ON STRUCTURE OF MAGNETIC FIELD OF THE SUN

By I. K. CSADA

Institute of Theoretical Physics, The University, Szeged

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The form of polar rays will be discussed based on the coronal photographs made by the expeditions of solar eclipses from 1896 to 1954. It is assumed that the polar rays coincide with the lines of force of the general magnetic field. The ratio of the coefficients of the second and the first terms, respectively, is determined for the vector potential represented in a series of Legendre-functions. An analysis of periods suggests that the ratio changes with a period of 22,85 years, which we may regard as a variation parallel to the solar magnetic cycle.

§ 1. *Introduction*

The highly ionized gases of the solar corona may be arranged by a very weak magnetic field so far that tubes of different electron densities can occur. These tubes are observed in time of total eclipses as beautiful light rays in the polar areas. Some theoretical investigations conclude that these polar rays coincide with the lines of force of the magnetic field. In the following this assumption will be accepted.

The solar magnetograms show that the magnetic field of the Sun has random character. In the polar areas effects can be observed showing a magnetic field of definite orientation. As a matter of fact it can be supposed that the solar magnetic field has a component of *permanent character* and that of *random character*. The average of actual field defines a permanent field *i. e.* the *general field*. The lines of force of the actual field show stochastic oscillations and their mean form gives the lines of force of the general field. The polar rays observed correspond to the actual field and the lines of force of the general field will be defined by the mean form of the polar rays. We may assume that the structure of the general magnetic field may be deduced on the basis of the analysis of the form of the polar rays.

§ 2. *General magnetic field*

Let us consider a system of polar rays traced from a coronal photograph. A system of straight lines can be constructed in such a way that the lines coincide so accurately as it is possible with the tangent of the polar rays at the solar surface. The meeting point of the system of straight lines will be excentrically placed between the centre and the pole, Denoting the displacement of the meeting point from the centre by q, the formula

$$q/r_{\odot} = \frac{\sin \alpha}{\sin (\alpha + \vartheta)} \tag{1}$$

can be obtained, where α denotes the angle between the radius and the tangent of the polar rays at the solar surface and ϑ is the polar distance of the radius.

Values of $q/r_{\odot}$ determined from the corona, observed in course of the eclipses of 1900, 1901 and 1941, were published by VAN DE HULST [1] and are summarized in Table I.

It will be deduced $q/r_{\odot}$ for the magnetic field represented by the vector potential

$$\mathfrak{A}_{\varphi} = \sum_{n=1}^{\infty} \frac{h_n P_n^{(1)}(\vartheta)}{r^{n+1}} \tag{2}$$

Table I

$q/r_{\odot}$	Year
0,70	1900
0,65	1901
0.56	1941

where h_n denote arbitrary coefficients, $P_n^{(1)}$ the associated LEGENDRE functions as well as r the distance from the solar centre.

First of all we have to calculate α. It can be easily proved that

$$tg\, \alpha = r \frac{d\vartheta}{dr}, \tag{3}$$

where the derivative $d\vartheta/dr$ from the equation of the lines of force

$$\sum_{n=1}^{\infty} \frac{h_n P_n^{(1)} \sin \vartheta}{r^n} = \text{const.}$$

can be deduced. Owing some recurrence formulae[1] for the LEGENDRE-functions we obtain

$$\operatorname{tg} \alpha = \frac{\sum \frac{n\, h_n}{r^n} P_n^{(1)}}{\sum \frac{n(n+1) h_n}{r^n} P_n}. \tag{4}$$

Substituting (4) into (1) we may write

$$q/r_{\odot} = \frac{\sum \frac{n\, h_n}{r^n} P_n^{(1)}}{\sum \frac{n\, h_n}{r^n} P_{n+1}^{(1)}}.$$

In the case of a dipole ($n = 1$) the ratio $q/r_{\odot}$ reduces to the simple form

$$q/r_{\odot} = \frac{1}{3 \cos \vartheta},$$

[1] In these deductions and in the following ones the recurrence formulae have been used:

$$(2n+1) P_n^{(1)} \sin \vartheta = n(n+1)(P_{n-1} - P_{n+1}), \qquad (2n+1) P_n \cos \vartheta = n P_{n-1} - (n+1) P_{n+1}$$

$$(2n+1) P_n \sin \vartheta = -(P_{n-1}^{(1)} - P_{n+1}^{(1)}), \qquad (2n+1) P_n^{(1)} \cos \vartheta = (n+1) P_{n-1}^{(1)} - n P_{n+1}^{(1)}$$

and when ϑ is small we have

$$x/r_{\odot} = 0{,}33.$$

Comparing this value with the data given in Table I, it can be undoubtedly seen that the solar magnetic field is not represented by a simple dipole.

