CLASSICAL THEORY OF PHYSICAL FIELDS OF SECOND KIND IN GENERAL SPACES

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A general scheme of a classical theory of physical fields of second kind is elaborated in the space of line-elements. The broad outlines of the geomerty of the space of line-elements the foundations of which have been established previously is reviewed and the differential structure of the field of higher order are elaborated.

The world continuum in which the physical phenomena take place represented by the field of second kind[1] is usually the four-dimensional pseudo-EUCLIDian metrical space of points. The quantities which determine the state of the physical field are in the case of the current — so-called — local field theory ordinary space-time functions with a defined law of transformations.

In the course of the last years the field theories of second kind were discussed from very different points of view.

In this paper we shall generalize the field theories of second kind in a quite natural way. Namely, we shall regard the metrical geometrical space in which the field is generated as a generalized one, the geometry of which is determined by an arbitrary metrical fundamental tensor. Such investigations were known previously in the case of relativistic electrodynamics in a gravitational field, when the relativistic covariant formalism of the MAXWELLian theory was elaborated in RIEMANNian space. However, our investigations are not due to a pure mathematical generalization of the theories of second kind, but are supported also from the physical point of view.

The metrical point geometry of spaces is the geometrical modell for isotropic spaces. If the basic geometrical space is a space of line-elements, such as *e. g.* in the case of the FINSLERian geometry [2] and of the geometry

[1] To distinguish between the different kinds of field theories we have proposed [5] recently the expressions: *field theory of first and of second kind.* In the case of a field theory of second kind the geometrical basis is the four-dimensional EUCLIDian (respectively pseudo-EUCLIDian) space and the physical fields are described by potentials and field functions resp., which are ordinary space time functions. If, on the other hand, the physical properties of the field are, according to the ideas of RIEMANN characterized by the geometrical structure of the space, we shall call the field theory that of the first kind. From such a point of view EINSTEIN's theory of gravitation, *e. g.*, is a field theory of first kind, and the electromagnetic and mesonic theories respectively, are field theories of second kind.

elaborated previously [7], our geometry represents an anisotropic space. This aspect is interesting from the point of view of the relativistic electrodinamics in dielectricum as well as that of classical bilocal theory of fields [12], where the so-called YUKAWAian variables can be regarded as the point co-ordinates and the directional co-ordinates of the line-elements [7].

It is well known that in anisotropic spaces the differential structure of the field is the most interesting problem. By definition of the LAGRANGian of the field in anisotropic spaces too, we shall derive the covariant field equations and conservation laws. The tensor of energy and impulse will be derived in a quite general form on the basis of the results mentioned above and for the different well known cases elaborated previously by D. HILBERT [4], M. BORN [1], L. ROSENFELD [9] and J. S. de WETT [3], respectively, it can be obtained directly by specialization.

§ 1. *Geometrical Preliminaries*

1. General definitions. The ground element of our line-element space $\mathfrak{L}$ is a line-element defined by its four dimensional space co-ordinates x^μ and by a contravariant vector v^μ ($\mu = 0, 1, 2, 3$), the direction of which corresponds to that of our line-element. Since only a direction is defined by the vector v^μ it is evident that the components of v^μ are not independent and only their proportion has meaning. *The ensemble of the ground elements* (x, v)[2] *is the so-called line-element space* $\mathfrak{L}$.

The geometrical structure of space $\mathfrak{L}$ is defined by a metrical fundamental tensor $g_{\mu\nu}(x, v)$ which will be in the following a given function of the line-elements (x, v) being homogeneus function of the variable v^μ of zero degree.

The one-parametric sequence of the line-elements

$$x^\mu = x^\mu(t), \quad v^\mu = v^\mu(t) \qquad (t_1 \leqq t \leqq t_2)$$

is defined as *a curve of space* $\mathfrak{L}$ *for the direction field* $v^\mu(t)$.

The line-elements (x, v) will be changed by transformation of the co-ordinates as follows

$$x^{\mu'} = x^{\mu'}(x), \; v^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^\mu} v^\mu, \quad \Delta \overset{\text{def}}{=} \det \left| \frac{\partial x^{\mu'}}{\partial x^\mu} \right| \neq 0$$

and *the law of transformation of tensors is given by*

$$T^{\alpha'\beta'}_{\cdot\,\cdot\,\gamma'} = \frac{\partial x^{\alpha'}}{\partial x^\alpha} \frac{\partial x^{\beta'}}{\partial x^\beta} \frac{\partial x^\gamma}{\partial x^{\gamma'}} T^{\alpha\beta}_{\cdot\,\cdot\,\gamma}.$$

Particulary, the law of transformation of our well defined metrical fundamental tensor is

$$g_{\mu'\nu'} = \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} g_{\mu\nu}.$$

[2] x and v are the abbreviations for x^μ and v^μ, respectively.

The length of the arch of our curve $x^\mu = x^\mu(t)$ *for the regarded fields of directions* $v^\mu = v^\mu(t)$ *is defined by*

$$s = \int_{t_1}^{t_2} \{g_{\mu\nu}(x, v)\, \dot{x}^\mu \dot{x}^\nu\}^{1/2} dt \qquad \left(\dot{x}^\mu \overset{\text{def}}{=} \frac{dx^\mu}{dt}\right).$$

Now, let be given a scalar *fundamental function* by

$$F(x, v) \overset{\text{def}}{=} \{g_{\mu\nu}(x, v) v^\mu v^\nu\}^{1/2},$$

which is a homogeneous function of the variable v^μ of first degree. With the help of our fundamental function $F(x, v)$ we can define the vector l^μ of unit length in the direction of the line-element (x, v) by

$$l^\mu \overset{\text{def}}{=} F^{-1} \cdot v^\mu, \tag{1, 1}$$

which is naturally also a homogeneous function of zero degree of v^μ.

In our space $\mathcal{L}$ *the invariant differential* of a vector ξ^μ is defined by

$$D\xi^\mu \overset{\text{def}}{=} d\xi^\mu + C_{\varkappa.\lambda}^{\mu} \xi^\varkappa dv^\lambda + \Gamma_{\varkappa.\lambda}^{\mu} \xi^\varkappa dx^\lambda,$$

where $C_{\varkappa.\lambda}^{\mu}$ and $\Gamma_{\varkappa.\lambda}^{\mu}$ are the "components of connection" of the space which can be calculated explicitly if $g_{\mu\nu}$ is known. These formulae as well as their laws of transformation can be found in our previous paper cited [7].

