A POSSIBLE GEOMETRICAL INTERPRETATION OF THE ISOSPACE AND OF ITS TRANSFORMATIONS*

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Due to experiments proving the violation of parity conservation the assumption will be advanced that the structure of the space-time world determined by physical interactions may be anisotropic. The natural geometrical model of anisotropic spaces is the line-element geometry being an ensemble of line-elements (x^μ, v^μ) characterized by their position co-ordinates x^μ and homogeneous direction co-ordinates v^μ $(\mu = 0, 1, 2, 3)$, respectively. The latter ones can be substituted by inhomogeneous direction co-ordinates x'^μ or rather by the quantities $\xi_{(i)} = \cos\theta_i$ $(i = 1, 2, 3)$ related to an orthogonal trieder (λ-trieder). The co-ordinates x^μ can be interpreted as the co-ordinates of external and the direction co-ordinates θ_i (or $\xi_{(i)}$) as the co-ordinates of the internal degrees of freedom of the physical field considered. This means that the supposition that the physical field would be excited in anisotropic spaces can be substituted by the more physical term that in the case of field theories also the internal degrees of freedom of the fields have to be considered. Due to this supposition the components of isovectors and isopseudovectors, *etc.*, can easily be defined and the transformations of the isotopic space can be represented by rotation and reflexions of the λ-trieder. Finally, a method will be proposed for the determination of the metrical fundamental tensor of the anisotropic space-time world.

§ 1. *Introduction*

The development of quantum theory of fields achieved in the last few years several important and interesting results. In the topic of the theory of elementary particles, *e. g.*, the qualitative features — above all the selection rules — of the processes of interactions of elementary particles have been carefully investigated and in spite of the lack of knowledge of the exact form of interaction law the comparison of the results of such investigations with the experiments could be used as a first qualitative test of the systematizations and theories of these particles [1].

In spite of these great successes one can, however, object to two — otherwise mostly accepted — elements of the theory of elementary particles:

(*i*) Taking the different methods of the theory of elementary particles into account, it can be noticed that these methods have, from certain points of view, two essentially different features. Some of them are closely connected with geometry and obtain such physical laws as conservation of energy and momentum,

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etc.; the others, nevertheless, are rather based on the abstract concept of the isobaric space (in the following: *isospace*) than on current geometrical terms and results in such physical laws as the conservation of charge or of the baryon number, *etc.*, which have not acquired yet any generally accepted geometrical interpretation. In other words, *some of the groups of transformations* — like translations, rotations, and inversions in the four-dimensional space-time world — *posses an immediate geometrical meaning but some of the others* — such as *e. g.* gauge transformations, charge conjugation, charge symmetry and mesoparity transformation — *possess none.* This situation is from the point of view of the unified theory of fields unsatisfactory [2].

(*ii*) A generally accepted starting point of the theory is the *a priori* supposition that the structure of the space-time continuum — as a geometrical background of physical processes — would be homogeneous and isotropic.

We would like to suggest that this second supposition, in spite of its general acceptability, can be objected from philosophical point of view. Owing to the metaphysical concept of space-time world the structure of space and time, respectively, would be an *a priori* cathegory of human mind. It is, however, well known that this view was refused in the last century by the famous geometrical investigations of BOLYAI and LOBATSHEWSKI. At the beginning of the great development of natural sciences J. BOLYAI was the first scientist who already hundred years ago had suggested that the structure of space is determined by objective physical interactions of the matter. This idea was recalled by RIEMANN and finally as the fundamental idea of EINSTEIN's theory of gravitation has been scored its revolutional success.

As a matter of fact the gravitational interaction can be neglected in the case of elementary particles. But if we take seriously into account the above point of view then one can say that the homogenity and isotropy of the space-time world is rather a consequence of symmetry properties of the strong and electromagnetic interactions, respectively, than an *a priori* property of the space-time world. This means, however, that if the violation of parity conservation respectively as a special property of the weak and universal weak interactions can be regarded, due to the view that the structure of the space would be determined by material interactions, it seems that from the anisotropy of the weak interactions, several times proved by careful experiments, the anisotropy of the space can be concluded. The reason that the structure of the space-time world is isotropic in the case of strong and electromagnetic interactions it seems to be that the anisotropic weak interactions are overlapped by these stronger interactions. In other words: the question may arise whether the insistence on the *a priori* pseudo-Euclidian structure of the space-time world is not a rest of the metaphysical concept of space and time?

This is the reason that in this paper the radical idea will be suggested that the second fundamental element of the quantum theory of field mentioned above is responsible for the problems connected with the violation of parity conservation predicted by LEE and YANG [3].

The violation of parity conservation was recently investigated from the point of view of relativistic invariance by WIGNER [4]. As it is well known, the consequence of the cobalt experiment of WU and her collaborators, as well as others of the same type, may be briefly summarized that the symmetry of the real world is smaller than it had been thought. Namely, the whole experimental arrangement, *e. g.*, in the case of Co^{60} has at the beginning of the experiment a symmetry plane, which would remain valid throughout the further fate of the system. Nevertheless, the intensity of the β-radiation is larger on one side of this plane than on the other one. Should it, however, be true that a symmetry plane always remains a symmetry plane, the initial state of the system in the case of cobalt experiment could not have contained a symmetry plane. Therefore, the radical solution of this paradoxial situation was suggested by WIGNER that the polar and axial transfomation character of the electric and magnetic vector of the electromagnetic field has to be changed which has the immediate consequence that the charge density would become a pseudo-scalar rather than a scalar as in the current theory. By this means the problem concerning the cobalt experiment would be solved, as well as the *CP* invariance predicted by LANDAU [5] would also be explained, however, this proposal should entail further consequences that are at present difficult to forsee and it is inconceivable — at least for us — that such radical change in the case of the theory of the electromagnetic field had not been suggested by any previous experimental effect.