Let us try to interpret the solar magnetic field taking the second ($n = 2$) or the third ($n = 3$) terms of the series (2) into account. The second term corresponds to a quadrupole which has equal polarity at the two poles and therefore it could not be called into play.

The next term of (2) corresponds to an octupole. This three-order pole has a family of lines of force resembling a six-petal rosette. It has opposite polarity at the two poles but the polarity changes in each hemisphere near the equator. Calculating $q/r_{\odot}$ for this field we get near the poles

$$q/r_{\odot} = 0{,}60.$$

It appears as a very important fact that this term is nearly equal to the value of $q/r_{\odot}$ derived from observations. There exist, however, small deviations which cannot be explained only by one-term potential. Therefore we shall use two terms of the series (2), notably the first and the third terms, as follows:

$$\mathfrak{A}_{\varphi} = h_1 \frac{P_1^{(1)}}{r^2} + h_3 \frac{P_3^{(1)}}{r^4}. \qquad (6)$$

Owing to this representation near the polar areas we have

$$q/r_{\odot} = \frac{36 + k}{60 + 3k},$$

where

$$k = \frac{2 r_{\odot}^2 h_1}{h_3}$$

is a parameter runnig over all numbers between $-\infty$ and $+\infty$. Its value has to be chosen in such a way that the calculated ratio of h_3/h_1 should be in agreement with the observed value. The numerical results are summarized in Table II.

Table II

k	$h_3/h_1 r_{\odot}^2$	Year
$-5{,}45$	$-0{,}37$	1900
$-2{,}53$	$-0{,}79$	1901
$3{,}52$	$0{,}57$	1941

The method presented above has, however, the disadvantage that it would be very difficult to estimate its range of approximation. Therefore in the next paragraph there will be proposed a more exact method for determination of these parameters.

§ 3. *Determination of h_3/h_1*

The determination of the characteristics of the solar magnetic field discussed in the previous paragraph can be applied only in the neighbourhood of the poles. The accuracy of the determination of the meeting point of the straight lines is limited by the fact that the meeting point of the tangents of the magnetic lines being farther off the poles do not exactly coincide with that of the tangents of magnetic lines which are in the neighbourhood of the poles. As a matter of fact the determination of the ratio h_3/h_1 depends on the polar distance of the magnetic lines used in the method. To avoid this difficulty in the following an other method will be suggested to calculate the ratio h_3/h_1 having no systematic errors of this kind.

The equation of magnetic lines taking only dipole and octupole terms into account by the help of the transformation

$$r = r_{\odot}\left(\frac{36}{7k}\right)^{1/2} x$$

can be put into the following typical forms

$$\frac{1}{x}(P_0 - P_2) + \frac{1}{x^3}(P_2 - P_4) = C', \tag{7a}$$

$$\frac{1}{x}(P_0 - P_2) - \frac{1}{x^3}(P_2 - P_4) = C'', \tag{7b}$$

where C' and C'' are new parameters.

This transformation corresponds to a linear enlargement reducing the parameters C and h_2/h_1 of the equation (6) to C' and C'', respectively, for the two independent family of (7a) and (7b). All trajectories represented by the variations of the parameters of the superposed dipole and octupole fields correspond to different enlargement of (7a) and (7b), respectively. The Figures 1 and 2 show the diagrams in the cases when h_1 and h_3 have the same and opposite signs, respectively. In order to determine the ratio h_3/h_1 one has to estimate the best coincidence between the trajectories and the polar rays. This can be carried out practically in such a way that we project the magnified picture of (7a) and (7b), respectively, to the drawing of the corona and read the scale.

A simple treatment shows us that coincidence takes place only at the family (7a) and $h_3/h_1 > 0$. These results seem to be at first moment in contradiction with VAN DE HULST's remark on the structure of the polar rays;

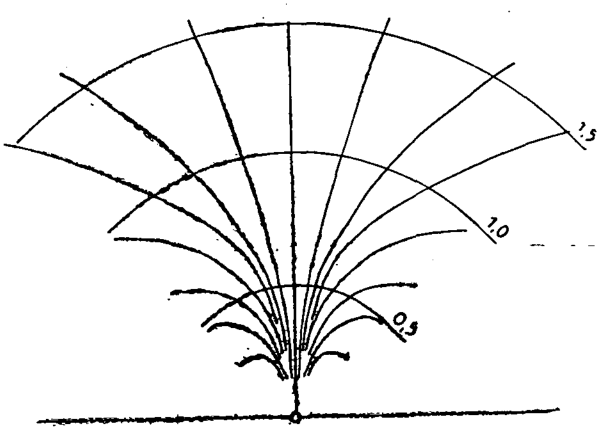

Fig. 1

namely the ratio h_3/h_1, introduced in the previous paragraph corresponding with his data in the years 1900 as well as 1901, is negative. This contradiction proves rather the large systematic error of the method discussed above than the unacceptability of the present one.

