic properties of orthogonal polynomials.

Among the applications and interrelation with other branches of mathematics are: coding theory and algebraic combinatorics (E. Bannai), Padé approximation and Julia sets (D. Bessis), digital signal processing (P. Delsarte and Y. Genin), functional analysis (J. Dombrowski), numerical analysis (W. Gautschi), Schroedinger equation (R. Haydock), birth and death processes (M. Ismail, J. Letessier, D. M. Masson and G. Valent), Hopf algebras and quantum groups (T. Koornwinder), group representation (D. Stanton).

A sketchy list of the topics dealing mainly with questions of the theory is the following: characterization theorems for orthogonal polynomials (W. A. Salam), three term recurrence relations and spectral properties (T. S. Chihara; W. Van Assche), rational function extensions on the unit circle (M. M. Djrbashian), special functions and symbolic computer algebraic systems (G. Gasper), moment problems and orthogonal polynomials with respect to exponential weights (D. Lubinsky), root systems (I. G. Macdonald), extensions of the beta integral (M. Rahman), orthogonal matrix polynomials (L. Rodman), complex methods (E. B. Saff), potential theory and n-th root asymptotics (H. Stahl and V. Totik).

This excellent book should serve as a standard reference for researchers in the field, but it can also be recommended to students because many of the papers are of introductory nature.

Vilmos Totik (Szeged)

Gilles Pisier, The Volume of Convex Bodies and Banach Space Geometry, Cambridge Tracts in Mathematics, +250 pages, Cambridge University Press, Cambridge—London—New York—Port Chester—Melbourne—Sydney, 1989.

During the last decade, considerable progress was achieved in the Local Theory, i.e. the part of Banach Space Theory which uses finite dimensional tools to study infinite dimensional spaces. One of the leading schools of this subject is the Israel Seminar on Geometric Aspects of Functional Analysis (three Springer Lecture Notes volumes mark their works). The author of the present book is an outstanding researcher of this topic. The aim of this book is to present a self-contained discussion of a number of recent results. A very powerful method is introduced which is a combination of the classical theory of convex sets, probability theory and approximation theory. One of the main ideas is to get quantitative versions of theorems on convex bodies. For example the quantitative version of the famous result of Dvoretzky, due to V. D. Milman, is the following:

Let B be the unit ball of an n-dimensional Banach space. Given $\varepsilon>0$, there exists a subspace F with dimension $[\varphi(\varepsilon)\log n]$ ($\varphi(\varepsilon)>0$ depending only on ε) and an ellipsoid $D\subset F$ such that

$$D\subset B\cap F\subset(1+\varepsilon)D.$$

The book is divided into two parts. The object of the first part (Chapters 1 to 9) is to give detait led proofs of three fundamental results:

(I) The quotient of subspace Theorem due to Milman: For each $0<\delta<1$ there is a constant $C=C(\delta)$ such that every n-dimensional normed space admits a quotient of a subpace $F=E_1/E_2$ (with $E_2\subset E_1\subset E$) with dimension dim $F\geqq\delta n$ which is C-isomorphic to a Euclidean space.

(II) The inverse Santalo inequality due to Bourgain and Milman: There are positive constants α and β (independent of n) such that for all balls $B\subset\mathbf{R}^n$ we have

$$\alpha/n\leqq(\operatorname{vol}(B)\operatorname{vol}(B^0))^{1/n}\leqq\beta/n.$$

(The upper bound goes back to a 1949 article by Santalo.)

(III) The inverse Brunn—Minkowski inequality due to Milman: Two balls B_1, B_2 in $\mathbf{R}^n$ can always be transformed (by a volume preserving linear isomorphism) into balls $\tilde{B}_1$, $\tilde{B}_2$ which satisfy

$$\operatorname{vol}(\tilde{B}_1+\tilde{B}_2)^{1/n}\leqq C[\operatorname{vol}(\tilde{B}_1)^{1/n}+\operatorname{vol}(\tilde{B}_2)^{1/n}]$$

where C is a numerical constant independent of n. Moreover, the polars $\tilde{B}_1^0$, $\tilde{B}_2^0$ and all their multiples also satisfy a similar inequalty.

The second part (Chapters 10 to 15) is devoted to the discussion of recently introduced classes of Banach spaces of weak cotype 2 and weak type 2 and the intersection of these classes, the weak Hilbert spaces.

The book is recommended to researchers in functional analysis but it may be useful to convex geometers, too.

J. Kincses (Szeged)

Philip Protter, Stochastic Integration and Differential Equation. A New Approach (Application of Mathematics, 21), X+302 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1990.