Taking the above considerations into account the conclusion may be drawn again that some physical factors related to the space-time structure have not yet been considered and due to the lack of the expected symmetry plane of the cobalt experiment, *the structure of our physical world seems to be richer than it was previously believed.*

It is remarkable that both objections mentioned above may be eliminated by the radical interpretation of the cobalt experiment that

(1) *as a consequence of weak interactions the structure of the space-time continuum becomes homogeneous but anisotropic.*

(2) *isotropy and homogenity of the space-time world, which has been previously accepted* a priori *without any criticism in current field theories, should be only an approximate one and by the decrease of the strength of interactions the anisotropy and inhomogenity of the space-time world become more and more dominant.*

The natural geometrical model of an anisotropic space is the line-element geometry being the geometry of an ensemble of line-elements rather than that of points in the space as in current geometries. Such a geometry was first proposed by FINSLER [6], then, some years ago, FINSLER's exposition was generalized to fit it more adequate to physical applications [7]. In the course of more recent investigations also the differential structure of classical theory of fields excited in an anisotropic space-time continuum was elaborated; it means that the field equations deduced from a general and also from a special Lagrangian density, respectively, as well as the deduction of the differential conservation laws by means of PAULI's method of infinitesimal transformations were discussed [8]. The latter investigations have been suggested by the recognition that YUKAWA's bilocal theory of fields corresponds to a theory of fields excited in such an anisotropic world.

§ 2. Geometrical Characterization of the Space-time Anisotropy

The fundamental elements of a line-element space — corresponding to an anisotropic space-time continuum — are *line-elements* defined by their four-dimensional space co-ordinates x^μ and by contravariant vectors v^μ ($\mu = 0, 1, 2, 3$) the direction of which correspond to that of the line-elements. The ensemble of the fundamental elements (x^μ, v^μ) is the so-called *line-element space* $\mathfrak{L}$.

Since only a direction is defined by the vector v^μ, the components v^μ are not independent and their proportion has only meaning. Therefore, *it will be supposed that the metrical fundamental tensor of the space,* i. e. $g_{\mu\nu} = g_{\mu\nu}(x^\mu, v^\mu)$, *the different geometrical and physical quantities, respectively, are homogeneous functions of the so-called direction co-ordinates* v^μ *of zero degree.*

The geometrical structure of such a space may be determined by the *metrical fundamental tensor* $g_{\mu\nu}$. Owing to the general idea of the suggested theory the explicit form of the metrical fundamental tensor has to be determined by the anisotropy of the physical interaction. Since as a result of the interactions of elementary particles the space-time world becomes homogeneous the metrical fundamental tensor $g_{\mu\nu}$ does not depend on the co-ordinates x^μ, but due to the anisotropy of interactions it can depend on the direction co-ordinates v^μ, this means that $g_{\mu\nu} = g_{\mu\nu}(v^\mu)$. The geometry of such a space is elaborated in details in recent papers [7, 8].

It seems, however, that the geometry of the space has to be slightly changed. Let us, namely, suppose, that the direction of the line-elements is characterized rather by the contravariant vector-density of order (-1), u^μ:

$$u^{\mu'} = \Delta^{-1} \frac{\partial x^{\mu'}}{\partial x^\mu} u^\mu, \qquad \left(\Delta \equiv \det \left| \frac{\partial x^{\mu'}}{\partial x^\mu} \right| \right) \tag{1}$$

than by the contravariant vector v^μ with the transformation law

$$v^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^\mu} v^\mu. \tag{2}$$

The consequences of this new supposition are not interesting from the point of view of the present investigations, therefore it will be dealt with elsewhere.

Let us only mention that a vector-density of unit length in the direction of the line-element (x^μ, u^μ) can be introduced by the definition

$$l^\mu = F^{-1} u^\mu, \quad (F = \{g_{\mu\nu}(u^\mu) u^\mu u^\nu\}^{1/2}) \tag{3}$$

F being the so-called *scalar fundamental function of the space*, and the structure of the space at a point $P(x^\mu)$ is characterized by the *Carathéodoryan indicatrix of the space* which is contructed as follows: Consider all directions crossing over the point P. The end points of the unit vectors l^μ directed towards the different directions starting from P form a hyper-surface in the four-dimensional space $\mathfrak{L}$ having the equation

$$F(x^\mu, l^\mu) = 1. \tag{4}$$

Since the space-time world is in our case homogeneous the Carathéodoryan indicatrices at different points of the space are congruent.

In order to take in a covariant way explicitly into account that the geometrical and field quantities, respectively, depend only on three independent direction co-ordinates, let us introduce three linearly independent vectors $\underset{(i)}{\lambda^\mu}$ $(i = 1, 2, 3)$ of unit length determining three different, otherwise arbitrary, directions. Then the three angles

$$\theta_i = \text{arc}\cos\{g_{\mu\nu} l^\mu \underset{(i)}{\lambda^\nu}\} \quad (g_{\alpha\beta} \underset{(i)}{\lambda^\alpha} \underset{(i)}{\lambda^\beta} = 1, \text{ no summation over } i) \tag{5}$$

can be obtained regarded as *inhomogeneous directional co-ordinates in all points of the space*. The angles θ_i were introduced in an invariant way. This means that the angles θ_i are pseudo-scalars of the general group of co-ordinate transformations.

The directions of the vectors $\underset{(i)}{\lambda^\mu}$ can be given quite arbitrarily. For sake of simplicity one can, however, suppose that the $\underset{(i)}{\lambda^\mu}$ are by pairs orthogonal, *i. e.*

$$g_{\mu\nu} \underset{(i)}{\lambda^\mu} \underset{(j)}{\lambda^\nu} = \delta_{ij} \tag{6}$$

forming an orthogonal trieder. In the case when the orientation of this trieder coincides with that of the co-ordinate axes it will be denoted as λ^+*-trieder* and in the contrary one as λ^-*-trieder.*

Due to the definition of the inhomogeneous direction co-ordinates θ_i one can see immediately that these angles depend on the orientation of the λ-trieder. Let us suppose that the orientation of the λ^+-trieder would be *a priori* defined in a suitable way. For sake of simplicity — in this paragraph and also in the following ones — one can briefly denote the λ^+-trieder as λ-trieder.

