EXPANSION OF METEOROLOGICAL FIELDS BY MACROSYNOPTIC AVERAGE FIELDS

by

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Meteorológiai mezők sorfejtése makroszinoptikus átlagmezők alapján. A tanulmány az atlanti-európai térségben 5° szélességkülönbséggel és 10° hosszúságkülönbséggel felvett pontok által meghatározott rácson az AT 500-as geopotenciál mezők optimális reprezentációjával foglalkozik s kísérletet tesz arra is, hogy megalkossa a makroszinoptikus mezőknek a Péczely-féle típusokkal analóg módon definiált alaprendszerét.

A dolgozatban megoldott feladat a közép- és hosszútávú előrejelzés szempontjából gyakorlati jelentőségű is.

The study is dealing with optimal representation of the AT 500 geopotential fields given by grid points with 5° latitude difference and 10° longitude difference. At the same time it attempts to compose a basic system of the macrosynoptic fields defined on the analogy of the Péczely-types.

The problem solved in the study has got practical importance in the respect of middle-range and long-range weather forecast.

Introduction

Generally it is difficult to describe meteorological fields, weather situations with high accuracy, in practice they are given as interpolated values on grid points or as values measured on meteorological stations. They are demonstrated by isolines on weather maps. In consequence of the increasing role of computers, spreading of the computer science and data banks it became an important requirement to represent meteorological fields with numbers, that is to associate a vector $(a_0, a_1, a_2, \ldots, a_N)$ with a meteorological object. Usually when choosing codes to be used the following factors are considered:

i) Using the codes the field should be reconstructable with required accuracy.

ii) Possibly only a few numbers should be used in coding.

iii) The codes should be easily calculated.

iv) The field should be interpretable, physically explainable and analyzable in coded form, too.

Accordingly, function series are applied as field representation methods, such as Fourier-expansion, approximation with Chebishev series, Shannon—Kotelnikov representation, and mainly in meteorology, natural orthogonal expansion. Applying the different expansions the coding requirements cannot be fulfilled simultaneously, because these requirements are usually contradictive. The known expansions are generally favourable from one point of view, while the other requirements are usually partially fulfilled. Thus in meteorology the often used natural orthogonal expansion

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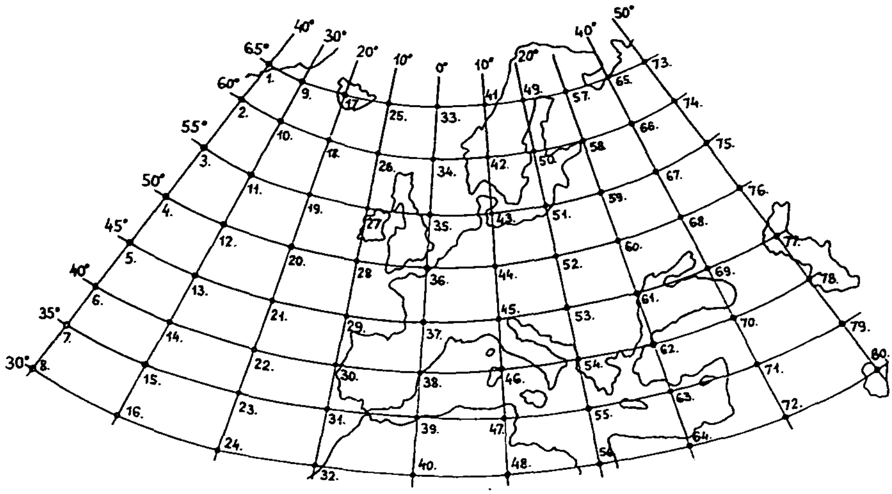

Fig. 1. Grid network ower the Atlantic European area

(*Craddock—Flood*, 1969) is optimal as to the numbers of codes, while at the same time, it is not simple to calculate the coefficients and to interprete the field in coded form.

This paper describes an expansion method which produces the field with relatively few coefficients, by a physically well interpretable system. The procedure was used for describing the 500 mbar isobaric surface, (hPa).

The system of average fields

In the suggested method the field

$$\xi(\mathbf{r}) = a_0 + a_1 e_1(\mathbf{r}) + a_2 e_2(\mathbf{r}) + \ldots + a_N e_N(\mathbf{r}), \tag{1}$$

may be given by the coefficient system:

$$(a_0, a_1, a_2, \ldots, a_N) \tag{2}$$

where $\{e_i(\mathbf{r})\}$ is a given field defined on R^3. In case of natural orthogonal expansion $\{e_i(\mathbf{r})\}$ consists of the eigenvectors of the covariance matrix of the field, and in our suggested method $\{e_i(\mathbf{r})\}$ equals to the 500 mbar isobaric field, averaged over the days with equivalent macrosynoptic code. As later is shown, it is not an orthogonal system, but linearly independent. Our results show — though it is surprising — that relatively small numbers of codes are needed to produce practical accuracy and the expansion has physical content at the same time.

500 mbar isobaric surface were used during the investigations by grid points, on an area represented on the chart below:

The data were obtained from the data bank of the British Meteorological Service. All 500 mbar isobaric surface daily values were used between 1955—1966. The data were read from the charts of Deutscher Wetterdienst, and after being interpolated for the grid points and being arranged they were recorded on magnetic tape.