The novelty of this introductory book is that the author defines a semimartingale as a stochastic process wich is a "good integrator" on an elementary class of processes, rather than as a process of general Walsh series is equivalent to the study that can be written as the sum of a local martingale and a finite variation process.

At first an intuitive Riemann-type definition of the stochastic integral as the limit of sums is given for the adapted processes having left continuous paths with right limits. This is sufficient to prove many theorems including Itô's formula. Then it is shown that the "good integrator" definition of a semimartingale is equivalent to the usual one and a general theory of semimartingales are developed. Finally, the author extends the stochastic integral by continuity to predictable integrands, making the stochastic integral a Lebesgue-type integral. These integrands give rise to a presentation of the theory of semimartingale local times. The book is concluded by an introduction to stochastic differential equations and to the theory of flows (existence and uniqueness of solutions, stability, Markov nature of solutions).

The book allows a rapid introduction to some of the deepest theorems of the subject. It is highly recommended both to instructors and students in probability and statistics.

L. Hatvani (Szeged)

q-**Series and Partitions,** Edited by D. Stanton (The IMA Volumes in mathematics and its applications, 18), X+212 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong, 1989.

This is the Proceedings of the Workshop on q-Series and Partitions held at the Institute for Mathematics and its Applications, Minnesota, USA on March 7—11, 1988. It contains up to date research papers on q-series, unimodality, q-special functions and q-orthogonal polynomials.

What is a q-series? In the theory of partitions it is customary to write q as the argument in the generating functions, but many "ordinary" objects in mathematics have their q-analogues. For example the q-analogue of the binmial coefficient $\binom{n}{k}$ is

$$\frac{(1-q)(1-q^2)\dots(1-q^n)}{(1-q)(1-q^2)\dots(1-q^k)(1-q)(1-q^2)\dots(1-q^{n-k})}$$

(note that for $q\to 1-0$ we get back the original definition of the binomial coefficient). G. Gasper's paper in the proceedings under review discusses many such q-analogues.

Identities in terms of q-series often have interpretation in terms of partitions. Perhaps one of the most famous q-identities are the two Rogers—Ramanujan identities the first of which reads as

$$1+\sum_{n=1}^{\infty}\frac{q^{n^2}}{(1-q)(1-q^2)\dots(1-q^n)}=\prod_{n=0}^{\infty}\frac{1}{(1-q^{5n+1})(1-q^{5n+4})}.$$

What does this have to do with partitions? If we look at the coefficiens of q^n on both sides then this identity has the interpretation that the partitions of n into parts which differ by at least 2 are equinumerous with the partitions of n into parts congruent ± 1 modulo 5 (try to verify this "translation"; there is twist in the proof!). G. Andrews's paper discusses diffeent proofs of the Rogers—Ramanujan identities. The paper by D. Zeilberger attempts to classify identities with regard to computer time required for their verification using computer algebra. Computers and symbolic computations appear in other papers in the proceedings, as well.

The papers by D. M. Bressoud, F. M. Goodman and K. M. O'Hara, D. Zeilberger and I. G. Macdonald are related with the recent combinatorial proof of K. M. O'Hara for the unimodality of the Gaussian polynomials, which asserts that the coefficients in the polynomials

$$\frac{(1-q^{n+1})(1-q^{n+2})\dots(1-q^{n+k})}{(1-q)\dots(1-q^k)}$$

are increasing up to a point and decreasing after that. Earlier proofs used very advanced techniques and even K. M. O'Hara's proof was rather involved. Using her ideas a relatively simple elementary proof can be found in Zeilberger's and Macdonald's papers.

The papers by F. G. Garvan, L. Habsinger and D. St. P. Richards discuss integrals in several variables and their q-analogues that are related to Selberg's integral

$$\int_{[0,1]^n} \prod_{i=1}^{n} x_i^{a-1}(1-x_i)^{b-1}|D(x)|^{2c}\,dx = \prod_{i=1}^{n} \frac{\Gamma(a+(n-i)c)\Gamma(b+(n-i)c)\Gamma(ic+1)}{\Gamma(a+b+(2n-i-1)c)\Gamma(c+1)},$$

where $x=(x_1, \ldots, x_n)$ and $D(x)=\prod_{i<j}(x_j-x_i)$ is the Vandermonde determinant.

D. Stanton writes on an elementary approach to the Macdonald identities which expand products of the form

$$\prod_{\substack{a>0\\ a\in S}} (1-e^a)$$

as certain sums.