For any other orientations of the λ-trieder — which will be noted by λ_i'-trieder — their axes $\underset{(i)}{\lambda'^\mu}$ relative to its *a priori* position can be determined by the Eulerian angles ϕ, ψ, ϑ. Then, for the relations between the quantities

$$\xi_{(i)} = \cos\theta_i = g_{\mu\nu} l^\mu \underset{(i)}{\lambda^\nu} \text{ and } \xi'_{(i)} = \cos\theta'_i = g_{\mu\nu} l^\mu \underset{(i)}{\lambda'^\nu} \tag{7}$$

the well-known equations

$$\begin{aligned}
\xi'_{(1)} &= \{\cos\phi\cos\psi - \sin\phi\sin\psi\cos\vartheta\}\xi_{(1)} + \\
&+ \{\cos\phi\sin\psi + \sin\phi\cos\psi\cos\vartheta\}\cdot\xi_{(2)} + \sin\phi\sin\vartheta\,\xi_{(3)} \\
\xi'_{(2)} &= -\{\sin\phi\cos\psi + \cos\phi\sin\psi\cos\vartheta\}\xi_{(1)} + \\
&+ \{-\sin\phi\sin\psi + \cos\phi\cos\psi\cos\vartheta\}\cdot\xi_{(2)} + \cos\phi\sin\vartheta\,\xi_{(3)} \\
\xi_{(3)} &= \sin\psi\sin\vartheta\;\xi_{(1)} - \cos\phi\sin\vartheta\;\xi_{(2)} + \cos\vartheta\;\xi_{(3)}
\end{aligned} \tag{8}$$

can be obtained.

§ 3. *General Feature of Theories of Fields Excited in Anisotropic Spaces*

In current field theories the physical fields are characterized by one or several space-time functions: $\psi(x^\mu)$, $\psi_\alpha = \psi_\alpha(x^\mu)$ *etc.*, — fulfilling certain partial differential equations, the so-called field equations — which have to satisfy definite laws of transformations. In the case of physical fields excited in anisotropic spaces, the fields are analogously characterized by such quantities fulfilling the field equations [8] and having also definite laws of transformations, however, these functions depend on the line-elements (x^μ, u^μ), *i. e.* $\psi = \psi(x^\mu, u^\mu)$, $\psi_\alpha = \psi_\alpha(x^\mu, u^\mu)$ *etc.*, being — due to the above considerations — homogeneous functions of the direction co-ordinates u^μ of zero degree.

Instead of the homogeneous direction co-ordinates u^μ let us introduce the inhomogeneous direction co-ordinates θ_i or — for the sake of appropriateness — rather the quantities $\xi_{(i)} = \cos\theta_i$, then the field quantity depends on the position co-ordinates x^μ, as well as on the quantities $\xi_{(i)}$, *i. e.*

$$\psi = \psi(x^\mu; \xi_{(1)}, \xi_{(2)}, \xi_{(3)}). \tag{9}$$

To use simple terms one can denote the co-ordinates x^μ as the *co-ordinates of the external degrees of freedom of the fields* and analogously the homogeneous direction co-ordinates u^μ and the quantities $\xi_{(i)}$, respectively, as the *co-ordinates of the internal degrees of freedom of the field.* This means last of all that the supposition introduced above — according to which the structure of the physical field would be anisotropic — in other, perhaps more physical, terms corresponds to the fundamental assumption that *the physical fields have also internal degrees of freedom characterized by the internal co-ordinates.*

Now, let us take into account that the variables x^μ and $\xi_{(i)}$ of the field quantity ψ, *i. e.* the external and internal co-ordinates of the field, respectively, have independent laws of transformation:

(*i*) The external co-ordinates x^μ are transformed in the case of all co-ordinate transformations in the usual way, however, the internal co-ordinates ξ_i are either invariants or pseudo-scalars of the general group of co-ordinate transformations. Let us denote the general group of co-ordinate transformations in the following by $\mathfrak{G}_x$.

(*ii*) On the other hand, the internal co-ordinates $\xi_{(i)}$ change in the case of rotations of the λ-trieder, as well as in the case of inversions in respect to the λ-trieder, respectively, but this group of transformations — being isomorph with the three-dimensional rotary reflexion group of transformations — does not involve any changes in the case of external co-ordinates x^μ. The general group of transformations of the internal co-ordinates will be denoted in the following by $\mathfrak{G}_\xi$.

As a matter of fact, in the case of fields excited in anisotropic space-time continuums — or in other words in the case of physical fields with internal degrees of freedom — the general group of transformations is a product of those of external and of internal co-ordinates, *i. e.*

$$\mathfrak{G} = \mathfrak{G}_x \mathfrak{G}_\xi, \tag{10}$$

must be considered. *Both groups $\mathfrak{G}_x$ and $\mathfrak{G}_\xi$ evidently have representations with immediate geometrical meaning and even this recognition would be the main point of the geometrization of the isospace and its transformations* suggested in the next paragraph.

If these considerations are true at all and would be accepted, one has to conclude that *the integral of action of the field* — being the fundamental quantity of the Lagrangian formalism of the field — *must be the invariant of the general group $\mathfrak{G}$ introduced above.* This means that due to the suggested theory more detailed feature of the general law of interaction can be concluded as will be published elsewhere.