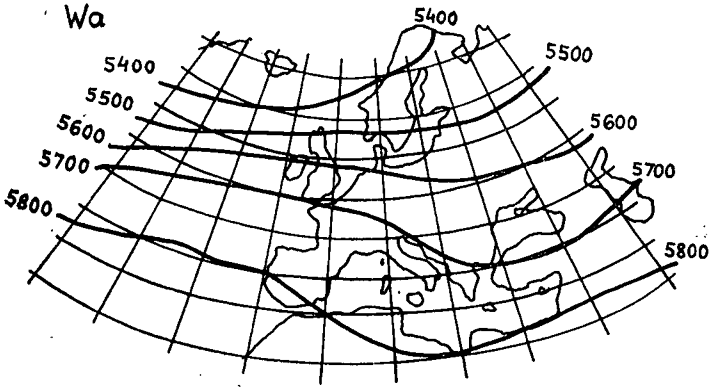

Fig. 2.1. Wa, Anticyclonic Western situation

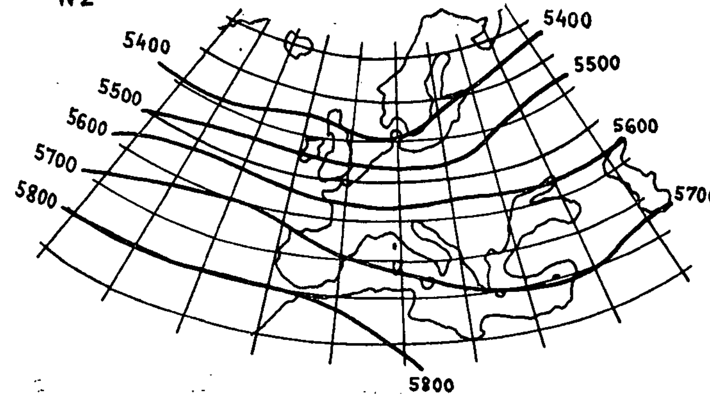

Fig. 2.2. Wz, Cyclonic Western situation

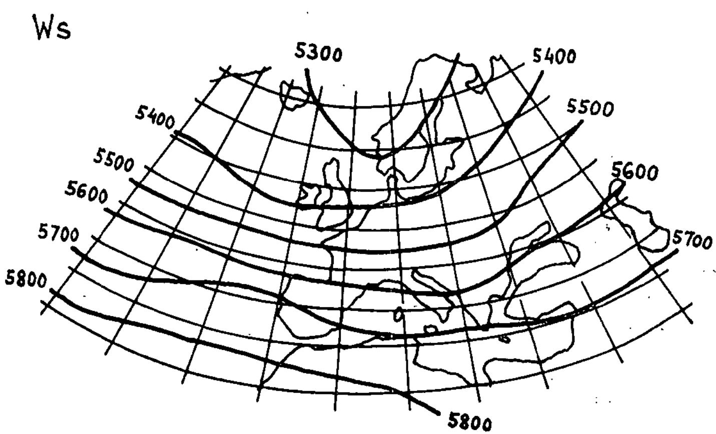

Fig. 2.3. Ws, South-Western situation

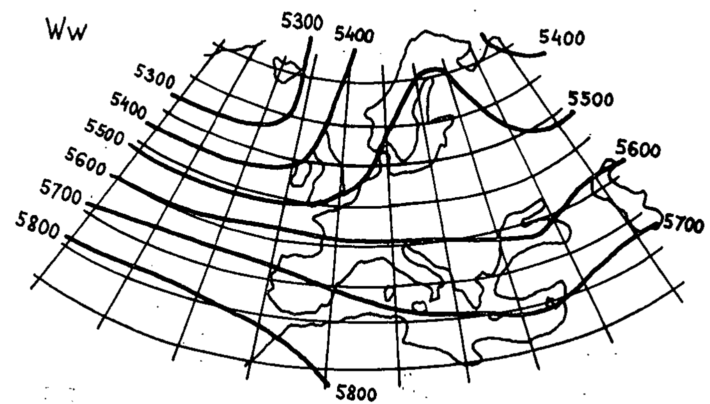

Fig. 2.4. Ww, Sharply edged Western situation

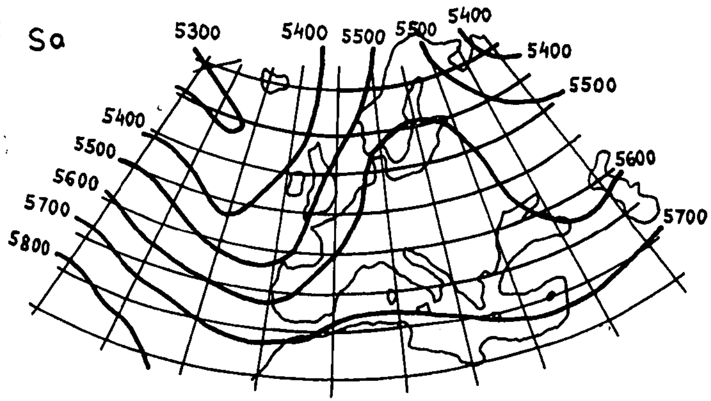

Fig. 2.5. Sa, Anticyclonic Southern situation

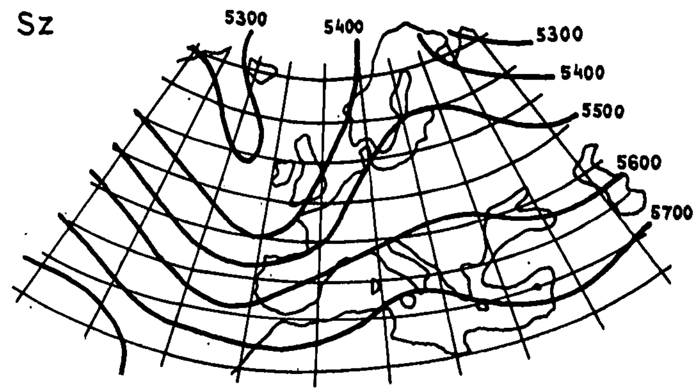