The volume ends with four papers by R. Askey, I. M. Gessel, M. H. Ismail and W. Miller, about orthogonal polynomials, their zeros and recurrence coefficients and their q-analogues (such as q-Hermite polynomials).

The book under review is an excellent source for a flourishing and very exciting area and can be recommended both to researchers and to advanced students in analysis, combinatorics and number theory.

Vilmos Totik (Szeged)

Rewriting Techniques and Applications, Proceedings, Chapel Hill 1989. Edited by Nachum Dershowitz (Lecture Notes in Computer Science 355), VII+579 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1989.

This volume contains the proceedings of the Third International Conference on Rewriting Techniques and Applications (RTA—89). The conference was held April 3—5, 1989, in Chapel Hill, North Caroline, U.S.A.

This book contains 34 papers in the following areas: Term rewriting systems, Conditional rewriting, Graph rewriting and grammars, Algebraic semantics, Equational reasoning, Lambda and combinatory calculi, Symbolic and algebraic computation, Equational programming languages, Completion procedures, Rewrite-based theorem proving, Unification and matching algorithms Term-based architectures.

Also included in this volume are short description of a dozen of the implemented equational reasoning systems demonstrated at the meeting.

This book is recommended to everybody working in the theory of Rewrite Systems.

Sándor Vágvölgyi (Szeged)

F. Schipp—W. R. Wade—P. Simon (with the assistance from **J. Pál**), **Walsh series, an introduction to dynamic harmonic analysis,** X+560 pages, Akadémiai Kiadó, Budapest, 1990.

The Walsh system is the simplest nontrivial model for harmonic analysis but shares many properties with the trigonometric system. It has been used to solve some fundamental problems in analysis, e.g., the basis problem. It has played a role in the development of other areas of mathematics, e.g., the fundamental theorem of martingales was proved first by Paley for the Walsh system.

The Walsh functions can be applied in many situations, among others, in data transmission, image enhancement, pattern recognition, etc. Since the Walsh functions take on only the values $+1$

and -1, they are easy to implement on high speed computers and can be used with very little storage space.

This is the first systematic and detailed exposition of the subject, from the foundations up to the most recent results, including many which were not previously published. The book can serve both as an excellent reference book and as a textbook. The reader is merely assumed to be familiar with the notion of and basic theorems on Lebesgue integration. Except for this material, concepts are developed as needed and the book is nearly self-contained. In particular, it is accessible to beginning graduate students and doctoral candidates in various specialities in mathematics and engineering.

The abundance and variety of the material presented in the book makes an exhausting description in a short review quite impossible. Thus, we can only comment on the general plan of the book, and mention some samples of the most characteristic results contained.

Chapter 1 contains a systematic account of the dyadic group, the definition of the Walsh functions in various enumerations, such as they were introduced by Walsh in 1923, by Paley in 1932, and by Kaczmarz in 1948. Separate sections are devoted to the transformations and rearrangements of the Walsh system, showing, in particular, that the Haar and Walsh systems are Hadamard transforms of each other; to Walsh—Fourier series, the Walsh—Dirichlet kernel, Walsh—Fejér kernel, dyadic derivative, and Cesàro summability.

The first half of Chapter 2 presents results which estimate the growth order of Walsh-Fourier coefficients for various classes of functions, e.g., L^p functions, continuous functions, etc., while the second half identifies conditions sufficient for pointwise convergence and absolute convergence of Walsh—Fourier series.

The Walsh functions provide a vehicle to link harmonic analysis and probability theory. The basic tool in this interrelation is the dyadic martingales which play an important role in the development of new spaces such as the dyadic Hardy spaces and dyadic BMO (=bounded mean oscillation). These results are dealt with in Chapter 3. Dyadic Hardy spaces are characterized in two ways: by means of martingale maximal function and of the atomic decomposition. It then proceeds to give an account of duality relations. Among others, H_0' (the dual of H_0, i.e., the collection of bounded linear functionals of H_0) is isometric and homeomorphic to BMO, and VMO$'$ (=vanishing mean oscillation) is isometric and homeomorphic to H_0, whereas the proofs are heavily relied on the dyadic version of the famous Fefferman inequality. The chapter ends with the study of martingale trees, i.e., martingales indexed by the tree-like collection of dyadic intervals. By introducing these nonlinear martingales and generalizing the Burkholder—Gundy theory of martingale transforms, the reader sees that the inequalities of Khintchin, Paley, and Sjölin as well as a.e. convergence of Walsh-Fourier series are all parts of a general theory of nonlinear martingale transforms.

