

# DETERMINATION OF THE DIFFUSION COEFFICIENT IN A LACQUER LAYER BY MEANS OF A RADIOACTIVE SUBSTANCE

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A method of calculating the diffusion coefficient, adjusted to a method of measurement important for the application, is given. The method is based upon an exact solution of the differential equation describing the diffusion in the homogeneous substance filling up the half-space and the diffusion coefficient is obtained graphically.

Several methods to characterize lacquer layers used for protecting metals are known. One of these [1, 2] uses the diffusion coefficient of the damaging substance diffusing into the lacquer layer to characterize the protecting effect. In the process of measurement, for a certain time, the lacquer layer is brought into contact with a corroding substance containing a radioactive isotope. Thereafter layers of definite thickness are ground off the lacquer layer and activity is measured at the new surfaces obtained. However, a method of calculation which could yield a theoretically correct diffusion constant has not been available so far. In the following such a way of calculation is described.

Let us start with considering a well known diffusion problem [3]. Let the half-space  $x > 0$  be filled by a homogeneous substance. At the moment  $t = 0$  let it be brought into contact with a solution of concentration  $c_0$ , filling the half-space  $x < 0$ . The molecules of the dissolved substance diffuse into the substance in the half-space  $x > 0$ . The phenomenon can be treated by the following differential equation:

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \quad (1)$$

where  $D$  is the diffusion coefficient. The well known solution of the equation for  $t > 0$  and  $x > 0$ , with given initial conditions  $c = c_0$  for  $x < 0$  and  $c = 0$  for  $x > 0$  is:

$$c(x, t) = \frac{c_0}{2} \left\{ 1 - \Phi \left( \frac{x}{2\sqrt{Dt}} \right) \right\} \quad (2)$$

where  $\Phi$  is the error integral defined by the equation

$$\Phi(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-t^2} dt. \quad (3)$$

The process of diffusion is interrupted at a time moment  $t > 0$ , by removing the solution in half-space  $x < 0$ . Then the concentration distribution for  $x > 0$  is given by (2). Let the concentration of the radioactive substance be designed by  $c$ . If  $\mu$  is the absorption coefficient of the homogeneous substance, its activity measured on the surface  $x = 0$  will be

$$N_0 = \alpha \int_0^{\infty} c(\xi, t) e^{-\mu\xi} d\xi. \quad (4)$$

Here the factor  $\alpha$  depends on the finite dimension of the surface  $x = 0$  and on the relatively small time of measuring compared to  $t$ . After removing a layer  $(0, x)$ , the activity measured at the new surface  $x > 0$  is given by

$$N(x) = \alpha \int_x^{\infty} c(\xi, t) e^{-\mu(\xi-x)} d\xi = \alpha \frac{c_0}{2} e^{\mu x} \int_x^{\infty} \left\{ 1 - \Phi \left( \frac{\xi}{2d} \right) \right\} e^{-\mu\xi} d\xi; \quad (5)$$

wherein the notation

$$d = \sqrt{Dt} \quad (6)$$

is introduced. The result of the integration of (5) is:

$$N(x) = \frac{A}{\mu} \left\{ 1 - \Phi \left( \frac{x}{2d} \right) - e^{\mu^2 d^2 + \mu x} \left[ 1 - \Phi \left( \frac{x}{2d} + \mu d \right) \right] \right\} \quad (7)$$

where  $A = \alpha c_0 / 2$ .  $N(x)$  can be measured by grinding off layers of different thickness  $(0, x)$ . The characteristic  $d$  or  $D$  cannot be expressed from (7) in explicit form. Besides, due to causes inherent in the method of measurement,  $N(0) = N_0$  is uncertain, therefore (4) also yields uncertain values for  $\alpha$ . The required result can be obtained from (7) and the measured  $N(x)$  in the following way. In the case of a given  $\mu$  we can write the following function

$$\frac{N(x_1)}{N(x_2)} = f(x_1, x_2, d) \quad (8)$$

which, at fixed  $x_1$  and  $x_2$  depends only on  $d$ , thus it can be plotted. (8) is a monotonous function of  $d$ . We read  $N(x_1)$  and  $N(x_2)$  from  $N(x)$  determined by measurements. Their quotient gives  $f(x_1, x_2, d)$  thus  $d$  can be read from the curve and  $D$  is given by (6).

In case of substances interesting for practice  $\mu = 2.124$ . In carrying out the measurements  $x_1 = 0.001$ ,  $x_2 = 0.002$  and  $x_3 = 0.003$  cm, respectively. The values of  $f(x_1, x_2, d)$  and  $f(x_2, x_3, d)$  in the range from  $d = 0.01$  to  $0.00063$  are listed in Table I.

Table I

$d$	$f(x_1, x_2, d)$	$f(x_2, x_3, d)$
0.01	1.096	1.101
0.008	1.124	1.129
0.0063	1.162	1.172
0.005	1.213	1.231
0.004	1.280	1.307
0.0032	1.376	1.422
0.0025	1.525	1.623
0.002	1.735	1.913
0.0016	2.071	2.404
0.00125	2.748	3.443
0.001	4.021	5.771
0.0008	6.407	12.491
0.00063	14.936	43.670

## References

- [1] *Anderson, J., J. Richards*: J. Chem. Soc. 537 (1946).
- [2] Investigation of Lacquer Layer by Measuring Diffusions of Acides (Report of CMEA, Moscow-Prague, manuscript, 1966).
- [3] *Frank, Ph., R. v. Mises*: Die Differential- und Integralgleichungen der Mechanik und Physik, II. Teil (Vieweg, Braunschweig, 1961).

ОПРЕДЕЛЕНИЕ ДИФФУЗИОННОГО КОЭФФИЦИЕНТА В СЛОЕ  
ЛАКА РАДИОАКТИВНЫМ ВЕЩЕСТВОМ*Ф. Й. Гилде*

Автором дается метод вычисления диффузионного коэффициента применяя к методу измерения важному для употребления в практике. Метод основан на точном решении дифференциального уравнения диффузии относительно вещества заполняющего пространство. Диффузионный коэффициент получается графическим методом.