Fig. 2.6. Sz, Cyclonic Southern situation

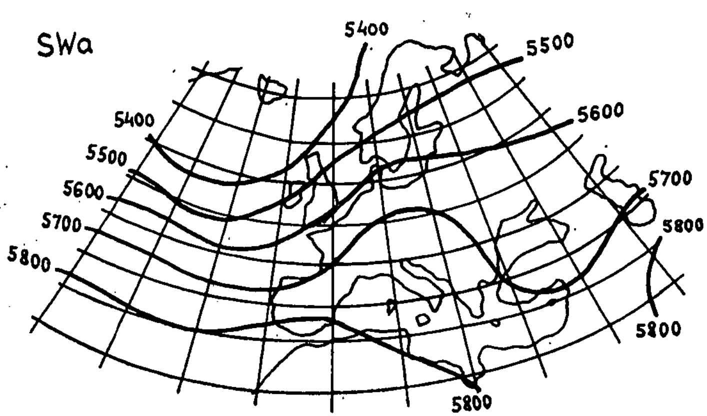

Fig. 2.7. SWa, Anticyclonic South-Western situation

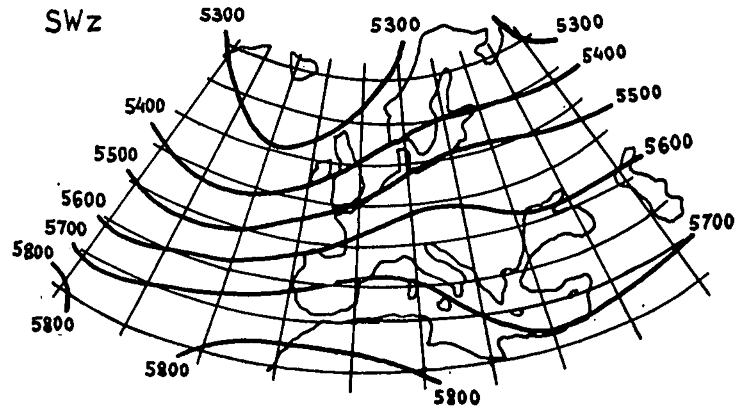

Fig. 2.8. SWz, Cyclonic South-Western situation

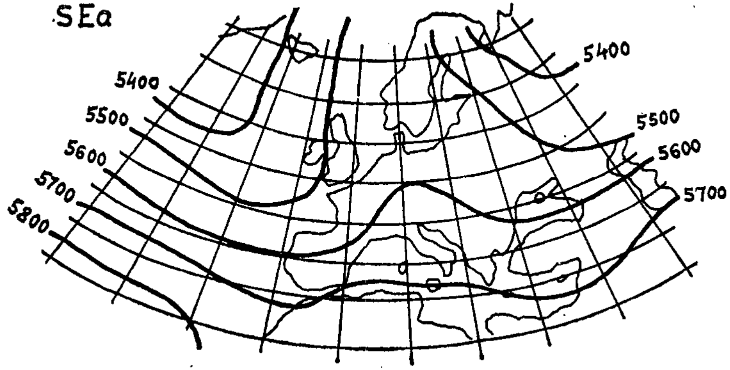

Fig. 2.9. SEa, Anticyclonic South-Eastern situation

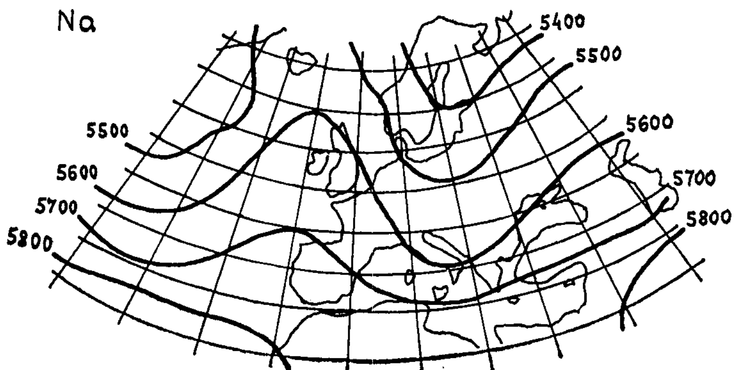

Fig. 2.10. SEz, Cyclonic South-Eastern situation

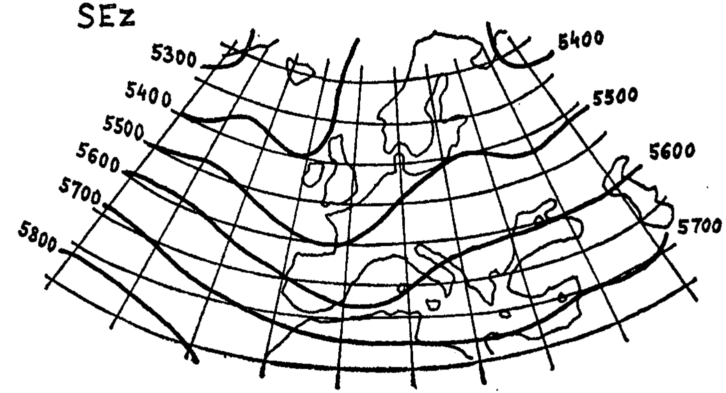

Fig. 2.11. Na, Anticyclonic Northern situation