Chapter 4 is devoted to study of convergence in L^p-norm, $p\geqq 1$, and uniform convergence of Walsh—Fourier series. The treatment of summability of Walsh—Fourier series in homogeneous Banach spaces and of sets of divergence is a certain adaptation of the corresponding technique developed by Kahane and Katznelson. Likewise, the adjustment of an integrable function f on a set of small measure in order to obtain a new function whose Walsh—Fourier series converges uniformly is modelled after Menshov's celebrated one for trigonometric series.

The first part of Chapter 5 touches the problem of approximation by Walsh polynomials. In great lines, it follows the trigonometric analogue. The major part of Chapter 5, however, presents the Haar, Walsh, Faber— Schauder, Franklin, and Ciesielski systems as bases and identifies for each of them the subspace of L^1 in which the given system is a basis. By indexing the Haar and Franklin systems in a natural way to make them nonlinear sequences, the authors find that the corresponding canonical isomorphisms induce explicit isomorphisms from the dyadic Hardy spaces and dyadic BMO to their classical trigonometric counterparts. This approach gives a natural way to get classical results from dyadic ones and vice versa. Then the authors show that the Haar and Franklin systems are equivalent bases in L^p for $1<p<\infty$. On the other hand, it turns out that the trigonometric and Walsh—Paley system are not equivalent bases in L^p for $1<p<\infty$, except for $p=2$. Finally, they also answer the long-standing problem of Banach by constructing a separable Banach space, similar in spirit to the dyadic Hardy space, which fails to have a basis. However, their decisive step in the construction is due to Enflo.

In Chapter 6 the authors collect several sufficient conditions ensuring the a.e. convergence of a Walsh-Fourier series. Using the notion of the so-called logarithm spaces, the sharpest result is due to Sjölin which says: If $f \in L \log^+ L \log^+ \log^+ L$, then the Walsh—Fourier series of f converges a.e. Then they prove the Walsh analogue of the famous Kolmogorov example of divergent Fourier series. On the other hand, the Walsh—Fourier series of an integrable function is Cesàro summable a.e. This is proved by exploiting the intimate connection between summability and pointwise dyadic derivative.

A fundamental problem in the theory of general Walsh series is the problem of uniqueness. To go into details, a set E is called a U-set (set of uniqueness) if every Walsh series converging to 0 outside E vanishes identically. Otherwise, E is called an M-set (set of multiplicity). It follows that every countable set is a U-set, while every set of positive measure is an M-set. Thus, it remains a delicate problem, not yet solved, to distinguish among sets of measure zero not normally made in Lebesgue analysis. The up-to-date approach of Chapter 7 is based on the observation that the study of general Walsh series is equivalent to the study of Walsh—Fourier—Stieltjes series of quasimeasures (i.e., finitely additive, real-valued set functions) defined on the dyadic intervals. This allows certain problems to be recast as measure theoretic questions. In some cases this perception provides simple explanations of known results, while in other cases it gives new insight into the nature of the problem itself. For example, the fact that no Walsh series can diverge to $+\infty$ on a set of positive measure is a reflection of the fact that a quasimeasure is either a.e. differentiable or has upper derivative $+\infty$ and lower derivative $-\infty$ a.e.

Chapter 8 is dedicated to the problem of representing measurable functions by Walsh series. This is connected with the term by term dyadic differentiation and the behavior of Walsh series with monotone coefficients, where a Sidon type inequality proved jointly by Schipp and the reviewer plays a crucial role. Then the representation problem is considered here in the more general framework of normalized convergence systems studied mainly by Talaljan.

Chapter 9 treats the questions of the Walsh—Fourier transform, which is the counterpart of the classical (trigonometric) Fourier transform. The fast Walsh transform seems to be more appropriate to implement on a computer than the fast Fourier transform. The inverse dyadic derivative plays a central role in the treatment. The various applications of the Walsh functions are only outlined, since several books have been written about them. For further reading we suggest the books by H. F. Harmuth, K. G. Beauchamp, C. A. Bass, M. Maqusi, etc.

Each chapter ends with Exercises ranging from fairly routine applications of the text material to those that extend the coverage of the book. For the reader's convenience there are seven Appendices containing a number of auxiliary topics at the end of the book. Historical Notes to each chapter separately, References to about 450 papers or books, Author, Subject, and Notational Index complete the book.

The book is carefully and accurately written. The presentation is concise but always clear and well-readable.