§ 4. Geometrical Interpretation of the Isospace and its Transformations

In order to carry out our programme of geometrization of the isospace quite generally, the structure of the anisotropic spacetime world will not yet be fixed, but the following general assumptions would be suggested:

(*a*) *Due to the anisotropy of the space-time continuum some (at least one but at most four) distinguished direction should be determined being also distinguished directions of the Carathéodoryan indicatrix of the space at a point* $P(x^\mu)$.

Then, in § 7 an explicit method will be proposed to determine the metrical fundamental tensor if the Carathéodoryan indicatrix is known.

(*b*) *Let us take into account a special orthogonal frame of reference assuming that the directions of its co-ordinate axes coincide with the distinguished directions mentioned in* (*a*). One can easily see that in this frame of reference the Carathéodoryan indicatrix has its most simple explicit form in so far as that only the components $g_{\mu\mu}$ of the metrical fundamental tensor do not vanish (*i. e.* $g_{\mu\mu} \neq 0$, $g_{\mu\nu} = 0$, if $\mu \neq \nu$).

(*c*) Let us furthermore suppose too that *the vectors* $\underset{(i)}{\lambda}^{\mu}$ *are parallel with the space-axes of the special frame of reference introduced above.* This means on the one hand that in this special frame of reference each vector $\underset{(i)}{\lambda}^{\mu}$ has only space-like components, *i. e.* $\{\underset{(i)}{\lambda}^{\mu}\} = \{0, \underset{(i)}{\lambda}^{1}, \underset{(i)}{\lambda}^{2}, \underset{(i)}{\lambda}^{3}\} = \{0, \underset{(i)}{\boldsymbol{\lambda}}\}$, and on the other hand the orientation of the λ^+-trieder would *a priori* be fixed in contemporary points relative to the special frame of reference distinguished by the anisotropy of the space-time world.

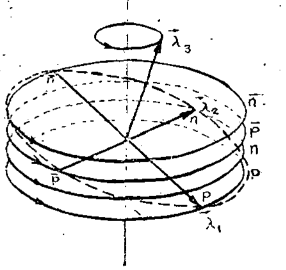

Fig. 1

(*d*) Finally, let us suppose that the scalar product of the spin vector and the unit vector directed in the direction of the line-elements is an adequate constant. The reason for introducing the characterization of the anisotropy of the space-time world instead of the contravariant direction coordinates v^μ, the pseudovector u^μ is that this product is — owing to the transformation character of the spin vector — a scalar.

Sometimes the spin of the field (or in other words: the elementary particles) is defined as an antisymmetrical tensor of second order. In this case it is more convenient to characterize the anisotropy of the space-time world by a surface-element $a^{\mu\nu}$ being an antisymmetric tensor of second order too, the normal vector of which is just a vector-density of order (-1). Such a space is then an ensemble of $(x^\mu, a^{\mu\nu})$ and usually called as Cartanian space. We will not use this more difficult geometrical method, since the more elementary treatment seems to be adequate for the geometrization of the isospace and its transformations.

The last supposition means, however, that

(*i*) in the case of fields (or elementary particles being the quanta of the fields regarded) with zero spin the condition introduced in (*d*) is meaningless, which means that no correlation between the internal and external degrees of freedom of the fields exists and the isospace is three-dimensional.

(*ii*) in the case of fields with non-zero spin the condition introduced in (*d*) reduces the internal degrees of freedom by one and therefore the isospace is quasi-two-dimensional. It is suitable to suppose in this case that the $\underset{(3)}{\lambda^{\mu}}$ axis of the λ^{+}-trieder makes a precession around the spin axis with angle $\theta_3 = const.$ and the two degrees of freedom are characterized by two internal co-ordinates $\xi_{(1)}$ and $\xi_{(2)}$.

§ 5. *The Isotriplets of the Pions*

To demonstrate the method of geometrization of the isospace and its transformations, let the pions be regarded as being particles with zero spin.

Let $P_x \equiv P$ be the transformation corresponding to the inversion in respect to the origin then follows from (*c*) that $P\lambda^{+} \to \lambda^{-}$ and *vice versa.*

Furthermore, the fundamental supposition would be proposed that

$$\psi_{\mathrm{I}} \equiv \psi(x^{\mu}; 1, 0, 0\}, \ \psi_{\mathrm{II}} \equiv \psi(x^{\mu}; 0, 1, 0), \ \psi_{\mathrm{III}} \equiv \psi(x^{\mu}; 0, 0, 1) \tag{11}$$

— being the values of the field component $\psi(x^{\mu}; \xi_{(1)}, \xi_{(2)}, \xi_{(3)})$ in the directions of the axes of the λ^{+}-trieder — *represent the components of the isovector or that of the isopseudovector* $\boldsymbol{\psi}$ *of the field according to that the field is scalar or pseudoscalar.*

One can immediately see that in each direction — determined by the quantities $\xi_{(1)}, \xi_{(2)}, \xi_{(3)}$ — the field quantity $\psi = \psi(x^{\mu}; \xi_{(1)}, \xi_{(2)}\xi_{(3)})$ can easily be determined as a linear combination of ψ_A-s (A = I, II, III).