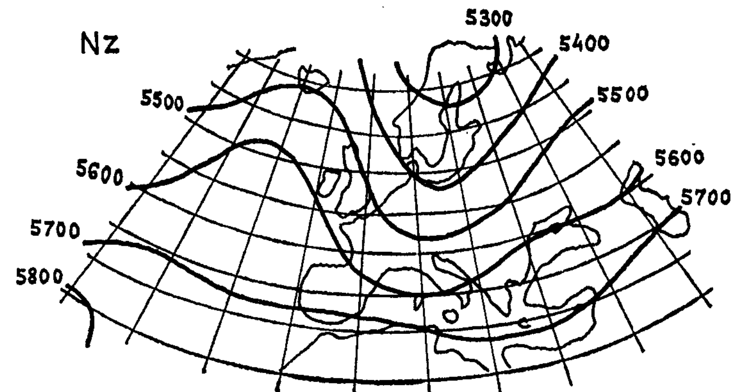

Fig. 2.12. Nz, Cyclonic Northern situation

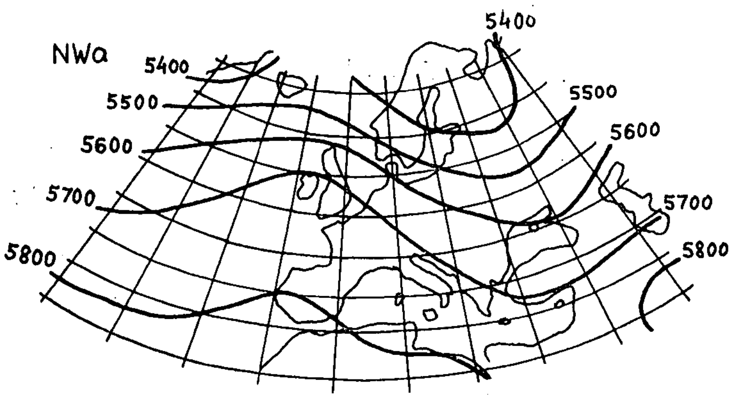

Fig. 2.13. NWa, Anticyclonic North-Western situation

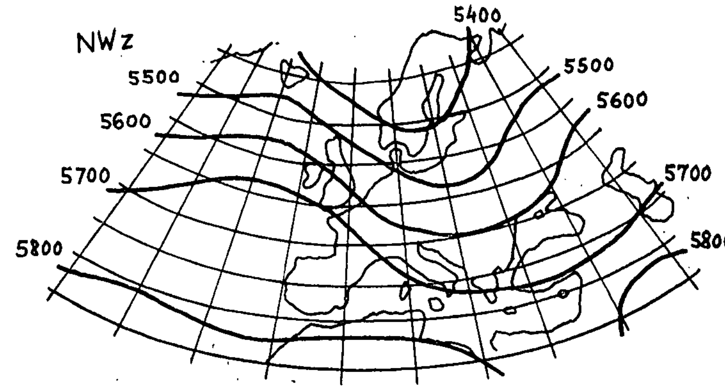

Fig. 2.14. NWz, Cyclonic North-Western situation

Fig. 2.15. NEa, Anticyclonic North-Eastern situation

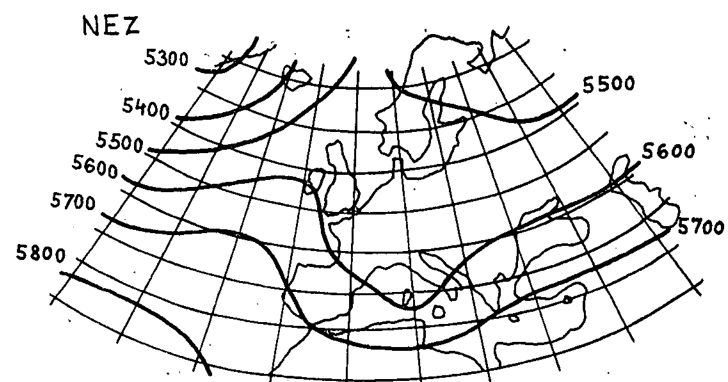

Fig. 2.16. NEz, Cyclonic North-Eastern situation

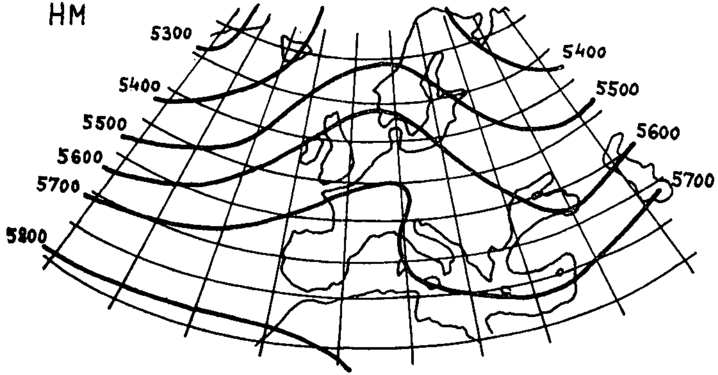