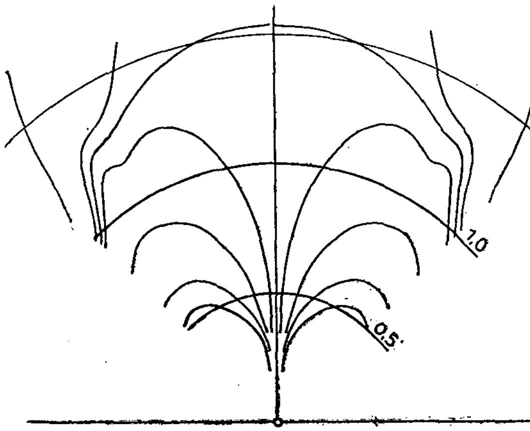

Fig. 2

Naturally the present method has a systematic error too. But, it can be easily estimated and can be proved that this systematic error does not cover the observed effect itself. To determine the systematic error of the present method one has to take into account that the trajectories discussed may coincide only with the rays starting from the apparent solar limb. In projection one observes, however, rays which arise from the solar surface before and behind the limb, respectively. Their projected form can be determined by the transformation $r' = r/\sqrt{1-\sin^2\vartheta\cos^2\varphi}$; $r'\sin\vartheta' = r\sin\vartheta\sin\varphi$; $r'\cos\vartheta' = r\cos\vartheta$.

In course of the determination of the coincidence between the trajectories and the polar rays it is impossible to distinguish the polar rays starting at the limb from the other ones; this means that the scale of enlargement may have an error varying along the polar ray considered. This variation depends on the polar distance ϑ. However, this variation depending on ϑ can be neglected taking only rays in the neighbourhood of the pole into account. As matter stands, it is sufficient to estimate the error of enlargement at the starting point of the polar rays. One can easily see that in the case of a starting point with the polar co-ordinates r and ϑ the magnitude of enlargement is $r/\sqrt{1-\sin^2\vartheta\cos^2\varphi}$ instead of r. Denoting the affected values of h_3/h_1 by $[h_3/h_1]$, the relation between the real and affected values of h_3/h_1 can be given in the form

$$\left[\frac{h_3}{h_1}\right] = \frac{h_3}{h_1}(1-\sin^2\vartheta\cos^2\varphi)$$

and for the deviation

$$\delta = \left[\frac{h_3}{h_1}\right] - \frac{h_3}{h_1} = -\frac{h_3}{h_1}\sin^2\vartheta\cos^2\varphi$$

can be obtained. Owing to the fact that the starting points of the polar rays cover, on the solar surface, an area determined by the polar distance of $\vartheta = 30°$, the mean value of the deviation can be given by

$$\bar{\delta} = -\frac{h_3}{h_1} \frac{\int\limits_0^{30°}\int\limits_0^{\pi} \sin^3\vartheta \cos^2\varphi \, d\vartheta \, d\varphi}{\int\limits_0^{30°}\int\limits_0^{\pi} \sin\vartheta \, d\vartheta \, d\varphi} = -0{,}065 \frac{h_3}{h_1}.$$

In order to eliminate the systematic error $\bar{\delta}$ the observed value of $[h_3/h_1]$ has to be multiplied by a coefficient of $1/(1-0{,}065) = 1{,}070$.

§ 4. *Observational material*

Corona photographs have been collected since the eclipse of 1896 and have been selected merely that of minimum activity, because polar rays are observed only at this phase. All the pictures have been prepared by drawing lines to represent the polar rays and so 18 drawings are possessed including the eclipse of 1954. In some papers schemata of the coronal structure resembling our drawings have been found and these were directly used (see Fig. 3).