Namely, due to (8) for the components of the isovector or isopseudovector $\psi'(x^{\mu}; \xi'_{(1)}, \xi'_{(2)}, \xi'_{(3)}) = \{\psi'_{\mathrm{I}}(x^{\mu}; 1, 0, 0), \psi'_{\mathrm{II}}(x^{\mu}; 0, 1, 0), \psi'_{\mathrm{III}}(x^{\mu}; 0, 0, 1)\}$ related to the axes of a λ'^{+}-trieder having other orientation than the original one — determined by the Eulerian angles ϕ, ψ, ϑ — the formula of transformation

$$\boldsymbol{\psi}' = \boldsymbol{\psi}\mathbf{M} \tag{12}$$

can be obtained, where $\mathbf{M}$ means the matrix of the transformation which was explicitly given by equation (8). This means, *e. g.*, that

$$\psi'_{\mathrm{I}}(x^{\mu}) = \{\cos\phi\cos\psi - \sin\phi\sin\psi\cos\vartheta\}\,\psi_{\mathrm{I}}(x^{\mu}) + \tag{13}$$
$$+ \{\cos\phi\sin\psi + \sin\phi\cos\psi\cos\vartheta\}\,\psi_{\mathrm{II}}(x^{\mu}) + \sin\phi\sin\vartheta\psi_{\mathrm{III}}(x^{\mu}).$$

But, by a suitable rotation the λ^{+}-trieder its first axis can be put into the direction determined by the internal co-ordinates $\{\xi_{(1)}, \xi_{(2)}, \xi_{(3)}\}$ so far that $\psi(x^{\mu}; \xi_{(1)}, \xi_{(2)}, \xi_{(3)}) = \psi'(x^{\mu}; 1, 0, 0)$ and in this way the explicit dependence of $\psi(x^{\mu}; \xi_{(1)}, \xi_{(2)}, \xi_{(3)})$ on the components ψ_A is obtained by (12).

(*i*) Let

$$\Psi_{\lambda^{+}} \equiv \frac{1}{\sqrt{2}}\{\psi_{\mathrm{I}} + i\psi_{\mathrm{II}}\}, \ \Psi^{*}_{\lambda^{+}} \equiv \frac{1}{\sqrt{2}}\{\psi_{\mathrm{I}} - i\psi_{\mathrm{II}}\}, \ \Psi^{0}_{\lambda^{+}} \equiv \psi_{\mathrm{III}} \tag{14}$$

be the usual components of the combined, charged and neutral field, as well as let us also suppose that the plane determined by the vectors $\underset{(1)}{\boldsymbol{\lambda}}$ and $\underset{(2)}{\boldsymbol{\lambda}}$ of the $\boldsymbol{\lambda}^{+}$-trieder coincides with a complex plane — being $\underset{(1)}{\boldsymbol{\lambda}}$ and $\underset{(2)}{\boldsymbol{\lambda}}$, respectively, the unit vectors parallel to the real and imaginary axes — then it can be proposed that in the case of a rotation by an angle α on this plane around the $\underset{(3)}{\boldsymbol{\lambda}}$-axis the transformation formulae of the components of the charged and that of the neutral field are

$$\Psi'_{\lambda^+} = \Psi_{\lambda^+} e^{-i\alpha}, \quad \Psi'^*_{\lambda^+} = \Psi^* e^{\alpha i}, \quad \Psi'^0_{\lambda^+} = \Psi^0_{\lambda^+}. \tag{15}$$

This means that *the gauge transformation of first kind can be geometrically interpreted as a rotation of the λ-trieder around one of its axes* (in our case around the $\underset{(3)}{\lambda}$-axis).

(*ii*) Furthermore, if

$$\overset{(\pm)}{\psi}_{\mathrm{I}} = \frac{1}{\sqrt{2}}\{\Psi^*_{\lambda^\pm} + \Psi^*_{\lambda^\pm}\}, \quad \overset{(\pm)}{\psi}_{\mathrm{II}} = \frac{1}{\sqrt{2}i}\{\Psi_{\lambda^\pm} - \Psi^*_{\lambda^\pm}\}, \quad \overset{(\pm)}{\psi}_{\mathrm{III}} = \Psi^0_{\lambda^\pm} \tag{16}$$

denote the components of the isovector or that of the isopseudovector of the field related to the λ^+-and λ^--trieder, respectively, then one can prove that by the transformation C which changes, *e. g.* the orientation of the $\underset{(2)}{\lambda}$-axis of the λ^+-trieder, without changing the co-ordinates x^μ, in the case of scalar fields the transformation formulae

$$C\overset{(\pm)}{\psi}_A = \overset{(\pm)}{\psi}_A \ (A = \mathrm{I, II, III}) \text{ or } C\Psi_{\lambda^\pm} = \Psi_{\lambda^\pm}, \ C\Psi^*_{\lambda^\pm} = \Psi^*_{\lambda^\pm}, \ C\Psi^0_{\lambda^\pm} = \Psi^0_{\lambda^\pm} \tag{17}$$

and in the case of pseudoscalar field the transformation formulae

$$C\overset{(\pm)}{\psi}_{\mathrm{I}} = \overset{(\pm)}{\psi}_{\mathrm{I}}, \ C\overset{(\pm)}{\psi}_{\mathrm{II}} = -\overset{(\pm)}{\psi}_{\mathrm{II}}, \ C\overset{(\pm)}{\psi}_{\mathrm{III}} = \overset{(\pm)}{\psi}_{\mathrm{III}} \text{ or } C\Psi_{\lambda^\pm} = \Psi^*_{\lambda^\pm}, \ C\Psi^*_{\lambda^\pm} = \Psi_{\lambda^\pm}, \ C\Psi^0_{\lambda^\pm} = \Psi^0_{\lambda^\pm}, \tag{18}$$

respectively, can be obtained. *This means*, however, *for pseudoscalar field that the changing of the orientation of the λ-trieder represents the charge conjugation.*

Taking the equations (16) into account one can see that *the charge conjugation cannot be explained in the case of scalar boson field in this way*. Therefore, the suggested theory can only be accepted if pions are pseudo-scalar which seems to be in accordance to other theories [1, 2].

(*iii*) Taking the definitions of the transformations P and C, respectively, into account we have

$$CP\Psi_{\lambda^\pm} = C\Psi_{\lambda^\mp} = \Psi_{\lambda^\pm} \tag{19}$$

which is just the *geometrical interpretation of* LANDAU'*s theory* [5].