The parallel displacement of vectors in sense of LEVI—CIVITA *is defined in our space by*

$$D\xi^\mu = 0.$$

The covariant derivative of the tensor $T_{\alpha.\gamma}^{\beta}$ is given by

$$\nabla_\lambda T_{\alpha.\gamma}^{\beta} = \partial_\lambda T_{\alpha.\gamma}^{\beta} - (\partial_{v\varrho} T_{\alpha.\gamma}^{\beta}) \Gamma^{*}{}_{0.\gamma}^{\varrho}$$
$$- \Gamma^{*}{}_{\alpha.\lambda}^{\varrho} T_{\varrho.\gamma}^{\beta} + \Gamma^{*}{}_{\varrho.\lambda}^{\beta} T_{\alpha.\gamma}^{\varrho} - \Gamma^{*}{}_{\gamma.\lambda}^{\varrho} T_{\alpha.\varrho}^{\beta},$$

where the abbreviations

$$\partial_\lambda \overset{\text{def}}{=} \frac{\partial}{\partial x^\lambda}, \quad \partial_{v\varrho} \overset{\text{def}}{=} \frac{\partial}{\partial v^\varrho}, \quad \partial^{*}_{v\varrho} \overset{\text{def}}{=} F \frac{\partial}{\partial v^\varrho}, \quad A_{\alpha.\tau}^{\varrho} \overset{\text{def}}{=} F C_{\alpha.\tau}^{\varrho},$$

$$\Gamma^{*}{}_{\alpha.\lambda}^{\varrho} \overset{\text{def}}{=} \Gamma_{\alpha.\lambda}^{\varrho} - A_{\alpha.\tau}^{\varrho} \Gamma_{0.\lambda}^{\tau}$$

are introduced and the index "0" means contraction with the vector l^α, *e.g.*

$$T_{0.\gamma}^{\beta} \overset{\text{def}}{=} T_{\alpha.\gamma}^{\beta} l^\alpha, \quad \Gamma^{*}{}_{0.\lambda}^{\varrho} = \Gamma^{*}{}_{\alpha.\lambda}^{\varrho} l^\alpha,$$

respectively, used consequently in the following.

The curvature of space $\mathcal{L}$ is given by the tensors

$$R_{\varkappa.\varrho\tau}^{\mu} = R^{*}{}_{\varkappa.\varrho\tau}^{\mu} + A_{\varkappa.\sigma}^{\mu} R^{*}{}_{0.\varrho\tau}^{\sigma},$$

$$P_{\varkappa.\varrho\tau}^{\mu} = \partial^{*}_{v^\tau} \Gamma^{*}{}_{\varkappa.\varrho}^{\mu} - \nabla_\varrho A_{\varkappa.\tau}^{\mu} + A_{\varkappa.\sigma}^{\mu} (\partial_{v^\tau} \Gamma^{*}{}_{\nu.\varrho}^{\sigma}) l^\nu,$$

$$S_{\varkappa.\varrho\tau}^{\mu} = 2 A_{\varkappa.[\tau}^{\sigma} A_{|\sigma|.\varrho]}^{|\mu|},$$

where

$$R^{*}{}_{\varkappa.\varrho\tau}^{\mu} = -2\partial_{[\varrho} \Gamma^{*}{}_{\tau].\varkappa}^{\mu} + 2\Gamma^{*}{}_{0.[\varrho}^{\sigma} \partial^{*}_{|v^\sigma|} \Gamma^{*}{}_{\tau].\varkappa}^{\mu} - 2\Gamma^{*}{}_{\sigma.[\varrho}^{\mu} \Gamma^{*}{}_{\tau].\varkappa}^{\sigma} \tag{1, 2}$$

is *the tensor of principal curvature,* and $T_{[\mu|\nu|\varrho]}$ is generally an abbreviation introduced by J. A. SCHOUTEN:

$$T_{[\mu|\nu|\varrho]} \overset{\text{def}}{=} \frac{1}{2}\{T_{\mu\nu\varrho} - T_{\varrho\nu\mu}\}.$$

By contraction of the indices μ and τ of $R_{\varkappa\cdot\varrho\tau}^{\ \ \mu}$ *the* EINSTEIN—RICCI'*s tensor*

$$R_{\varkappa\varrho} \overset{\text{def}}{=} R_{\varkappa\cdot\varrho\mu}^{\ \ \mu}, \tag{1, 3}$$

furthermore, by contraction with $g^{\varkappa\varrho}$ *the scalar curvature of the space*

$$R \overset{\text{def}}{=} g^{\varkappa\varrho} R_{\varkappa\varrho} \tag{1, 4}$$

can be derived, where $g^{\varkappa\varrho}$ is the contravariant component of the metrical fundamental tensor.

The equations of the extremal curves, or the *geodetical lines of the space:*

$$\frac{d^2x^\lambda}{ds^2} + \Gamma^{*\ \lambda}_{\mu\cdot\nu} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0 \tag{1, 5}$$

are the EULER—LAGRANGE equations of the variational principle

$$\delta\int_{s_1}^{s_2} \{g_{\mu\nu}(x, \dot{x})\dot{x}^\mu \dot{x}^\nu\}^{1/2} ds = 0.$$

2. *Volume integrals and tensor densities in the space of line-elements.* In classical physics the field is described by one or several (real) space-time functions $\psi_\mu = \psi_\mu(x, v)$ which satisfy certain partial differential equations, the so-called field equations. A current alternative procedure is to start with a variational principle chosen in such a way that its EULER—LAGRANGE differential equations are identical with the field equations. This method renders the so-called canonical formalism of the field possible.

The canonical formalism of the theory starts with the definition of the LAGRANGian $\mathfrak{L}^*$ of the field. In the usual point spaces the LAGRANGian is a scalar density and therefore the volume integral

$$I = \int_\Omega \mathfrak{L}^*(x)\, d^4x \tag{1,6}$$

— the so-called *integral of action* — is an invariant of the transformations of co-ordinates.

The scalar and tensor densities, respectively, can also defined in the space of line-elements by the usual law of transformation:

$$\mathfrak{L}' = \Delta^{-1}\mathfrak{L}$$

and

$$\mathfrak{T}_{\alpha'\cdot\gamma'}^{\ \ \beta'} = \Delta^{-1} \frac{\partial x^\alpha}{\partial x^{\alpha'}} \frac{\partial x^{\beta'}}{\partial x^\beta} \frac{\partial x^\gamma}{\partial x^{\gamma'}} \mathfrak{T}_{\alpha\cdot\gamma}^{\ \ \beta},$$

respectively, and the LAGRANGian $\mathfrak{L} = \mathfrak{L}(x, v)$ can be introduced without any difficulties, however the definition of the integral of action

$$I = \int_{\mathfrak{L}} \mathfrak{L}(x, v) d^4 x \qquad (1, 7)$$

corresponding to (1, 6) *has only a definit meaning in the usual sense if in all x points of the space a direction is given by* $v^\mu = v^\mu(x)$. In this case — as one says — *the integral of action* (1, 7) *refers to the field of direction* $v^\mu = v^\mu(x)$.