Finally, may the reviewer venture to express his particular desire to take some time a second volume in his hands comprising the latest research done in the field of multiple Walsh series as well as providing a rigorous mathematical treatment of the concrete questions occurring in the vast and diverse field of pratical applications. The reviewer's hope is that all this enormously arge material in the authors' unified presentation would prove to be more accessible to anyone interested in dyadic harmonic analysis. Of course, this desire does not affect the value of this almost perfect work at all.

To sum up, this book fills in a gap in the literature. It provides in a polished form a rich and up-to-date material of a fast-growing field whose significance is becoming basic for practice. It is perhaps not exaggerated to assert that this book is of fundamental importance for everybody who wants to keep pace with modern developments in the Dyadic and Classical Analysis.

Ferenc Móricz (Szeged)

C. L. Siegel, Lectures on the Geometry of Numbers, X+160 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1989.

This is the printed version of Siegel's lectures at New York University during 1945—46. The original notes by B. Friedman were rewritten by K. Chandrasekharan with the assistance of R. Suter.

"Geometry of Numbers" is a subject dealing mainly with lattices and their points in prescribed sets in R^n. Its fundamental theorem is Minkowski's First Theorem: A convex body in R^n, having a centre at the origin and having a volume larger than 2^n, must contain at least one point other than the origin with integer coordinates. The first chapter of the book is devoted to this theorem and its generalization involving the so called successive minima of even gauge functions.

The second chapter starts out with the discussion of vector groups which are nothing else than the subgroups of the additive group of R^n. Discerte vector groups correspond to lattices and so to matrices. Such concepts as basis, ranks, characters duals etc. are treated in detail. As an application of the duality theorem Kronecker's approximation theorem is proved together with one-of its generalization. Further applications are given concerning periods of real and complex functions, parquets formed by parallelepipeds. The rest of the chapter deals with the minimum of products of linear forms and of positive definite quadratic forms on lattice points different from the origin. The exact minimum value in R^2 is determined and it yields a proof of Hurwitz' theorem according to which to every irrational a there are infinitely many pairs (p, q) of integers with

$$\left|a-\frac{p}{q}\right| \leqq \frac{1}{\sqrt{5}q^2}.$$

A lattice is a geometric object but for its analytic description we use matrices. Several matrices correspond to the same lattice and these are connected by unimodular transformations. In other words, several lattices have the same set of points as a geometrical entity and it would be advantageous to single out one lattice from the class of all lattices which are equivalent under a unimodular transformation. The problem of finding such a representative for every class of equivalent lattices is called the problem of reductions, and Chapter III is devoted to the theory of reduction. Some applications, as for instance closest packing in two, three and four dimensions are also covered.

The lectures on the geometry of numbers provide an excellent source of learning for undergraduate and graduate students and the book can serve as a basis for a course in the field. Some of the lectures contain more material than what is appropriate for a single lecture, so the active participation of the students seems to be absolutely necessary if one would like to keep up with the pace suggested by the table of contents. The only criticism I make is that some of the proofs are unnecessarily detailed and some parts of Chapter III may seem to be more specialized and less exciting for an average reader than the material in the first two chapters.

Vilmos Totik (Szeged)

R. Silhol, Real Algebraic Surfaces (Lecture Notes in Mathematics, 1392), X+215 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1989.

If a book is written by a specialist then it is in a great danger of becoming too dry or too special for a general reader. Fortunately this book has avoided these trips inspite of the fact that its subject is far away from the basics of mathematics.

The basic idea in this book is to consider the real algebraic surfaces and to regard them as complex algebraic varieties with an antiholomorphic involution. From this point of view there are two classes of the real algebraic surfaces as the Galois group Gal $(\mathbf{C}|\mathbf{R})$ on $H^*(X(\mathbf{C}, \mathbf{Z}))$ determines or only estimates the dimension of $H^*(X(\mathbf{R}), \mathbf{Z}/2)$. The previous type of the surfaces, such as rational surfaces and Abelian surfaces etc., are under a detailed analysis in this book. The main result is the complete classification of these surfaces.

We have to mention two great advantages of the book finally. First of all the two introductory chapters are extremely useful because they make really possible to read the book for non-specialists and graduate students in algebraic geometry. Also the examples throughout the book are useful to understand better the new notions.

Árpád Kurusa (Szeged)

James K. Stayer, Linear Programming and Its Applications (Undergraduate Texts in Mathematics), XII+265 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong, 1989.

This book is devoted to serve as an introductory text in linear programming. It is divided into two main parts.