(*iv*) Finally, let I_{II} be an inversion with respect to the origo in the plane determined by the axes $\underset{(1)}{\lambda}$ and $\underset{(3)}{\lambda}$ of the λ^+-trieder then the *mesoparity transformation* $P_g = P' = CI_{\mathrm{II}}$ can be interpreted as the geometrical transformation which is composed of an inversion with respect to the origo in one of the planes determined by the axes of the λ^+-trieder (in our case in the $(\underset{(1)}{\lambda}, \underset{(3)}{\lambda})$ plane) and the changing of the orientation of the axes of the λ^+-trieder being orthogonal to the plane of inversion. It can be namely seen that P'_i in the case of pseudo-scalar fields changes the sign of each component of its isopseudovectors:

$$P'\psi_A = -\psi_A. \tag{20}$$

One can immediately see that due to the definition of the transformations P'_i and P, respectively, analogously to LANDAU's theorem the relation

$$P'P\Psi_{\lambda^\pm} = P'\Psi_{\lambda^\mp} = \Psi_{\lambda^\pm} \tag{21}$$

can be obtained being *a new law of invariance* which has not been observed previously.

In course of the above considerations the $\underset{(3)}{\lambda}$ axis of the λ^+-trieder has been distinguished by the supposition that the field component of the neutral field

$$\Psi^0_{\lambda^\pm} \equiv \psi_{\rm III} \equiv \psi(x^\mu; 0, 0, 1) \tag{22}$$

means the value of the field quantity ψ in its direction. In the case of scalar of pseudoscalar fields this distinction has not any further consequence.

§ 6. *The Isodublets of Spin One-half Particles*

Owing to our supposition (*d*) in § 4 the isospace is in this case quasi-two-dimensional, *i. e.*, the internal degrees of freedom of the fields are reduced to two. This means, however, that the field components ψ can be written in the form $\psi = \psi(x^\mu; \xi_{(1)}, \xi_{(2)})$. For sake of simplicity let only the fields of nucleons $\{p, n\}$ as well as that of the leptons, respectively $\{e, \bar{\nu}\}$ and $\{\mu, \bar{\lambda}\}$, be taken into account supposing that ψ fulfils either the second order equations of FEYNMAN [9] or rather that of MARX [10]. The isospace of photons and xions can be analogously treated without any difficulty.

It is supposed that ψ is a two-component spinor the components of which belong to the two spinstates of the mentioned particles. To distinguish the operators of *charge symmetry* and *mesoparity* let us suppose that $\psi(x^\mu; \xi_{(1)}, \xi_{(2)})$ corresponds to the different particle states as follows:

Components of isospinors	Nucleons	Leptons	
$\psi_{11}(x^\mu) \equiv \psi(x^\mu; 1, 0)$	p-state	e^--state	μ^--state
$\psi_{12}(x^\mu) \equiv \psi(x^\mu; -1, 0)$	$\bar{n}$-state	ν-state	λ-state
$\psi_{21}(x^\mu) \equiv \psi(x^\mu; 0, 1)$	n-state	$\bar{\nu}$-state	$\bar{\lambda}$-state
$\psi_{22}(x^\mu) \equiv \psi(x^\mu; 0, -1)$	$\bar{p}$-state	e^+-state	μ^+-state

Due to the two-dimensional character of the isospace the geometrical interpretation of the different isotransformations can be easily illustrated graphically (Fig. 2):

(*i*) The transformation of *charge symmetry* (Z) means a reflexion with respect to a plain being orthogonal to the two-dimensional isospace (*i. e.*, to the plain containing the $\underset{(1)}{\lambda}$ and $\underset{(2)}{\lambda}$ axes of the λ-trieder) which bisects the angle between the $\underset{(1)}{\lambda}$ and $\underset{(2)}{\lambda}$ axes. This definition of the charge symmetry transformation corresponds its usual definition according to which the proton (antiproton) is transformed into neutron (antineutron).

(*ii*) *Charge conjugation* (C) means a reflexion with respect to a plain being also orthogonal to the two-dimensional isospace which bisects the angle between the axes $\underset{(1)}{\lambda}$ and $-\underset{(2)}{\lambda}$ of the λ-trieder. This definition of charge conjugation corresponds to its usual definition according to which the proton (neutron) is transformed into antiproton (antineutron).

(*iii*) *Mesoparity* or *isoparity* transformation (P') exchanges the orientation of the λ^+-trieder; i. e., the λ^+-trieder is transformed into the λ^--trieder. It can be immediately proved that the mesoparity transformation can be composed by the transformations of parity, charge conjugation and charge symmetry operations: $P' = PCZ$ (Fig. 3).

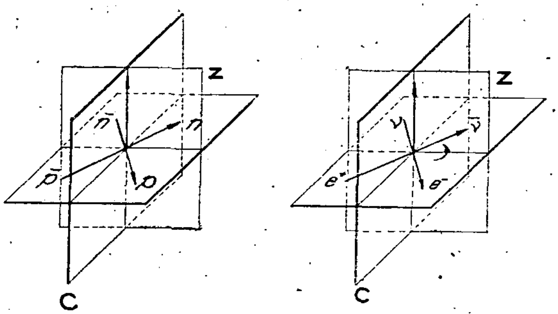

Fig. 2

Remarks. (1) The graphical description of isospace of this paragraph does not differ essentially from that of § 5. Namely, due to FEYNMAN's theory the charge symmetry operator transforms $\pi^{\pm 0}$ into $\pi^{\pm 0}$. This means

$$\psi(x^\mu;\ 1, 0, 0) = \psi(x^\mu;\ -1, 0, 0) \equiv \psi_{\mathrm{I}},$$
$$\psi(x^\mu;\ 0, 1, 0) = \psi(x^\mu;\ 0, -1, 0) \equiv \psi_{\mathrm{II}},$$
$$\psi(x^\mu;\ 0, 0, 1) = \psi(x^\mu;\ 0, 0, -1) \equiv \psi_{\mathrm{III}}.$$

But, it can be proved that in this case the charge symmetry operator is equivalent with the operator of charge conjugation (Fig. 4). As a matter of fact this situation caused previously some misunderstandings.