In the following we shall not deal with the definitions quite generally associated unnecessary difficulties [11], but only in a special case which seems to be sufficient for our purpose:

Let a field of direction be $v^\mu = v^\mu(x)$ satisfying the differential equation

$$\frac{d l^\mu}{d x^\varrho} + \Gamma^{*\ \mu}_{0.\varrho} = 0, \qquad (1, 8)$$

where

$$\frac{d l^\mu}{d x^\varrho} \overset{\text{def}}{=} \partial_\varrho l^\mu (x^\alpha, v^\alpha(x)). \qquad (1, 9)$$

The condition of integrability of the differential equation (1, 8) is

$$\partial_\tau \partial_\varrho l^\mu = \partial_\varrho \partial_\tau l^\mu.$$

However, based on (1, 8) it becomes

$$\partial_\varrho l^\mu (x, v(x)) = - \Gamma^{*\ \mu}_{0.\varrho}(x, l(x))$$

and in the following manner

$$\partial_\tau \partial_\varrho l^\mu - \partial_\varrho \partial_\tau l^\mu = - \Gamma^{*\ \mu}_{0.\varrho} + \frac{\partial \Gamma^{*\ \mu}_{0.\varrho}}{\partial l^\sigma} \Gamma^{*\ \sigma}_{\theta.\tau} + \partial_\varrho \Gamma^{*\ \mu}_{0.\tau} - \frac{\partial \Gamma^{*\ \mu}_{0.\tau}}{\partial l^\sigma} \Gamma^{*\ \sigma}_{0.\varrho}.$$

But $\partial \Gamma^{*\ \mu}_{0.\varrho} / \partial l^\sigma$ is a homogeneous function of l^σ of zero order because of which we have

$$\partial_\tau \partial_\varrho l^\mu - \partial_\varrho \partial_\tau l^\mu = - R^{*\ \mu}_{0.\varrho\tau},$$

where

$$R^{*\ \mu}_{0.\varrho\tau} = -2 \partial_{[\varrho} \Gamma^{*\ \mu}_{\tau].0} + 2 \Gamma^{*\ \sigma}_{0.[\varrho} \overset{*}{\partial}_{|v^\sigma|} \Gamma^{*\ \mu}_{\tau].0}.$$

The condition of integrability of our equation (1, 8) is also given by

$$R^{*\ \mu}_{0.\varrho\tau} = 0. \qquad (1, 10)$$

Based in the theorem of FROBENIUS [10], [7] the fulfilment of equation (1, 10) means that *in our space of line-elements there exists a parallel displacement of line-elements*. This is a restricting condition for the space, having immediate geometrical meaning that to a given direction in a space-time point in every other point of our space-time world a parallel direction in the sense of LEVI—CIVITA can be determined unambiguously.

Therefore in the following we shall define the integral of action (1, 7) for a field of direction $v^\mu = v^\mu(x)$ which fulfils equation (1, 8).

3. The osculate Riemannian space. It is well known that in the immediate surroundings of a point of the RIEMANNian space a pseudo-EUCLIDian metric can be introduced, that is to every point of the RIEMANNian space a "tangential pseudo-EUCLIDian space" can be given.

If in the space of line-elements absolute parallelism of the line-elements exists we can construct to all line-elements of our space, an osculate Riemannian space fulfilling the following conditions:

a) *The metrical fundamental tensor* $\gamma_{\mu\nu}$ *of the osculate Riemannian space is identical with the metrical fundamental tensor of the original line-element space,* that is

$$\gamma_{\mu\nu}(x) = g_{\mu\nu}(x, v(x)).$$

Owing to the homogenity of zero order of $g_{\mu\nu}$ in the variable v^μ

$$\gamma_{\mu\nu}(x) = g_{\mu\nu}(x, l(x)). \tag{1, 11}$$

b) *The geodetical lines of both spaces osculate each other.*

c) *The invariant differential and the covariant derivative of the vectors* ξ^μ *are identical in both spaces.*

d) *The tensors of principal curvature of both spaces are the same.*

This construction of the osculate Riemannian space differs essentially from the VARGAian one [11] being far simpler and it is based on the existence of absolute paralellism in the space.

To prove the correctness of our construction we have to calculate the parameter of connection in the osculate RIEMANNian space:

$$\tilde{\Gamma}_{\alpha\tau\beta} \stackrel{\text{def}}{=} \frac{1}{2}\{\partial_\beta \gamma_{\alpha\tau} + \partial_\alpha \gamma_{\tau\beta} - \partial_\tau \gamma_{\alpha\beta}\} =$$
$$= \frac{1}{2}\{\partial_\beta g_{\alpha\tau} + \partial_\alpha g_{\tau\beta} - \partial_\tau g_{\alpha\beta}\} + \frac{1}{2}\left\{\frac{\partial g_{\alpha\tau}}{\partial l^\sigma}\partial_\beta l^\sigma + \frac{\partial g_{\tau\beta}}{\partial l^\sigma}\partial_\alpha l^\sigma - \frac{\partial g_{\alpha\beta}}{\partial l^\sigma}\partial_\tau l^\sigma\right\}.$$

Based on (1, 8) and taking into account that owing to the homogeneity of $(-1)^{\text{th}}$ order in l^ϱ

$$\frac{\partial g_{\alpha\tau}}{\partial l^\sigma} = F\frac{\partial g_{\alpha\tau}}{\partial v^\sigma}\,\partial^*_{v^\sigma} g_{\alpha\tau},$$

we obtain immediately

$$\tilde{\Gamma}_{\alpha\tau\beta} = \Gamma^*_{\alpha\tau\beta}(x, l(x)). \tag{1, 12}$$

Furthermore,

$$\partial_\tau \tilde{\Gamma}_{\alpha.\gamma}^{\ \beta} = \partial_\tau \Gamma^{*\ \beta}_{\alpha.\gamma} - (\partial_{v^\varrho} \Gamma^{*\ \beta}_{\alpha.\gamma})\Gamma^{*\ \varrho}_{0.\tau}$$

and based on the definition of the tensor of principal curvature

$$\tilde{R}_{\alpha.\varrho\tau}^{\ \beta} = R^{*\ \beta}_{\alpha.\varrho\tau}(x, l(x)),$$

where $\tilde{R}_{\alpha.\varrho\tau}^{\ \beta}$ is RIEMANN's tensor of curvature of the osculate RIEMANNian space.

However, in our case in the line-element space absolute paralellism of the line-elements exist, hence,

$$R_{\alpha.\varrho\tau}^{\ \beta} = R^{*\ \beta}_{\alpha.\varrho\tau},$$

therefore

$$\tilde{R}_{\alpha.\varrho\tau}^{\ \beta} = R_{\alpha.\varrho\tau}^{\ \beta}.$$

Qu. e. d.

4. *The infinitesimal transformation.* The infinitesimal transformation of the co-ordinates is defined also in the space of line-elements by

$$x^{\mu'} = x^{\mu} + \varepsilon \xi^{\mu}(x), \tag{1, 13}$$

where ε is an infinitesimal parameter and $\xi^{\mu}(x)$ is an arbitrary covariant vector which is continuous and a limited functions of the co-ordinates x.