Fig. 2.17. HM, High pressure over Central-Europe

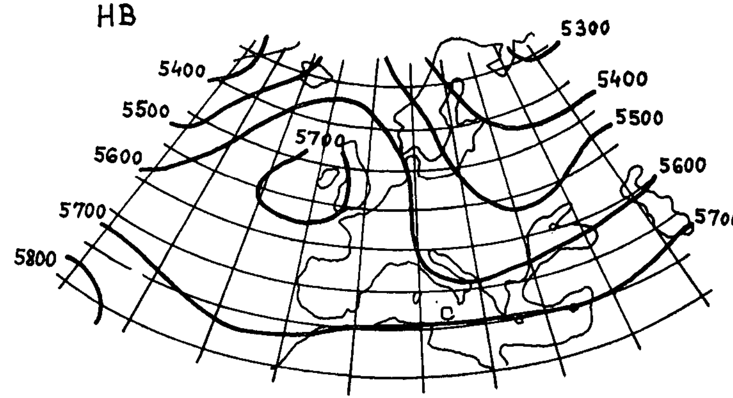

Fig. 2.18. HB, High pressure centre over Britain

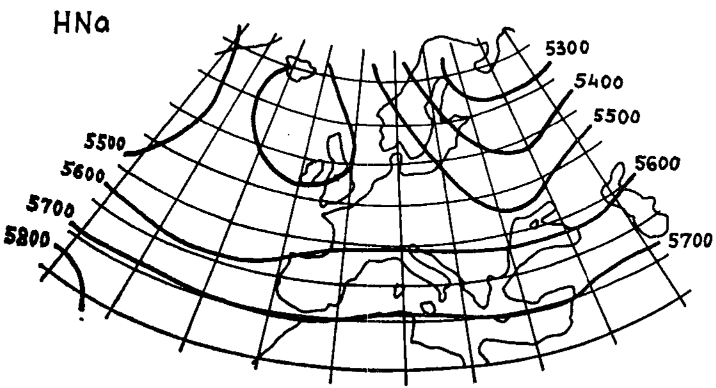

Fig. 2.19. HNa, High pressure centre on the Northern Sea, developing over Central-Europe

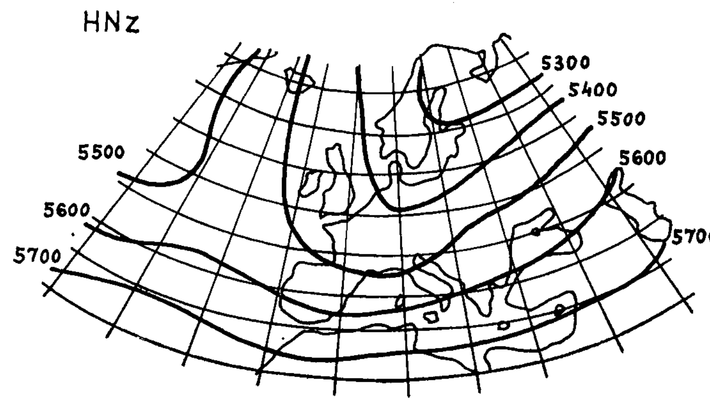

Fig. 2.20. HNz, High pressure on the Northern Sea

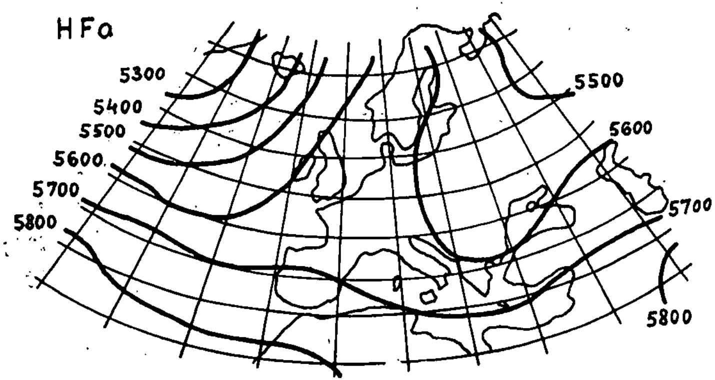

Fig. 2.21. HFa; High pressure on Fenno-Scandinavia developing over Central-Europe