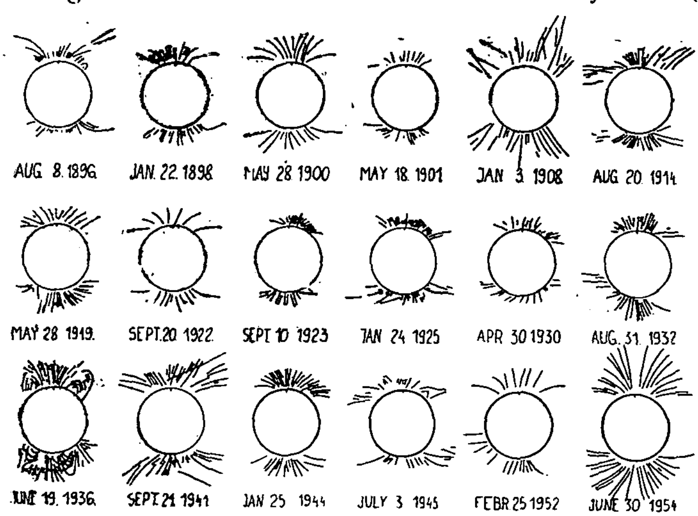

Fig. 3

To eliminate the personal uncertainty of the estimation of the scale three persons have independently determined the enlargement at which the greatest number of polar rays coincide with the trajectories.

These collections are summarized in Table III. In the first column the dates of eclipses are given, in the second the scales of enlargement as well as mean enlargements, finally the third contains references and the type of picture published (photo or drawing).

Table III

Date	Scale of enlargement				Lettery references	
	1	2	3	mean		
1896, Aug. 8.	1,18	0,91	0,99	1,027	St. I. **19**, 61	d.
1898, Jan. 22.	1,10	0,88	1,08	1,020	„ 62	d.
1900, May 28.	1,05	0,94	0,96	0,983	Sm. I. 1904	ph.
1901, May 18.	1,04	0,96	1,23	1,077	Pop. Astr. No. 91,	ph.
1908, Jan. 3.	0,75	0,88	1,10	0,910	St. I. **19**, 75	d.
1914, Aug. 20.	0,98	0,76	0,98	0,907	„ 78	d.
1919, May 28.	1,02	1,06	1,36	1,207	Obs. **17**, 405	ph.
1922, Sept. 20.	1,10	1,10	0,92	1,040	Pop. Astr. **31**, 144	ph.
1923, Sept. 10.	0,96	0,80	0,98	0,910	„ 629	ph.
1925, Jan. 24.	1,00	0,89	1,28	1,057	St. I. **19**, 79	d.
1930, Apr. 28.	1,00	1,19	1,19	1,127	Pop. Astr. **39**, 241	ph.
1932, Aug. 31.	1,10	1,00	0,95	1,017	P. A. S. P. **44**, 341	ph.
1936, June 19.	0,85	0,89	0,65	0,797	St. I. **19**, 32	d.
1941, Sept. 21.	1,20	0,85	1,40	1,150	„ 42	d.
1944, Jan. 25.	1,13	1,12	1,20	1,150	Sky and Tel. **3**, 7	ph.
1945, July 3.	1,24	0,99	1,19	1,137	St. I. **19**, 92	d.
1952, Febr. 25.	1,56	0,86	1,00	1,140	Sky and Tel. **12**, 8.	ph.
1954, June 30.	1,12	0,85	1,15	1,030	As. J. **32**, 4.	b.

Notes: St. I. = Trudi Astronomicheskogo Instituta in P. K. Sternberga Tom. **19** (Bogoslevska: Structura Solnechnoi Koroni) Sm. I. = S. P. Langley, The 1900 Solar Eclipse Expedition of the Astronomical Observatory of the Smithsonian Institution.

§ 5. The structure of the magnetic field

Let us take the mean value of the scales summarized in Table III. We get

$$r_{\odot} = r_{\odot}\left(\frac{36}{7k}\right)^{1/2} 1{,}038, \quad \frac{1}{k} = 0{,}722;$$

and eliminating the systematic error we obtain

$$\frac{h_3}{h_1} = 0{,}361 \times 1{,}070\, r_{\odot}^2 = 0{,}386\, r_{\odot}^2 .$$

Then the vector potential of the external field has the form

$$\mathfrak{A}_{\varphi} = h_1^{i}\left(\frac{P_1^{(1)}}{r^2} + 0{,}386\, r_{\odot}^2 \frac{P_3^{(1)}}{r^4}\right).$$

The magnetic field represented by this vector potential differs from the dipole field near the solar surface. In consequence of the second term the polarity changes in the equatorial areas opposite compared to the polarity at the poles of the same hemisphere. The second term vanishes very quickly with the increasing distance and the dipole becomes dominant of the two terms.

§ 6. *Expected variation of the magnetic field*

The above discussion appears as useful for analyzing a probable variation of the solar magnetic field. Before explaining this problem let us estimate the error of this method.