Let $T(x, \mu)$ be a quantity of the space having an arbitrary law of transformation then we define its *total* and *local variation,* respectively, as follows:

$$\begin{cases} \delta T(x, v) \overset{\text{def}}{=} T'(x', v') - T(x, v) \\ \delta^* T(x, v) \overset{\text{def}}{=} T'(x, v) - T(x, v). \end{cases}$$

Considering that based on law of transformation of v^{μ}

$$\delta v^{\mu} = \varepsilon (\partial_{\varrho} v^{\mu}) \xi^{\varrho} + 0(\varepsilon^2), \tag{1, 14}$$

the connection between the two types of variation is given by

$$\delta T = \delta^* T + \varepsilon \{(\partial_{\mu} T) \xi^{\mu} + (\partial_{v^{\mu}} T)(\partial_{\varrho} \xi^{\mu}) v^{\varrho}\} + 0(\varepsilon^2). \tag{1, 15}$$

In the case of the local variation the operations δ^* and ∂_{μ} and $\partial_{v^{\varrho}}$, respectively, can be exchanged, that is,

$$\delta^*(\partial_{\mu} T) = \partial_{\mu}(\delta^* T); \quad \delta^*(\partial_{v^{\varrho}} T) = \partial_{v^{\varrho}}(\delta^* T),$$

but in the case of the total variation

$$\delta(\partial_{\mu} T) = \partial_{\mu}(\delta T) - \varepsilon \{(\partial_{\varrho} T)(\partial_{\nu} \xi^{\varrho}) + (\partial_{v^{\varrho}} T)(\partial_{\mu} \partial_{\lambda} \xi^{\varrho}) v^{\lambda}\} + 0(\varepsilon^2) \tag{1, 16}$$

and

$$\delta(\partial_{v^{\mu}} T) = \partial_{v^{\varrho}}(\delta T) - \varepsilon (\partial_{v^{\varrho}} T)(\partial_{\mu} \xi^{\varrho}) + 0(\varepsilon^2). \tag{1, 17}$$

If the special law of transformation of T is given, it is possible — based on our above results — to calculate the total and local variation explicitly. *E. g.* if T is a covariant tensor of second order

$$\delta T^{\mu\nu} = \varepsilon \{(\partial_{\lambda} \xi^{\mu}) T^{\lambda\nu} + (\partial_{\lambda} \xi^{\nu}) T^{\mu\lambda}\} + 0(\varepsilon^2)$$

and owing to (1, 15)

$$\delta^* T^{\mu\nu} = -\varepsilon \{(\partial_{\lambda} T^{\mu\nu}) \xi^{\lambda} - (\partial_{\lambda} \xi^{\mu}) T^{\lambda\nu} - (\partial_{\lambda} \xi^{\nu}) T^{\mu\lambda} + (\partial_{v^{\varrho}} T^{\mu\nu})(\partial_{\lambda} \xi^{\varrho}) v^{\lambda}\} + 0(\varepsilon^2).$$

In the case of tensor densities we have to calculate the variation of $\sqrt{|g|}$, where

$$g \overset{\text{def}}{=} \det |g_{\mu\nu}|.$$

Since, as it is well known that

$$\delta \sqrt{|g|} = -\frac{1}{2} \sqrt{|g|}\, g_{\mu\nu} \delta g^{\mu\nu} = \frac{1}{2} \sqrt{|g|}\, g^{\mu\nu} \delta g_{\mu\nu},$$

we have

$$\delta \sqrt{|g|} = -\varepsilon \sqrt{|g|} (\partial_{\lambda} \xi^{\lambda}) + 0(\varepsilon^2).$$

Now, for a tensor, density

$$\delta \mathfrak{T} = \delta T \cdot \sqrt{|g|} + T \delta \sqrt{|g|}\,; \quad \delta^* \mathfrak{T} = \delta^* T \cdot \sqrt{|g|} + T \cdot \delta^* \sqrt{|g|}$$

and *e. g.*

$$\delta^* \mathfrak{g}^{\mu\nu} \stackrel{\text{def}}{=} \delta^*(\sqrt{|g|}\, g^{\mu\nu}) = \varepsilon \sqrt{|g|}\, \{(\partial_\lambda g^{\mu\nu}) \xi^\lambda - \\ - (\partial_\lambda \xi^\mu) g^{\lambda\nu} - (\partial_\lambda \xi^\nu) g^{\mu\lambda} + A^{\mu\nu}_{\cdot\cdot\,\cdot\lambda} (\partial_\tau \xi^\lambda) v^\tau\} + 0(\varepsilon^2). \quad (1, 18)$$

§ 2. *The deduction of the field equations*

Let

$$v^\mu = v^\mu(x) \quad (2, 1)$$

be a field of directions which fulfils our previous equation (1, 8). Then the integral of action of the field is given by

$$I = \int_\Omega \mathfrak{L}(x, v(x)) d^4 x, \qquad (\mathfrak{L} = \sqrt{|g|}\, L)$$

where Ω is a four-dimensional domain of integration and the LAGRANGian density of the field

$$\mathfrak{L} = \mathfrak{L}[g^{\mu\nu}(x, v(x)),\ \Phi_\mu(x, v(x)),\ \Phi_{\mu|\nu}(x, v(x)),\ \Phi_{\mu|\nu\lambda}(x, v(x))],$$

where Φ_μ are the components of the potentials of the field as well as

$$\Phi_{\mu|\nu} \stackrel{\text{def}}{=} \nabla_\nu \Phi_\mu = \frac{d}{dx^\nu} \Phi_\mu - \Gamma^{*\ \varrho}_{\mu\cdot\nu} \Phi_\varrho \qquad \left(\frac{d}{dx^\nu} \Phi_\mu \stackrel{\text{def}}{=} \partial_\nu \Phi_\mu + (\partial_{v\varrho} \Phi_\mu)(\partial_\nu v^\varrho)\right)$$

and

$$\Phi_{\mu|\nu\lambda} = \nabla_\lambda \Phi_{\mu|\nu} = \frac{d}{dx^\lambda} \Phi_{\mu|\nu} - \Gamma^{*\ \sigma}_{\mu\cdot\lambda} \Phi_{\sigma|\nu} - \Gamma^{*\ \sigma}_{\nu\cdot\lambda} \Phi_{\mu|\sigma},$$

respectively [7].[3]