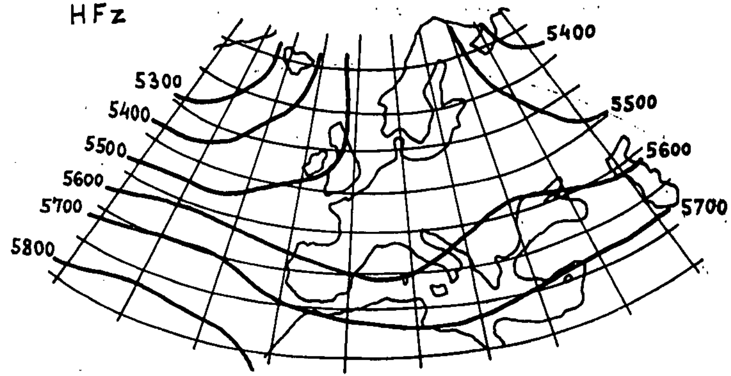

Fig. 2.22. HFz, High pressure on Fenno-Scandinavia and low pressure over Central-Europe

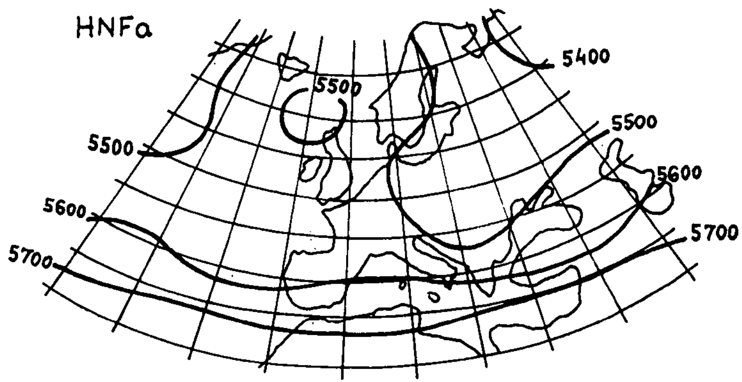

Fig. 2.23. HNFa, High pressure on the Northern Sea and Fenno-Scandinavia, developing over Central-Europe

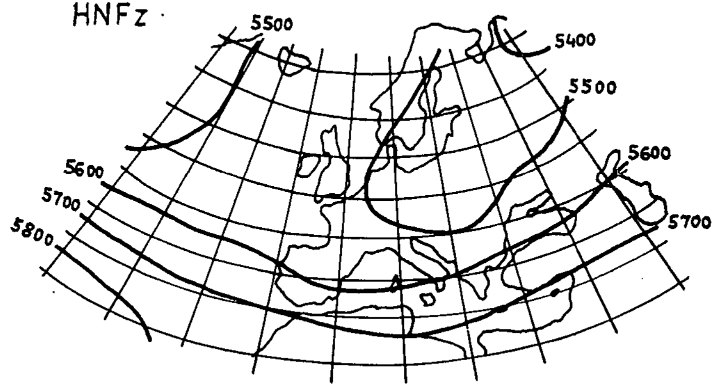

Fig. 2.24. HNFz, High pressure on the Northern Sea and Fenno-Scandinavia and low pressure over Central-Europe