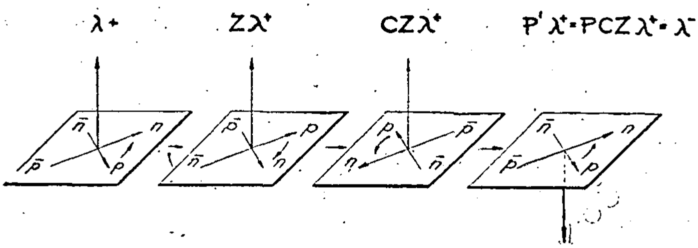

Fig. 3

(2) The electromagnetic interactions are not invariant against the charge symmetry operation. This means that the different members of the isodublets due to the electromagnetic interaction are different (*e. g.*, the mass difference between proton and neutron, etc.)

(3) It can be seen — as it was previously proved several time — that the number of invariants of weak interactions is not less than that of the electromagnetic inter-

action, but the invariants of the two different interactions are different. *E. g.*, the invariance against the mesoparity transformation P' corresponds in the case of electromagnetic interactions to that of the parity transformation as well as the invariance against the charge symmetry operation Z corresponds to that of charge conjugation C.

(4) Fig. 1 shows that because of the rotation of the λ-trieder the ψ_{AB}-s $(A, B = 1, 2)$ represent four different levels corresponding to the different particle states. Furthermore, it can be seen that in terms of the suggested theory the isospace is threedimensional in spite of the fact that the internal degree of freedom is only two.

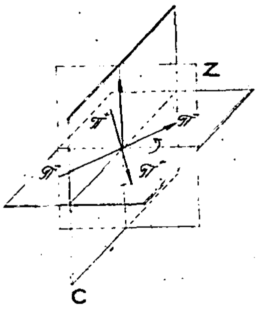

Fig. 4

Corrolaria. (1) It can be seen without any difficulty that LANDAU's PC theorem holds true since the λ^+-trieder is transformed by the operation PC into the λ^+-trieder (Fig. 5a). Analogously, the same theorem is valid for $P'Z$ (Fig. 5b).

(2) The invariance against the transformation PP', *i. e.* the PP' theorem — as it was shown in § 6 — is naturally valid also in this case (Fig. 6).

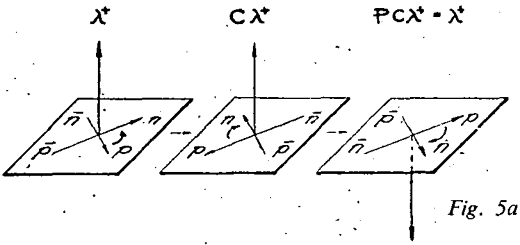

Fig. 5a

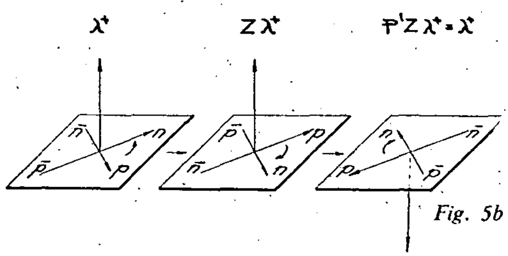

Fig. 5b

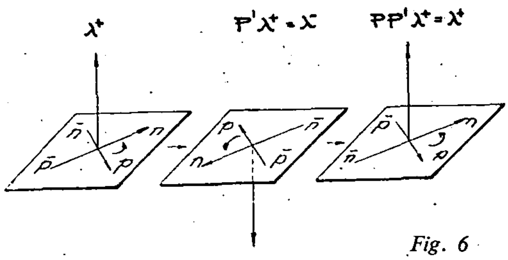

Fig. 6

§ 7. *A Method to Determine the Metrical Fundamental Tensor of the Line-element Space*

The metrical fundamental tensor $g_{\mu\nu}$ of the space — being for homogeneous and anisotropic spaces a function only of the homogeneous direction co-ordinates u^μ, *i. e.* $g_{\mu\nu} = g_{\mu\nu}(u^\mu)$ — was considered above as an *a priori* known function. Owing to the fundamental idea of the theory suggested, the anisotropy of the space-time world is induced by the anisotropy of the weak interactions. But, the anisotropy of the weak interactions is characterized, *e. g.*, by the longitudinal polarization of the emitted electrons. If one calculates with the general interaction the probability for an ordinary allowed β-transition in which the electron has a fixed energy W and its spin makes an angle ϑ with its momentum one obtains

$$N(\vartheta) = \text{const.}\{1 + \mathcal{A} \cos \vartheta\}\, d\Omega_e \qquad (23)$$

where $d\Omega_e$ denotes the width of the solid angle of the electron direction, furthermore

$$\mathcal{A} = d\xi \frac{v_e}{e} [\xi(1 + bm/W]^{-1}$$

$$\xi = (|g_S|^2 + |g_V|^2) |M_F|^2 + (|g_T|^2 + |g_A|^2) |M_{GT}|^2$$

$$b\xi = \left[1 - \left(\frac{e^2}{\hbar c} Z\right)^2\right]^{\frac{1}{2}} \{(g_S^* g_V + g_S S g_V^*) |M_F|^2 + (g_T^* g_A + g_T g_A^*) |M_{GT}|^2\}$$

$$d\xi = 2\,\mathrm{Re}\{g_S g_S'^* - g_V g_V'^*\} |M_F|^2 + 2\,\mathrm{Re}\{g_T g_T'^* - g_A g_A'^*\} |M_{GT}|^2 -$$

$$- i \frac{e^2}{\hbar c} \frac{Zm}{p} [2\,Im\,\{g_S g_V'^* + g_S' g_V^*\} |M_F|^2 + 2\,Im\,\{g_T g_A'^* + g_T' g_A^*\} |M_{GT}|^2]$$

$$M_F = \sum_{n=1}^{A} \int \phi_f^* \tau_-^{(n)} \phi_i dV, \quad M_{GT} = \sum_{n=1}^{\infty *} \int \Phi_f^* \sigma^{(n)} \tau^{(n)} \phi_i dV,$$

respectively, m and v_e mean the mass and the velocity of the electron; M_F and M_{GT} are the so-called FERMI and GAMOW—TELLER nuclear matrix elements; at last respectively g_S, g_V, g_T and g_A are the coupling constants of the scalar, vector, tensor and axial vector interactions [11].