Varying the functions Φ_μ for the fixed region Ω of integration subject to the restrictions that the variations of the Φ_μ-s and their first derivatives at the boundary of the domain of integration vanish, one obtains

$$\delta I = \int_\Omega \left\{ \frac{\partial \mathfrak{L}}{\partial \Phi_\mu} \delta \Phi_\mu + \left[\frac{\partial \mathfrak{L}}{\partial \Phi_{\mu|\nu}} - \frac{\partial \mathfrak{L}}{\partial \Phi_{\alpha|\nu\lambda}} \Gamma^{*\ \mu}_{\alpha\cdot\lambda} - \frac{\partial \mathfrak{L}}{\partial \Phi_{\mu|\alpha\lambda}} \Gamma^{*\ \nu}_{\alpha\cdot\lambda} \right] \delta \Phi_{\mu|\nu} \\ + \frac{\partial \mathfrak{L}}{\partial \Phi_{\mu|\nu\lambda}} \frac{d}{dx^\lambda} \Phi_{\mu|\nu\lambda} \right\} d^4 x.$$

Based on the condition of stationarity

$$\delta I = 0,$$

[3] The field equations in the case of scalar fields as well as in the case of tensorial fields were deduced in [7] and in a paper communicated to the Hungarian Academy of Sciences (1955), respectively.

we have by repeated partial integration

$$\int\limits_{\Omega} \left\{ \frac{\partial \mathfrak{L}}{\partial \Phi_\nu} - \Theta^{\varrho\nu} \Gamma^{*\,\mu}_{\varrho\cdot\nu} - \frac{d}{dx^\nu} \Theta^{\mu\nu} \right\} \delta \Phi_\mu d^4 x = 0,$$

where

$$\Theta^{\mu\nu} \overset{\text{def}}{=} \frac{\partial \mathfrak{L}}{\partial \Phi_{\mu|\nu}} - \frac{\partial \mathfrak{L}}{\partial \Phi_{\alpha|\nu\lambda}} \Gamma^{*\,\mu}_{\alpha\cdot\lambda} - \frac{\partial \mathfrak{L}}{\partial \Phi_{\mu|\alpha\lambda}} \Gamma^{*\,\nu}_{\alpha\cdot\lambda} - \frac{d}{dx^\lambda} \frac{\partial \mathfrak{L}}{\partial \Phi_{\mu|\nu\lambda}}. \qquad (2, 2)$$

However, this equation is fulfilled for arbitrary variations of the Φ_μ-s which satisfy the above mentioned conditions and for an arbitrary choice of the integration region. Consequently for all space-time points:

$$\frac{\partial \mathfrak{L}}{\partial \Phi_\mu} - \Theta^{\varrho\nu} \Gamma^{*\,\mu}_{\varrho\cdot\nu} - \frac{d}{dx^\nu} \Theta^{\mu\nu} = 0. \qquad (2, 3)$$

To put the field equations (2, 3) in their explicit covariant form we write $\Theta^{\mu\nu}$ as

$$\Theta^{\mu\nu} = \mathfrak{F}^{\mu\nu} - \mathfrak{F}^{\alpha\nu\lambda} \Gamma^{*\,\mu}_{\alpha\cdot\lambda} - \mathfrak{F}^{\mu\alpha\lambda} \Gamma^{*\,\nu}_{\alpha\,\lambda} - \frac{d}{dx^\lambda} \mathfrak{F}^{\mu\nu\lambda},$$

where the following abbreviations are introduced:

$$\mathfrak{F}^{\mu\nu\lambda} \overset{\text{def}}{=} \frac{\partial \mathfrak{L}}{\partial \Phi_{\mu|\nu\lambda}}, \quad F^{\mu\nu\lambda} = \frac{1}{\sqrt{|g|}} \mathfrak{F}^{\mu\nu\lambda}$$

and

$$\mathfrak{F}^{\mu\nu} \overset{\text{def}}{=} \frac{\partial \mathfrak{L}}{\partial \Phi_{\mu|\lambda}}, \quad F^{\mu\nu} = \frac{1}{\sqrt{|g|}} \mathfrak{F}^{\mu\nu}.$$

Since

$$\nabla_\lambda F^{\mu\nu\lambda} = \frac{d}{dx^\lambda} F^{\mu\nu\lambda} + F^{\alpha\nu\lambda} \Gamma^{*\,\mu}_{\alpha\cdot\lambda} + F^{\mu\alpha\lambda} \Gamma^{*\,\nu}_{\alpha\cdot\lambda} + F^{\mu\nu\alpha} \Gamma^{*\,\lambda}_{\alpha\cdot\lambda}$$

and based on the equations (1, 9) and (1, 12)

$$\frac{d}{dx^\lambda} \sqrt{|g|} = \partial_\lambda \sqrt{|g|} - (\partial_{v\varrho} \sqrt{|g|} \, F\Gamma^{*\,\varrho}_{0\cdot\lambda} = \partial_\lambda \sqrt{|g|} + \frac{\partial \sqrt{|g|}}{\partial l^\sigma} \partial_\lambda l^\sigma =$$

$$= \frac{1}{2} \sqrt{|g|} \, g^{\sigma\lambda} \left\{ \partial_\lambda g_{\sigma\tau} + \frac{\partial g_{\sigma\tau}}{\partial l^\varrho} \partial_\lambda l^\varrho \right\} = \sqrt{|g|} \, \Gamma^{*\,\varrho}_{\lambda\cdot\varrho},$$

we have

$$\sqrt{|g|} \, \nabla_\lambda F^{\mu\nu\lambda} = \frac{d}{dx^\lambda} \mathfrak{F}^{\mu\nu\lambda} + \mathfrak{F}^{\alpha\nu\lambda} \Gamma^{*\,\mu}_{\alpha\cdot\lambda} + \mathfrak{F}^{\mu\alpha\lambda} \Gamma^{*\,\nu}_{\alpha\cdot\lambda}.$$

Therefore we obtain

$$\Theta^{\mu\nu} = \mathfrak{F}^{\mu\nu} - \sqrt{|g|} \, \nabla_\lambda F^{\mu\nu\lambda}.$$

Furthermore, introducing the notations

$$\mathfrak{F}^{\mu} \overset{\text{def}}{=} \frac{\partial \mathfrak{L}}{\partial \Phi_\mu}, \quad F^\mu = \frac{1}{\sqrt{|g|}} \mathfrak{F}^\mu,$$

we can put our equation (2, 3) in the form

$$\mathfrak{F}^{\mu}-[\mathfrak{F}^{\varrho\nu}-\sqrt{|g|}\,\nabla_{\lambda}F^{\varrho\nu\lambda}]\Gamma^{*\ \mu}_{\varrho\cdot\nu}-\frac{d}{dx^{\nu}}[\mathfrak{F}^{\mu\nu}-\sqrt{|g|}\,\nabla_{\lambda}F^{\mu\nu\lambda}]=0$$

and

$$\mathfrak{F}^{\mu}-\mathfrak{A}^{\varrho\nu}\Gamma^{*\ \mu}_{\varrho\cdot\nu}-\frac{d}{dx^{\nu}}\mathfrak{A}^{\mu\nu}=0,$$

respectively, where

$$\mathfrak{A}^{\mu\nu}\overset{\text{def}}{=}\mathfrak{F}^{\mu\nu}-\sqrt{|g|}\,\nabla_{\lambda}F^{\mu\nu\lambda}.$$