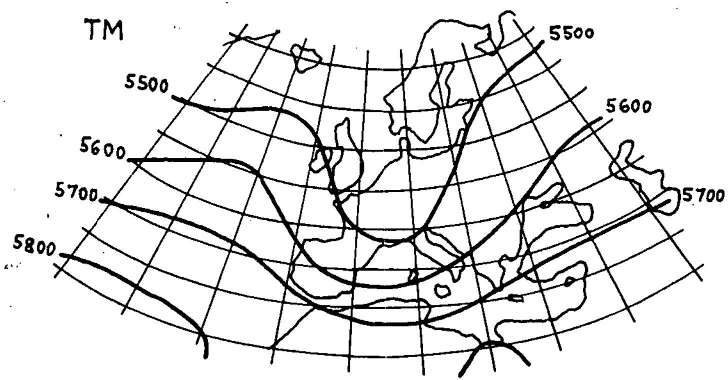

Fig. 2.25. TM, Low pressure over Central-Europe

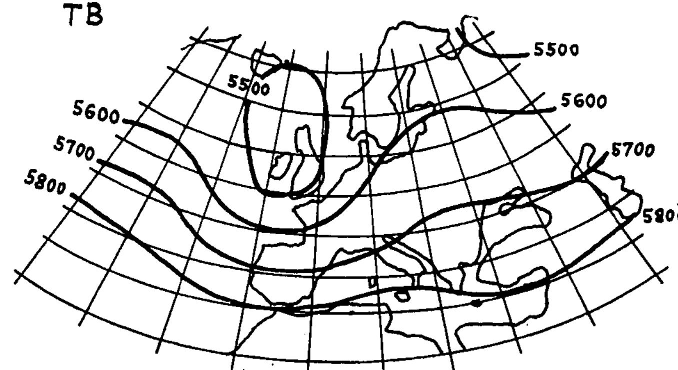

Fig. 2.26. TB, Low pressure centre over Britain

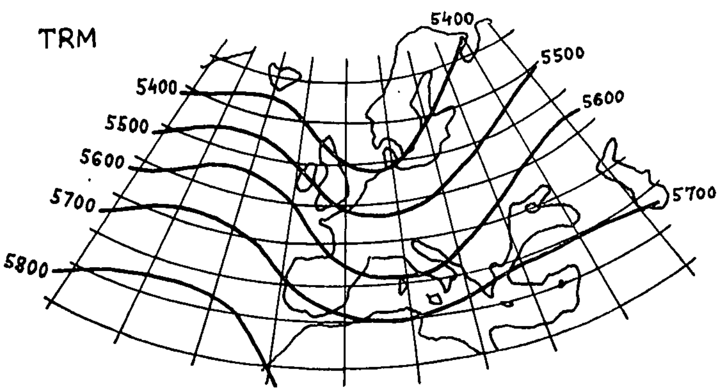

Fig. 2.27. TRM, Trough over Central-Europe

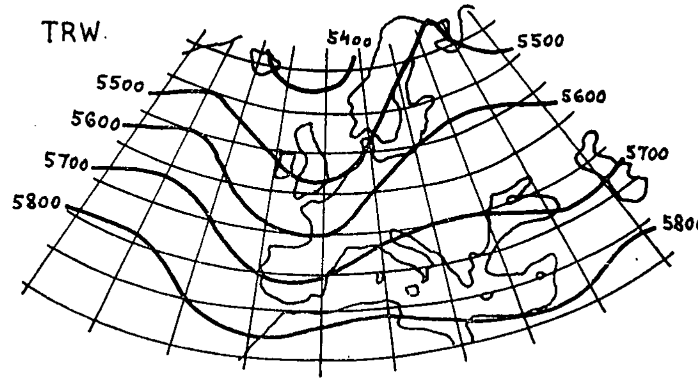

Fig. 2.28. TRW, Trough over Western-Europe

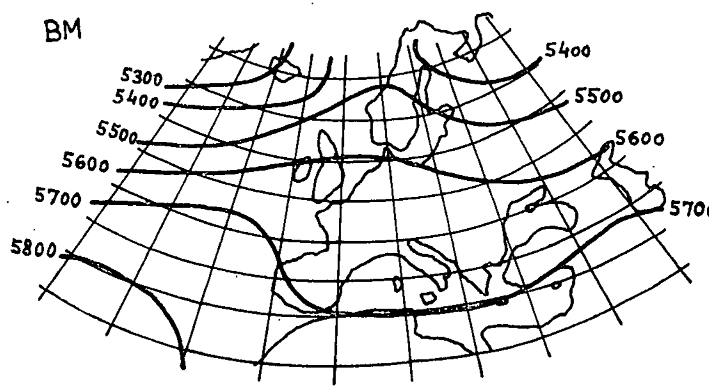

Fig. 2.29. BM, Zonal circulation with high ridge over Central-Europe

Therefore the i-th day can be given by a 80-vector:

$$\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{i80}), \quad i = 1, 2, \dots, 4018 \tag{3}$$

where x_{ij} denotes the value of the i-th day in the j-th grid point. The fields were grouped in according to macrosynoptic codes. The macrosynoptic codes were given from the above mentioned data bank, too. Being arranged according to the Hess—Brezowsky's macrosynoptic codes, (*Hess—Brezowsky*, 1952) the 500 mbar isobaric surface devided into 30 groups. The average fields, i. e. the fields averaged over each group were calculated and they served as the function system $\{e_i(\mathbf{r})\}_{i=1}^{30}$ in the new expansion. On the basis of this, each element of the system corresponds to a well defined meteorological situation. The following figures 2.1.—2.30. represent these Hess—Brezowsky's average fields. The obtained average fields $\{e_i(\mathbf{r})\}_{i=1}^{30}$ were analyzed and they were found suitable for expansion, and the fields were expansed by the function system $\{e_i(\mathbf{r})\}_{i=1}^{30}$.

Expansion by the average fields

Our task is to give the field $\xi(\mathbf{r})$ with the following series:

$$\xi(\mathbf{r}) = a_0 + a_1 e_1(\mathbf{r}) + a_2 e_2(\mathbf{r}) + \dots + a_N e_N(\mathbf{r}) \tag{4}$$

where $e_i(\mathbf{r})$ is the function system derived from 500 mbar AT fields averaged over the days with equivalent macrosynoptic codes. For the sake of possible interpretation of this series the function system should be linearly independent and complete. The coefficient system will be unique and the total accuracy can be achieved theoretically, too. The linear independence can be checked directly. As it is well known from the Vector Algebra the vector system $(e_1, e_2, \dots, e_N)$ is linearly independent if and only if the Gram determinant form of the scalar products (e_i, e_j) is not 0. This condition was tested, and it was proved correct.

The steps of the expansion:

i) The system of the average fields $\{e_i(\mathbf{r})\}$ belonging to the macrosynoptic codes is transformed into an orthonormal vector system $\{e_i^*(\mathbf{r})\}$ by means of Hilbert—Schmidt's orthogonal procedure (as it is known, the only essential and controlled condition is the linear independence of the system $\{e_i(\mathbf{r})\}$).