It was mentioned before that the local magnetic fluctuations are superposed on the solar permanent field. One can generally suppose that these fluctuations affect the orientation of the magnetic axis and the ratio h_3/h_1. Based on the suggested method, however, only the ratio h_3/h_1 can be determined because the estimation of the position of the axis is practically uncertain, namely, coincidence of acceptable accuracy can be estimated also in the case when a small deviation exists.

This random variation in position of the magnetic axis is, however, very small therefore it does not essentially affect the determination h_3/h_1. As a matter of fact the variation of the axis in inclination — even the annual one — can be neglected without running the risk of systematic errors. Owing to the above consideration the actual variation of the magnetic field observed consists of a periodical fluctuation superposing on a steady random fluctuation, where the periodical fluctuation is a stochastic one which is synchronous to the solar magnetic cycle. Let us approximate this periodical fluctuation by the following function:

$$[h_3/h_1] = 0{,}361 + A_0 + A_1 \cos\frac{2\pi}{T}t + A_2 \cos\frac{2\pi}{T}t. \quad (8)$$

The coefficients A_0, A_1, A_2 can be determined by the method of least squares but the existence of the magnetic period T has to be verified. Unfortunately

Table IV

T	A_0	A_1	A_2	δ
22,0	0,00983	−0,0373	−0,0410	0,1802
22,2	0,0110	−0,0415	−0,0398	0,1793
22,4	0,0121	−0,0455	−0,0362	0,1785
22,6	0,0132	−0,0492	−0,0328	0,1778
22,8	0,0171	−0,0562	−0,0384	0,1724
23,0	0,0152	−0,552	−0,0245	0,1766
∞	0,00	—	—	0,2043

the method of the least squares does not give simple schemata for calculation of this kind. Therefore, we have calculated the standard-deviation for several periods, choosing the most probable one. The coefficients as well as the standard-deviations of $[h_3/h_1]$ (δ) are summarized in Table IV.

It can be immediately noted that the most probable period is between 22,8 and 23,0. For the period as well as for the coefficients the values

$$T = 22{,}85 \quad A_0 = 0{,}0188 \quad A_1 = -0{,}0595 \quad A_2 = -0{,}0585$$

can be graphically obtained (see Fig. 4.) and finally taking of the correction of the systematic error into account we have

$$h_3/h_1 = r_\odot^2 \left[0{,}407 - 0{,}0742 \cos \left(\frac{2\pi}{22{,}85} t - 2{,}22 \right) \right].$$

The period 22,85 seems to be in relatively good agreement with the double of the solar cycle derived from the spot acivity. Based on the agreement between periodicities it would be difficult to draw any further conclusions and to establish connections between the two phenomena. Difficulties may be summarized as follows:

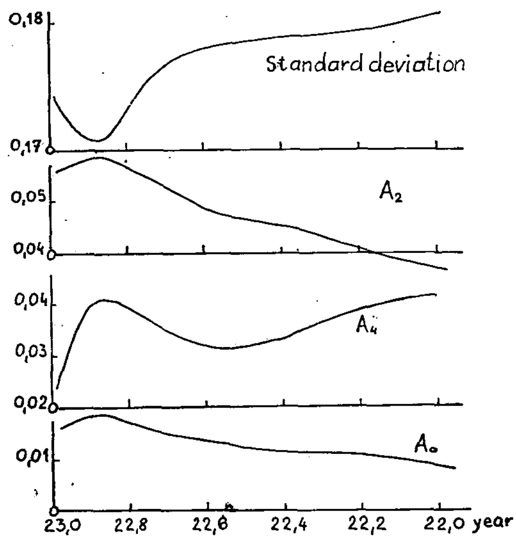

Fig. 4

Owing to the mean values of the period of spot activity, for the years 1900—1954, the minimum activities are expected on the years $1902{,}46 + 10{,}43\,n$ ($n = 0, 1, 2, \ldots$). One may note that the minimum spot activities coincide with the maximal elongation of h_3/h_1 in the years 1902 and 1913. It cannot, however, be drawn any conclusion, from this coincidence because in consequence of deviation between these periods in 1954 the maximum spot activity coincided with the maximal elongation. It would be difficult to explain the reason for this large displacement of phases, namely, the standard-deviation and the amplitude of h_3/h_1 have the same order of magnitude. As matters stand a further analysis of this problem — based on the present observational materials — should be meaningless. Nevertheless, we may conclude — taking into account our consideration at the beginning of this paragraph — that the existence of the period of the maximal elongation of h_3/h_1 is verified.

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References

[1] *Hulst, H. C. Van de:* B. A. N. **11**, 150 (1950).