Since

$$\nabla_{\nu}A^{\mu\nu}=\frac{d}{dx^{\nu}}A^{\mu\nu}+A^{\varrho\nu}\Gamma^{*\ \mu}_{\varrho\cdot\nu}+A^{\mu\varrho}\Gamma^{*\ \nu}_{\varrho\cdot\nu},$$

just as above, we have

$$\sqrt{|g|}\,\nabla_{\nu}A^{\mu\nu}=\frac{d}{dx^{\nu}}\mathfrak{A}^{\mu\nu}+\mathfrak{A}^{\varrho\nu}\Gamma^{*\ \mu}_{\varrho\cdot\nu}$$

and finally

$$F^{\mu}-\nabla_{\nu}\{F^{\mu\nu}-\nabla_{\lambda}F^{\mu\nu\lambda}\}=0.$$

Taking into account, that the determinant g of the metrical fundamental tensor $g_{\mu\nu}$ does not depend on Φ_{μ} and its derivatives, we have

$$\frac{\partial L}{\partial\Phi_{\mu}}-\nabla_{\nu}\left\{\frac{\partial L}{\partial\Phi_{\mu|\nu}}-\nabla_{\lambda}\frac{\partial L}{\partial\Phi_{\mu|\nu\lambda}}\right\}=0. \qquad (2, 4)$$

This equation gives for the field of direction $v^{\mu}=v^{\mu}(x)$ the explicit covariant form of the field equations of our vectorial field.

In RIEMANNian space equations (2, 4) has the form

$$\frac{\partial\tilde{L}}{\partial\tilde{\Phi}_{\mu}}-\tilde{\nabla}_{\nu}\left\{\frac{\partial\tilde{L}}{\partial\tilde{\Phi}_{\mu|\nu}}-\tilde{\nabla}_{\lambda}\frac{\partial\tilde{L}}{\partial\tilde{\Phi}_{\mu|\nu\lambda}}\right\}0, \qquad (2, 5)$$

where $\tilde{\nabla}_{\nu}$ is the differential operator of the covariant derivative in the RIEMANNian space.

In the case when in our space the absolute parallelism of the line-elements does not exists, based on the VARGAian methods of construction of the osculate RIEMANNian space, one another version of this theory can be elaborated. However, we shall not deal with this generalization because the supposition of the existence of the absolute parallelism of the line-elements seems to be realizable in the partically interesting cases.

§ 3. *The differential laws of conservation*

1. The fundamental identities deduced by the infinitesimal transformation. As it is well known, based on the infinitesimal transformation of co-ordinates some identities can be deduced which from the phisical point of view can be interpreted as the differential laws of conservation of the field [8].

If the variation of the LAGRANGian $\mathfrak{L}$ brought about by the change of co-ordinates is investigated, we must take into account the explicit dependence of the LAGRANGian on the contravariant components of the metrical fundamental tensor $g^{\mu\nu}$ and their derivatives too. The derivatives of $g^{\mu\nu}$ can be found in the parameters of connections of the space and in their derivatives. But these derivatives are the partial derivatives of the $g^{\mu\nu}$ owing to which our LAGRANGian density has the form

$$\mathfrak{L} = \mathfrak{L}[g^{\mu\nu}, g^{\mu\nu}_{\cdot\cdot(\alpha)}, g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}, \Phi_\mu, \Phi_{\mu|\nu}, \Phi_{\mu|\nu\lambda}] \tag{3, 1}$$

with

$$g^{\mu\nu}_{\cdot\cdot(\alpha)} \stackrel{\text{def}}{=} \partial_\alpha g^{\mu\nu}, \; g^{\mu\nu}_{\cdot\cdot(\alpha\beta)} \stackrel{\text{def}}{=} \partial_\beta g^{\mu\nu}_{\cdot\cdot(\alpha)} = \partial_\beta \partial_\alpha g^{\mu\nu}.$$

If in our space of line-elements the absolute paralellism of line-elements exists a field of directions

$$v^\mu = v^\mu(x) \tag{3, 2}$$

can be introduced satisfying our equation (1, 8) and the integral of action defined for this field of directions is

$$I = \int_\Omega \mathfrak{L}(x, v(x))\, d^4(x).$$

Now, we pass over to the osculate RIEMANNian space introduced above, which has the metrical fundamental tensor

$$\gamma^{\mu\nu}(x) = g^{\mu\nu}(x, v(x)),$$

therefore

$$\gamma^{\mu\nu}_{\cdot\cdot(\alpha)} = \partial_\alpha g^{\mu\nu} + (\partial_{v\varrho} g^{\mu\nu})\, \partial_\alpha v^\varrho = \partial_\alpha g^{\mu\nu} - (\overset{*}{\partial}_{v\varrho} g^{\mu\nu})\, \Gamma^{*\,\varrho}_{\,0\cdot\alpha} = \frac{d}{dx^\alpha} g^{\mu\nu}$$

and similarly

$$\gamma^{\mu\nu}_{\cdot\cdot(\alpha\beta)} = \frac{d^2 g^{\mu\nu}}{dx^\alpha dx^\beta}.$$

Furthermore, introducing the notations

$$\varphi_\mu(x) \stackrel{\text{def}}{=} \Phi_\mu(x, v(x)),$$

$$\varphi_{\mu;\nu}(x) \stackrel{\text{def}}{=} \tilde{\nabla}_\nu \varphi_\mu; \quad \varphi_{\mu;\nu\lambda}(x) = \tilde{\nabla}_\lambda \varphi_{\mu;\nu}$$

and

$$\mathfrak{L} \stackrel{\text{def}}{=} \mathfrak{L}(x, v(x)),$$

the integral of action in the osculate RIEMANNian space is

$$I = \int_\Omega \mathfrak{L}(x)\, d^4x.$$

The total variation of I subjected to the restrictions that the ξ^μ-s of the infinitesimal transformation (1, 13) at the boundary of the domain of integration Ω vanish is given by

$$\delta I = \int_\Omega \delta^* \mathfrak{L}\, d^4x,$$

where

$$\delta^* \mathfrak{L} = [\mathfrak{L}]_{\mu\nu} \delta^* \gamma^{\mu\nu} + \sqrt{|g|} \left\{ \frac{\partial \tilde{L}}{\partial \varphi_\mu} - \tilde{\nabla}_\nu \frac{\partial \tilde{L}}{\partial \varphi_{\mu;\nu}} + \nabla_\nu \tilde{\nabla}_\lambda \frac{\partial \tilde{L}}{\partial \varphi_{\mu;\nu\lambda}} \right\} \delta^* \varphi_\mu$$

being

$$[\mathfrak{L}]_{\mu\nu} \overset{\text{def}}{=} \frac{\partial \mathfrak{L}}{\partial \gamma^{\mu\nu}} - \frac{d}{dx^{\alpha}} \left[\frac{\partial \mathfrak{L}}{\partial \gamma^{\mu\nu}_{..(\alpha)}} - \frac{d}{dx^{\beta}} \frac{\partial \mathfrak{L}}{\partial \gamma^{\mu\nu}_{..(\alpha\mu)}} \right]$$

the LAGRANGian derivative of $\mathfrak{L}$.