ii) The actual examined field is expansed by the obtained system $\{e_i^*(\mathbf{r})\}$, thus

$$\xi(\mathbf{r}) = a_0^* + a_1 e_1^*(\mathbf{r}) + a_2 e_2^*(\mathbf{r}) + \dots + a_N e_N^*(\mathbf{r}) \tag{5}$$

where

$$a_i^* = (\xi(\mathbf{r}), e_i^*(\mathbf{r})) = \sum_{j=1}^{M} \xi(r_j) a_i^*(r_j). \tag{6}$$

iii) There is a linear relationship between the coefficients of the expansions e_i and e_i^*, which is described by the following equation system:

$$\begin{aligned}
(e_1^*, e_1)a_1 + (a_1^*, e_2)a_2 + \dots + (e_1^*, e_M)a_M &= a_1^* \\
(e_2^*, e_1)a_1 + (e_2^*, e_2)a_2 + \dots + (e_2^*, e_M)a_M &= a_2^* \\
\vdots \qquad\qquad &\quad \vdots \\
(e_M^*, e_1)a_1 + (e_M^*, e_2)a_2 + \dots + (e_M^*, e_M)a_M &= a_M^*.
\end{aligned} \tag{7}$$

Solving this equation system the coefficient vector will be given, and also the series determined by it:

$$\xi(\mathbf{r}) = a_0 + a_1 e_1(\mathbf{r}) + \ldots + a_M e_M(\mathbf{r}). \tag{8}$$

As regards the completeness it can be tested only in empirical way. It is clear that a system consisting of 30 elements cannot be complete in the 80 dimensional Euclidean space. But the fields taken into consideration do not fill up the whole space. However the obtained subspace cannot be defined exactly. That is why we had to restrict ourselves to carrying out the expansions in the examined fields — at least in a part of them — and to estimating the accuracy of the reproduction. This investigation gave much better results than we expected. In order to illustrate our results an original 500 mbar isobaric field and its reproduced form gained by the coefficient of the expansion in the Hess—Brezowsky's system (the date: January 1, 1962) is shown in *Figure 3.*

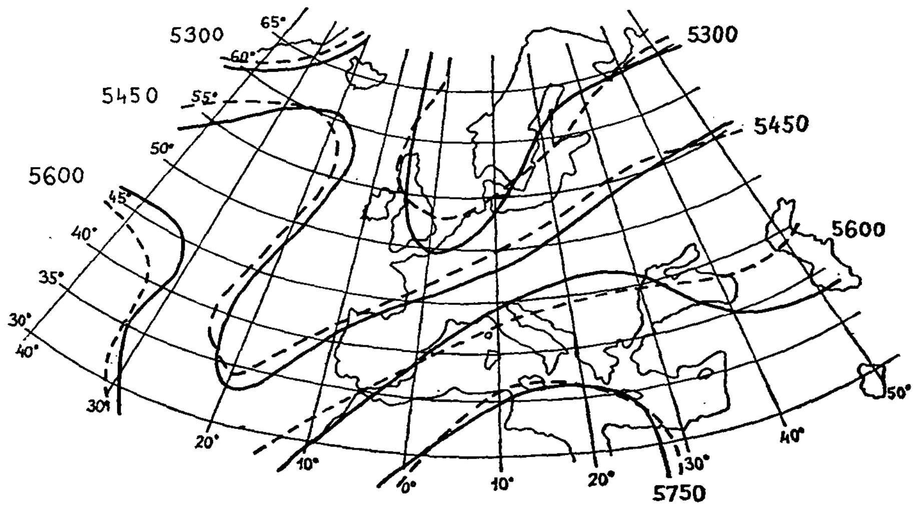

Fig. 3. 500 mbar isobaric field (1st January, 1962) before the expansion (black line), and its reconstructed form after the expansion (dotted line)

The expansional pattern was carried out for 30 days and it was found that the difference between the original field and the field reproduced by the obtained coefficients could be measured in decametres which does not exceed the inaccuracy in radiosonde measurements. (It is not necessary that the accuracy of the computed values should exceed the accuracy of the input field-data measured by radiosondes.) The expansion of the 500 mbar isobaric surface field was performed not only by the use of Hess—Brezowsky's average field system. The expansion was also done using Péczely's average field (*Péczely, 1957, Péczely, 1961*) system developed for macrosynoptic situations over the Carpathian basin. (This coding system separates 13 different situations.) According to our expectations there is significant difference between the accuracies of the expansions carried out by Péczely's and the Hess—Brezowsky's systems at daily fields reproduced by expansional coefficients. The Hess—Brezowsky's expansion representated by 29 coefficients is definitely more exact than the expansion carried out in accordance with the Péczely's system which was described by 13 coefficients. This can be explained by the more general and more differentiated character of the Hess—Brezowsky's system, furthermore with the fact the Péczely's system relates mainly to Hungary and our investigations were extended to the whole Atlantic-European area. The method presented in this paper does not use the expansion by a trigonometric function system (as for e. g. the Fourier's expansion system), nor by the eigen-system of the covariance matrix (as Karhunen—Loeve's natural orthogonal expansion), but it applies the expansion of the meteorological average fields of Hess—Brezowsky's macrosynoptic types. With the natural orthogonal expansion it came out that it is difficult to interpretate the obtained coefficients physically, and the practical solution of the eigenvalue problem is difficult even applying the modern methods of computer techniques. Using the expansion of meteorological average fields of Hess—Brezowsky's macrosynoptic types we do not have to face such difficulties because the function-system by which the expansion is carried out is given in advance.

The expansion and reproduction of the fields are being done by a sequence of programs. These programs were run partly on a Siemens type computer with operational system BS/1000—2000, and partly on IBM 370 in FORTRAN IV.

References

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