Let us suppose that the surface (23) should be identical with the Carathéodoryan indicatrix (4) of the anisotropic space. This means in terms of the theory suggested that the equation of this indicatrix in its parametric form is determined explicitly by

$$x^1 = \{1 + \mathcal{A} \cos \vartheta\} \sin \vartheta \cos \phi, \; x^2 = \{1 + \mathcal{A} \cos \vartheta\} \sin \vartheta \sin \phi, \qquad (24)$$
$$x^3 = \{1 + \mathcal{A} \cos \vartheta\} \cos \vartheta.$$

Then the components of the metrical fundamental tensor of the space is then given by

$$g_{00} = 1, \; g_{ik}(u^1, u^2, u^3) = \delta_{ik}\{1 + \mathcal{A} v^3 [(v^1)^2 + (v^2)^2 + (v^3)^2]^{-\frac{1}{2}}\}^{-2}. \qquad (25)$$

Such a space is the special case of the general line-element spaces[1].

Other method for explicit calculation of the metrical fundamental tensor based on the anisotropy of the Co^{60} experiment was previously proposed [8].

* * *

The author is indebted to Dr. G. MARX and to Dr. A. MÓOR for very valuable discussions.

[1] This space has some interest also from the geometrical point of view. This is, namely, an explicit example for a space, all its three curvature tensors vanish, but is non-Euclidian since its torsion tensor is different from zero.

References

[1] *Marx, G., G. Györgyi:* Suppl. Nuovo Cimento (X) **5**, 159 (1957).
Gell-Mann, M., A. H. Rosenfeld: Ann. Rev. of Nucl. Sci. **7**, 407 (1957).
Lee, R. D.: Conservation Laws in Weak Interactions (Notes on Four Lectures given at Harward Univ., 1957).
[2] *Rzewuski, J.:* Acta Phys. Polonica **16**, 435 (1957).
Tanaka, S.: Progr. Theoret. Phys. **18**, 295 (1957).
Rayski, J.: Acta Phys. Polonica **17**, 187 (1958).
[3] *Lee, T. D., G. N. Yang:* Phys. Rev. **104**, 234 (1956).
[4] *Wigner, E. P.:* Rev. Mod. Phys. **29**, 255 (1957).
[5] *Landau, L.:* Nuclear Phys. **3**, 127 (1957).
[6] *Finsler, P.:* Über Kurven und Flächen in allgemeinen Räumen (Diss. Göttingen, 1918).
Cartan, E.: Les Espaces de Finsler (Act. sc. et ind. No. 79, Paris, 1934).
[7] *Horváth, J. I., A. Moór:* Indag. Math. **17**, 421, 581 (1955).
[8] *Horváth, J. I.:* Acta Phys. et Chem. Szeged **4**, 3 (1958).
Horváth, J. I.: Suppl. Nuovo Cimento (X) **9**, 444 (1958).
[9] *Feynman, R. P.:* Rochester Conference of 1958.
Feynman, R. P., M. Gell-Mann: Phys. Rev. **101**, 193 (1959).
[10] *Marx, G.:* Nuclear Physics **9**, 337 (1958); **10**, 468 (1959).
[11] *Roman, P.:* Theory of Elementary Particles (North-Holland, Amsterdam, 1960).

ОБ ОДНОЙ ВОЗМОЖНОЙ ИНТЕРПРЕТАЦИИ ИЗОТОПНОГО СПИНОГО ПОЛЯ И ЕГО ПРЕОБРАЗОВАНИЯ

Я. И. Хорват

Автор делал выходы из экспериментов, утверждающих нарушение закон сохранения четности на анизотропность континуума пространства-времени, структура которого определяется физическими взаимодействиями. Геометрическая характеризация анизотропных пространств осуществляется геометрией линейного элемента, которая принимает пространство не как множество точек а множеством линейных элементов (x^μ, v^μ), где x^μ координата места линейного элемента, v^μ ($\mu = 0, 1, 2, 3$) гомогенные координаты определяющие направление линейного элемента. Эти последние могут быть замещены Θ_i ингомогенными координатами направления, или $\xi_{(i)} = \cos\Theta_i$ ($i = 1, 2, 3$) величинами, относящимися к одному пригодно выбранному трехграннику (λ-трехгранник). x^μ координатами места мы характеризуем внешние степени свободы физического пространства, а ингомогенными координатами (x^μ) направления (или с величинами ($\xi_{(i)}$) те же внутренние. Значит, наше представление о том что физическое пространство возбуждается в анизотропном континууме времени и пространства равно тем что при установлении теории элементарных частиц мы также должны иметь в виду внутренние степени свободы физических пространств. Это представление делает возможным приписывать простое геометрическое значение компонентам изовекторов, изопсеудовекторов и т. п., а также делать наглядным преобразования изотопного спиного поля, как вращение λ-трехгранника, или как отражение отнесенное к λ-трехграннику. Наконец, мы указываем на метод, при помощи которого метрический основной тензор анизотропного поля может быть явно определен.