Assuming that the potentials Φ_{μ} and the corresponding potentials φ_{μ} in the osculate RIEMANNian space, respectively, fulfil the equation of field

$$\frac{\partial \tilde{L}}{\partial \varphi_{\mu}} - \tilde{\nabla}_{\nu} \frac{\partial \tilde{L}}{\partial \varphi_{\mu;\nu}} + \tilde{\nabla}_{\nu} \tilde{\nabla}_{\lambda} \frac{\partial \tilde{L}}{\partial \varphi_{\mu;\nu\lambda}} = 0,$$

the variation of our integral of action is reduced to

$$\delta I = \int_{\Omega} [\mathfrak{L}]_{\mu\nu} \delta^{*} \gamma^{\mu\nu} d^{4}x.$$

I is, however, an invariant of the changes of co-ordinates, therefore

$$\delta I = 0$$

for the infinitesimal transformation of co-ordinates too. This means that

$$\int_{\Omega} [\mathfrak{L}]_{\mu\nu} \delta^{*} \gamma^{\mu\nu} d^{4}x = 0 \tag{3, 3}$$

for an arbitrary choice of the integration domain. But $\delta^{*}\gamma^{\mu\nu}$ is symmetrical in its indices μ and ν, therefore, the antisymmetrical part of the LAGRANGian derivatives of $\mathfrak{L}$ does not come into consideration. As matters stand we shall introduce the symmetric tensor density

$$\mathfrak{T}_{\mu\nu} \overset{\text{def}}{=} -2[\mathfrak{L}]_{(\mu\nu)} \equiv -\{[\mathfrak{L}]_{\mu\nu} + [\mathfrak{L}]_{\mu\nu}\} \tag{3, 4}$$

and based on (2, 5) we have

$$\int_{\Omega} \mathfrak{T}_{\mu\nu} \delta^{*} \gamma^{\mu\nu} d^{4}x = 0$$

and finally — using our equation (1, 18) for $\delta^{*}\gamma^{\mu\nu}$ — this integral can be written in the form

$$\varepsilon \int_{\Omega} \{\mathfrak{T}_{\mu\nu}(\partial_{\lambda}\gamma^{\mu\nu})\xi^{\lambda} - \mathfrak{T}_{\mu}{}^{\lambda}_{.}(\partial_{\lambda}\xi^{\mu}) - \mathfrak{T}^{\lambda}_{.\nu}(\partial_{\lambda}\xi^{\nu})\} d^{4}x = 0.$$

Subjected to the restriction that ξ^{μ} vanishes at the boundary of the domain of integration by partial integration it is obtained that

$$\varepsilon \int_{\Omega} \{\mathfrak{T}_{\mu\nu}(\partial_{\lambda}\gamma^{\mu\nu}) + 2\partial_{\mu}\mathfrak{T}_{\lambda}{}^{\mu}_{.}\} \xi^{\lambda} d^{4}x = 0.$$

However, this is an identity for arbitrary ξ^{μ}-s and for arbitrary choice of the integration domain, therefore, based on this consideration we have

$$\partial_{\mu}\mathfrak{T}_{\lambda}{}^{\mu}_{.} + \frac{1}{2}(\partial_{\lambda}\gamma^{\mu\nu})\mathfrak{T}_{\mu\nu} = 0, \tag{3, 5}$$

or

$$\partial_\mu \mathfrak{T}_{\lambda.}^{\mu} - \frac{1}{2}(\partial_\lambda \gamma_{\mu\nu})\mathfrak{T}^{\mu\nu} = 0 \tag{3, 6}$$

and

$$\partial_\mu \mathfrak{T}_{\lambda.}^{\mu} - \tilde{\Gamma}_{\lambda.\sigma}^{\varrho}\mathfrak{T}_{\varrho.}^{\sigma} = 0,$$

respectively.

These identities are deduced in the osculate RIEMANNian space. Now, returning from the osculate RIEMANNian space to our original space of line-elements we obtain based on (1, 15) that

$$\partial_\mu \mathfrak{T}_{\lambda.}^{\mu} - (\overset{*}{\partial}_{v^\tau} \mathfrak{T}_{\lambda.}^{\mu}) \Gamma^{*\,\tau}_{0.\mu} - \Gamma^{*\,\varrho}_{\lambda.\sigma} \mathfrak{T}_{\varrho.}^{\sigma} = 0 \tag{3, $\overset{*}{7}$}$$

for the field of direction satisfying our equation (1, 12).

Our equations (3, 7) are the required identities which will determine the laws of conservation for the physical field.

2. The metrical tensor of energy and impulse. The metrical tensor of energy and impulse of the field was originally defined by D. HILBERT [4] in the RIEMANNian space as the coefficients of the $\delta^* \tilde{g}_{\mu\nu}$-s in the integral

$$\delta \tilde{I} = 2 \int_\Omega [\tilde{\mathfrak{L}}]^{\mu\nu} \delta^* \tilde{g}_{\mu\nu} d^4 x, \tag{3, 8}$$

or explicitly

$$\tilde{T}^{\mu\nu} \overset{\text{def}}{=} \frac{2}{\sqrt{|g|}} [\tilde{\mathfrak{L}}]^{(\mu\nu)}. \tag{3, 9}$$

Now, based on the identity

$$\delta^* \tilde{g}_{\mu\nu} = -\tilde{g}_{\alpha\mu} \tilde{g}_{\beta\nu} \delta^* \tilde{g}^{\alpha\beta},$$

(3, 8) becomes

$$\delta I = -2 \int_\Omega [\tilde{\mathfrak{L}}]_{\mu\nu} \delta^* \tilde{g}^{\mu\nu} d^4 x \tag{3, 10}$$

and similarly the covariant components of the tensor of energy and impulse can be defined as

$$\tilde{T}_{\mu\nu} \overset{\text{def}}{=} -\frac{2}{\sqrt{|g|}} [\tilde{\mathfrak{L}}]^{\mu\nu}. \tag{3, 11}$$