The first part consisting of four chapters deals with methods for solving general linear programming problems. Chapter 1 exhibits the usual geometrical representation which is a good preparation for the later texts. The canonical forms are considered in Chapter 2 and the classical Dantzig's simplex algorithm is given as a solving method. The general problem of linear programming is treated in Chapter 3, and different methods are presented to solve it. Finally, Chapter 4 discusses the theory of duality showing the connection between the problems of maximization and minimization.

The second part presents several applications related to linear programming. Firstly, Chapter 5 deals with the two-person zero-sum matrix games. Such traditional applications of linear programming as transportation and assignment problems are treated in Chapter 6, and as solving algorithms the stepping stone method and the Hungarian method are given, respectively. Finally, Chapter 7 deals with networks. Algorithms are presented to solve the network-flow problem, the shortest-path network problem and the minimal-cost-flow network problem.

The book is well-written. It contains a rich collection of examples and exercises. Every algorithm is illustrated in a step-by-step manner. It can be recommended as an excellent text for an introductory course in linear programming.

B. Imreh (Szeged)

John Stillwell, Mathematics and Its History (Undergraduate Texts in Mathematics), X+371 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1989.

At almost all universities mathematics students are those who never get a course in mathematics. They get several separate courses such as calculus, algebra, geometry, topology and so on, and our usual teaching method seems to prevent these different topics from being combined in to a whole. Therefore there are several important questions which are not discussed in the proper place, e.g., the fundamental theorem of algebra, because that is analysis. Thus, if students are to feel they really know mathematics by the time they graduate, there is a need to unify the subject. This feeling is very important for future teachers of mathematics. A course on the history of mathematics does not take this job.

This book has grown from a course given to senior undergraduates at Monash University. The selection of the material has been a success. It covers almost all topics of primary importance. The emphasis is on history as a method for unifying and motivating mathematics. The twenty chapters are the following: The Theorem of Pythagoras, Greek Geometry, Greek Number Theory, Infinity in Greek Mathematics, Polynomial Equations, Analitic Geometry, Projective Geometry, Calculus, Infinite Series, The Revival of Number Theory, Elliptic Functions, Mechanics, Complex Numbers in Algebra, Complex Numbers and Curves, Complex Numbers and Functions, Differential Geometry, Noneuclidean Geometry, Group Theory, Topology, Sets, Logic, and Computation.

This is not a book on the history of mathematics, therefore it uses modern notations. This is debatable only in the first glance. For those readers, who want to read the original texts, there is a long reference at the end of the volume.

In each chapter we can find biographical notes and well selected exercises.

We warmly recommend this gap-filling book to any undergraduate course of mathematics, especially to teachers of mathematics.

Lajos Kulkovits (Szeged)

Josef Stoer, Numerishe Mathematik I (Springer Lehrbuch), XII+314 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1989.

A brief comparison to the previous English translation (Stoer—Bulirsch, Introduction to Numerical Analysis, Springer-Verlag, 1980) reveals two major changes beside a few technical updates.

Chapter 2 on interpolation has a comprehensive supplement on the formal properties and a simple recurrence relation of B-splines.

The second addition in Chapter 4 shows the pecularities of sparse matrix techniques. Efficient pivoting and storage schemes are demonstrated for the sparse Cholesky factorisation algorithm.

The standards of this book stand comparison with most new textbooks and, in the reviewer's opinion, this latest edition will not be the last one.

J. Virágh (Szeged)

J.-O. Strömberg—A. Torchinsky, Weighted Hardy Spaces, (Lecture Notes in Mathematics 1381), IV+193 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1989.

The development of harmonic analysis in the last few years has been centered around spaces of functions of bounded mean oscillation and the weighted inequalities for classical operators. The main goal of this book is to further develop some results in this topic in the general setting of the weighted Hardy spaces and to discuss some applications. The authors derive mean value inequalities for wavelet transforms and introduce halfspace techniques with, for example, nontangential maximal functions and g-functions. This leads to several equivalent definitions of the weighted Hardy spaces. Fourier multipliers and singular integral operators are applied to the weighted Hardy spaces and complex interpolation is considered.

Rich bibliography helps the reader in going back to the origin of the research of this topic. Apparently the book covers the whole spectrum of papers dealing with these very important spaces.

The book is highly recommended to research workers interested in the modern harmonic analysis.

J. Németh (Szeged)

J. L. Balcazar—J. Diaz—J. Gabarro, Structural Complexity I, (EATCS Monographs on Theoretical Computer Science, 11), IX+191 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1988.

There are many different yet related studies in the complexity of algorithms. The subject of structural complexity takes an abstract view of the complexity of computation by looking for inherent mathematical structures inside the problem classes. Many concepts of structural complexity originate in recursion theory, as is clearly demonstrated in the present book.

The first two chapters are included for the sake of completeness and to broaden the accessibility of the volume. Chapter 1 provides a brief exposition of models of computation, such as finite automata, and several versions of Turing machines. Some elementary properties of languages that represent decision problems and classes of languages are discussed. The main purpose is to explain the basic notions and to present a formalism for the remainder of the book. Enough references are provided for those who want a deeper background on the material covered in this chapter.

Chapter 2 starts with a survey of the rate of growth of functions and is followed by a discussion of the running time and work space of Turing machines. Some basic results are presented, e.g. the linear speed-up theorem and the tape compression theorem.

Then, after a thorough treatment of time and space constructible functions, complexity classes are defined in a general setting. This chapter ends with some simulation results, such as Savitch's theorem.

Central complexity classes form the subject matter of Chapter 3. Polynomial time (many-one) reducibility and logarithmic space reducibility are defined and related concepts (completeness, hardness, etc.) are discussed. Some well-known NP-complete problems are presented and QBF is shown to be PSPACE-complete. A separate section is devoted to padding arguments, which provide a useful tool for establishing inequalities between complexity classes.

Other types of reducibilities, namely polynomial time Turing reducibility and SN-reducibility are studied in Chapter 4, giving rise to relativizations of complexity classes. SN-reducibility is then related to self-reducible sets.

Finite sets can be accepted by deterministic finite automata in constant time with no work space whatsoever. The "intrinsically algorithmic approach" taken in preceding chapters thus fails when dealing with finite sets. The "uniform" approach of Chapter 5 measures the sizes of the algorithms accepting finite sets and associates with an infinite set the growth of the sizes of the algorithms that accept initial segments of the set. The unifying concept of "advice" functions is then used to relate the two approaches. Boolean complexity fits nicely in this framework.

The average case behavior of algorithms has become a topic of increasing interest in recent years. By using pseudo-random number generators, it is possible to design algorithms that solve problems with a reasonable rate of probability. Accordingly, Chapter 6 provides a glimpse of probabilistic algorithms. A basic theory of probabilistic complexity classes is developed.

A number of studies in complexity theory depend on the assumption that $P\neq NP$. Uniform diagonalization provides a powerful technique to prove e.g. that there are incomplete problems in $NP-P$, assuming that $P\neq NP$. Uniform diagonalization and its applications are discussed in

Chapter 7. The last chapter deals with the polynomial time hierarchy, the polynomial analogue of Kleene's hierarchy.

The book is well-written, the presentation of the material is sufficiently clear. The necessary prerequisites are a basic knowledge of automata and formal languages. Some acquaintance with recursion theory might be helpful. Each chapter ends with detailed bibliographical remarks and a number of exercises. The book can very well serve as a text for a graduate course in structural complexity.

Z. Ésik (Szeged)

Aimo Törn—Antanas Žilinskas, Global Optimization (Lecture Notes in Computer Science, 350), X+255 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1989.

Global optimization is a part of nonlinear programming: it is aimed at solving nonlinear optimization problems with many local minima. This problem is in general unsolvable, if the algorithm is based only on the evaluation of the objective function and its derivatives. Although such problems are rather frequent in practice, the traditional approach of the users is to accept the first local minimum found as an estimate of the global minimum.

The book by Törn and Žilinskas was the first to cover the broad field of global optimization. Since its publishing, some other volumes have been available, dealing mainly with different subproblems of global optimization (such as deterministic and stochastic methods).

After the definition and characterization of the global optimization problem, the book discusses the covering, the clustering and the random search methods, the method of generalized descent and the algorithms based on statistical models of the objective function. Testing is a crucial part of the evaluation of global optimization methods, since their reliability has to be measured somehow. The book devotes a section to questions arising in testing and applications. The test results are collected very carefully, thus the reader looking for a suitable method can rely on the tables given by the authors.

It must be mentioned that spelling errors make the text somewhat difficult to read. An extensive bibliography of more than 400 references completes the book.

The volume can be warmly recommended (beyond experts of the field) to everyone who must solve nonlinear optimization problems that can be multiextremal.

T. Csendes (Szeged)

L. Trave—A. Titli—A. Tarras, Large Scala Systems: Decentralization, Structure Constrainst and Fixed Modes (Lecture Notes in Control and Information Sciences, 120), XIV+384 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1989.