These considerations were valid in the RIEMANNian space. To define the metrical tensor of energy and impulse in the space of line-elements too — assuming that in the space of line-elements the absolute parallelism of the line-elements exists — we shall suppose that there is given a field of directions $v^\mu = v^\mu(x)$ fulfilling the equations (1, 8). Then we introduce the metrical tensor of energy and impulse — based on (3, 4) — by the definition

$$T_{\mu\nu} \overset{\text{def}}{=} -\frac{1}{\sqrt{|g|}} \left\{ \left(\frac{\partial \mathfrak{L}}{\partial g^{\mu\nu}} + \frac{\partial \mathfrak{L}}{\partial g^{\nu\mu}} \right) - \right. \tag{3, 12}$$

$$\left. - \frac{d}{dx^\alpha} \left[\left(\frac{\partial \mathfrak{L}}{\partial g^{\mu\nu}_{..(\alpha)}} + \frac{\partial \mathfrak{L}}{\partial g^{\nu\mu}_{..(\alpha)}} \right) - \frac{d}{dx^\beta} \left(\frac{\partial \mathfrak{L}}{\partial g^{\mu\nu}_{..(\alpha\beta)}} + \frac{\partial \mathfrak{L}}{\partial g^{\nu\mu}_{..(\alpha\beta)}} \right) \right] \right\}.$$

Taking into account that

$$\mathfrak{L} = \sqrt{|g|}\, L,$$

we have

$$\frac{\partial \mathfrak{L}}{\partial g^{\mu\nu}} = \sqrt{|g|}\, \frac{\partial L}{\partial g^{\mu\nu}} + L\, \frac{\partial \sqrt{|g|}}{\partial g^{\mu\nu}} = \sqrt{|g|} \left\{ \frac{\partial L}{\partial g^{\mu\nu}} - \frac{1}{2} g_{\mu\nu} \right\}$$

and

$$\frac{\partial \mathfrak{L}}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha)}} = \sqrt{|g|}\, \frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha)}}; \quad \frac{\partial \mathfrak{L}}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}} = \sqrt{|g|}\, \frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}},$$

respectively. Similarly

$$\frac{d}{dx^\alpha}\left(\frac{\partial \mathfrak{L}}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha)}}\right) = \frac{d}{dx^\alpha}\left\{ \sqrt{|g|}\, \frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha)}} \right\} = \sqrt{|g|} \left\{ \frac{d}{dx^\alpha} \frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha)}} + \Gamma^{*\,\sigma}_{\alpha.\sigma} \frac{\partial L}{g\partial^{\mu\nu}_{\cdot\cdot(\alpha)}} \right\}$$

and

$$\frac{d}{dx^\alpha} \frac{d}{dx^\beta}\left(\frac{\partial \mathfrak{L}}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}}\right) = \frac{d}{dx^\alpha}\left\{ \sqrt{|g|} \left[\frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha)}} + \Gamma^{*\,\sigma}_{\beta.\sigma} \frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}} \right] \right\} =$$

$$= \sqrt{|g|} \left\{ \frac{d}{dx^\alpha} \left[\frac{d}{dx^\beta} \frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}} + \Gamma^{*\,\sigma}_{\beta.\sigma} \frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}} \right] + \right.$$

$$\left. + \Gamma^{*\,\tau}_{\alpha.\tau} \left[\frac{d}{dx^\beta} \frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}} + \Gamma^{*\,\sigma}_{\beta.\sigma} \frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}} \right] \right\},$$

respectively. Therefore, we finally obtain:

$$\begin{aligned} T_{\mu\nu} = g_{\mu\nu} L - \left(\frac{\partial L}{\partial g^{\mu\nu}} + \frac{\partial L}{\partial g^{\nu\mu}} \right) + \frac{d}{dx^\alpha} \left\{ \left(\frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha)}} + \frac{\partial L}{\partial g^{\nu\mu}_{\cdot\cdot(\alpha)}} \right) - \right. \\ - \Gamma^{*\,\sigma}_{\beta.\sigma} \left(\frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}} + \frac{\partial L}{\partial g^{\nu\mu}_{\cdot\cdot(\alpha\beta)}} \right) - \left. \frac{d}{dx^\beta} \left(\frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}} + \frac{\partial L}{\partial g^{\nu\mu}_{\cdot\cdot(\alpha\beta)}} \right) \right\} + \\ + \Gamma^{*\,\tau}_{\alpha.\tau} \left\{ \left(\frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha)}} + \frac{\partial L}{\partial g^{\nu\mu}_{\cdot\cdot(\alpha)}} \right) - \right. \\ - \Gamma^{*\,\sigma}_{\beta.\sigma} \left(\frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}} + \frac{\partial L}{\partial g^{\nu\mu}_{\cdot\cdot(\alpha\beta)}} \right) - \left. \frac{d}{dx^\beta} \left(\frac{\partial L}{\partial g^{\mu\nu}_{\cdot\cdot(\alpha\beta)}} + \frac{\partial L}{\partial g^{\nu\mu}_{\cdot\cdot(\alpha\beta)}} \right) \right\} \end{aligned} \tag{3.13}$$

defined for the field of directions $v^\mu = v^\mu(x)$, where the differential operator d/dx^α is introduced by (2, 3).

3. The laws of conservation of the energy and impulse. The metrical tensor of energy and impulse of the field (3, 13) was defined on the basis of (3, 4) passing from the osculate RIEMANNian space to the space of line-elements. Therefore, the tensor (3, 13) satisfies the identities (3, 8) representing also the required laws of conservation of energy and impulse.

The above considerations where based on the assumption that in our space of line-elements there exists the absolute parallelism of line-elements. The tensor of energy and impulse was defined for this case and it was shown that this tensor fulfils the laws of conservation. If the absolute parallelism of line-elements does not exist the tensor of energy and impulse can be defined by (3, 13), however, this $T^{*}_{\mu\nu}$ does not fulfil the identity (3, 8) and, therefore, in this case it seems impossibble to give any physical meaning to $T^{*}_{\mu\nu}$.

References

[1] *Born, M.: Ann.* Inst. H. Poincaré, **7**. 155 (1937).
[2] *Cartan, E.:* Les espaces de Finsler (Act. sc. et ind. No. 79. Paris, 1934).
[3] *de Wett, J. S.:* Proc. Camb. Phil. Soc. **43**. 511. (1948).
[4] *Hilbert, D.:* Göttinger Nachr. 1915; p. 395.
[5] *Horváth, J. I.:* Bull. Acad. Pol. **3**. 153. (1955).
[6] *Horváth, J. I., A. Moór:* Zs. f. Phys. **131**. 544 (1952).
[7] *Horváth, J. I., A. Moór:* Proc. Kon. Nederl. Acad. Wettensch. (A) **58**. 421, 581. (1955).
[8] *Pauli, W.:* Relativitätstheorie. (*Enc. d. Math. Wiss.* B. V/2, Teubner, Leipzig, 1922).
[9] *Rosenfeld, L.:* Mem. de l'Acad. roy. de Belguique **5**. No **6** (1940).
[10] *Varga, O.:* Monatshefte f. Math. u. Phys. **50**. 165 (1941).
[11] *Varga, O.:* Proc. Kon. Nederl. Acad. Wettensch. **52**. 868 (1949).
[12] *Yukawa, H.:* Phys. Rev. **76**. 300, 1731 (1949); **77**. 219 (1950); **91**. 415, 416 (1950).