The models of the present day technological, environmental and societal processes are of high dimensions and complexity, which makes impossible to use the classical mathematical tools developed for system analysis and control. This book gives an excellent survey on the new techniques for the large scale systems characterized by a huge number of input and output variables on subsystems which are generally geographically distributed.

Chapter 1 presents an overview of the well-known results around the problem of stabilization and pole assignment of linear time — invariant dynamic systems subjected to centralized control. Chapter 2 deals with these problems when a specified restricted information pattern is required, which constraints the feedback control structure. Chapter 3 gives the different existing characterizations of fixed modes, namely characterizations in term of transmission zeros of subsystems, characterization in time-domain and in the frequency-domain, and graph-theoretic characterizations. In Chapter 4 it is shown that systems with unstable non structurally fixed modes can be stabilized by using time varying or non-linear feedback control laws which preserves the feedback structure constraints. Chapter 5 presents the different available methods for the design of an appropriate feedback control structure. Chapter 6 considers the problem of the synthesis of feedback gains under structural constraints. Chapter 7 is devoted to the problem of structural robustness.

The results are illustrated by significative examples which make easier their understanding. Some important algorithms are presented in a collection of program packages.

This book will be very useful both as a text and as a monograph in the control of large scale systems.

L. Hatvani (Szeged)

Ferdinand Verhulst, Nonlinear Differential Equations and Dynamical Systems, IX+277 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong, 1990.

Recently the theory and applications of nonlinear differential equations and dynamical systems have strongly attracted the attention of mathematicians and users of mathematics. The reason is that a lot of phenomena in the sciences and economy can be explained by modelling the processes by nonlinear differential equations and applying the new results of the nonlinear dynamics to these models. It is not easy to get acquainted with these results demanding deep mathematical prerequisites. This introductory text bridges the gap between elementary courses in ordinary differential equations and the modern research literature in the field of nonlinear dynamics.

The first part of the book — after giving the basic definitions — deals with the periodic phenomena. The reader can find here a very plastic proof for the Poincaré—Bendixson theorem on the existence of periodic solutions. The second part is devoted to the stability theory. The third part gives an overview on the methods for systems containing a small parameters (perturbation theory, Poincaré—Lindstedt method, averaging). In the last four chapters, which give the most interesting part of the book, more advanced topics like relaxation oscillations, bifurcation theory, chaos in mappings and differential equations, Hamiltonian systems are introduced.

The book is well-written and well-organized. Only the most important proofs are included; the results are illustrated by interesting and important examples from the real world. The chapters are concluded by exercises (at the end of book the reader gets answers and hints to them). After studying this book and solving the exercises the reader will be able to start working on open research problems.

This excellent textbook can be warmly recommended both to beginners and specialists interested in the modern theory of nonlinear differential equations and its applications.

L. Hatvani (Szeged)

Wolfgang Walter, Aanalysis I, zweite Anflage (Grundwissen Mathematic, 3) VIII+385 pages, Springer-Verlag, Berlin—Heidelberg—New York—Paris—Tokyo—Hong Kong, 1990. **Analysis II,** (Grundwissen Mathematic 4) VII+396 pages, Springer-Verlag, Berlin—Heidelberg—New York—Paris—Tokyo—Hong Kong, 1990.

The first volume consists of three parts. The first part summarizes the basic knowledges about real numbers, mathematical induction and polynomials. The second part introduces the concept of convergence of sequences and series of real numbers, defines the limit and continuity of functions, discusses power series and elementary transcendent functions. At the end of this part the complex numbers and functions are introduced. The third part is devoted to Riemannian integral and differentiation of functions. A lot of applications can also be found here. This part ends with complementary notes.

The second volume is divided into 10 paragraphs. The text starts with the introduction of metric spaces, basic topological concepts and continuity of functions in metric spaces. Then the differentiation theory of functions of several variables, the problem of implicite functions and extremal values of functions are developed. The general Moore—Smith convergence is introduced and the Riemannian integral as Moore—Smith limit is showed. The length and differentialgeometric concepts of curves are discussed, the equations of motions are developed and the classical two bodies problem is solved. A paragraph (the sixth) is devoted to Riemann—Stieltjes integral and line integrals. Two paragraphs deal with the Jordan mass, the Riemannian integral in n dimension and the Gauss, Green and Stokes integral theorems. The last two paragraphs introduce the Lebesgue integral, the Fourier series and develop the Hilbert-space theory of Fourier series.

The books are highly recommended to students in the first four semesters.

L. Gehér (Szeged)