

**BOND ANGLE DEPENDENT LIGAND FIELD MODEL FOR  
ELECTRONIC CONFIGURATIONS  $d^2$  AND  $d^3$ , I**  
**Theoretical Study of the Trigonal Distortion of  
Tetrahedral Complexes**

By

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The energy matrices of the electronic states arising from the symmetry splitting of configurations of  $d^2$  and  $d^3$  systems in trigonally distorted tetrahedral strong ligand fields are given in terms of Coulomb interaction and ligand field parameters and the angle of distortion.

In earlier studies [1—4], the theoretical discussion of the tetragonally distorted tetrahedral case has been presented on the basis of the bond angle-dependent ligand field model. Though this approach proved to be successful for the interpretation of several properties related with the changes in the electronic energies of a number of transition metal complexes [5—7] of the compositions  $MA_4$ ,  $MA_3B$  and  $MA_2B_2$ , nevertheless, later on it proved necessary to adapt the model to the trigonally and rhomboidally distorted tetrahedrons, which take more strictly into account the actual microsymmetries for complexes of  $MA_3B$  and  $MA_2B_2$  compositions.

In the present paper, the trigonally distorted tetrahedral case, and in the following one [8] the rhomboidally distorted tetrahedral case for the  $d^2(d^8)$  and  $d^3(d^7)$  configurations<sup>1</sup>, of greatest practical importance, have been treated theoretically. To do this, the procedure outlined in earlier papers has been followed.

In case of four-coordinate tetrahedrally oriented mixed ligand complexes of  $MA_3B$  composition three identical (A) ligands are located in three apices of the tetrahedron and one different (B) ligand in the fourth apex. The coordinate system is suitably oriented in such a way that its origin (O) lies in the centre of the tetrahedron and the Z-axis coincides with the O—B straight line. Each of the three identical A ligands encloses the angle  $\alpha$  with the +Z-axis<sup>2</sup>, so, except the cases  $\alpha = 109^\circ 28'$  ( $T_d$ ) and  $\alpha = 180^\circ$  ( $D_{\infty h}$ ), the symmetry is  $C_{3v}$  for any angle. The coordinates of the ligand positions with respect to the central metal ion are given in Table I.

<sup>1</sup> The discussion of all the remaining cases:  $d^4(d^6)$  and  $d^5$ , and further details of the theory can be found in [9].

<sup>2</sup> This Z-axis (the Z-axis of the "trigonally oriented tetrahedron") deviates, of course, by an angle of  $\arccos(3)^{-1/2}$  from the Z-axis of the "tetragonally oriented tetrahedron" (comp. with the case described in [1]). Throughout this paper  $\frac{\pi}{2} \cong \alpha \cong \pi$ .

Table I  
Ligand coordinates

Type of complex	Ligand	$R_k$ Metal-ligand distance	$\vartheta_k$ Polar angle (from the +Z-axis)	$\varphi_k$ Azimuthal angle (from the +X-axis)
$MA_3B$	$A^1$	$R_A$	$\alpha$	0
	$A^2$	$R_A$	$\alpha$	$2\pi/3$
	$A^3$	$R_A$	$\alpha$	$4\pi/3$
	$B$	$R_B$	0	0

By using the expression (1) of [1], the actual ligand field operator is of the form:

$$V(C_{3v}) = eq \left\{ \left[ \frac{3}{2} (3 \cos^2 \alpha - 1) r_2^A(r) + r_2^B(r) \right] P_2^0(\cos \vartheta) + \left[ \frac{3}{8} (35 \cos^4 \alpha - 30 \cos^2 \alpha + 3) r_4^A(r) + r_4^B(r) \right] P_4^0(\cos \vartheta) - \frac{1}{16} \sin^3 \alpha \cos \alpha r_4^A(r) P_4^3(\cos \vartheta) (e^{-3i\varphi} + e^{3i\varphi}) \right\}. \quad (1)$$

For  $\alpha = 109^\circ 28'$  and  $R_A = R_B$ , the perturbation potential valid in the regular ("trigonally oriented") tetrahedral ligand field will be effective:

$$V(T_d) = eq \left\{ \frac{28}{27} P_4^0(\cos \vartheta) + \frac{\sqrt{2}}{81} P_4^3(\cos \vartheta) (e^{-3i\varphi} + e^{3i\varphi}) \right\} r_4(r). \quad (2)$$

The single-electron trigonal ligand field matrix elements then are

$$\langle d_0 | V(C_{3v}) | d_0 \rangle = 2D_2^0(3) + 6D_4^0(3), \quad (3.1)$$

$$\langle d_{\pm 1} | V(C_{3v}) | d_{\pm 1} \rangle = D_2^0(3) - 4D_4^0(3), \quad (3.2)$$

$$\langle d_{\pm 2} | V(C_{3v}) | d_{\pm 2} \rangle = -2D_2^0(3) + D_4^0(3), \quad (3.3)$$

$$\langle d_{\mp 1} | V(C_{3v}) | d_{\pm 2} \rangle = \pm D_4^3(3), \quad (3.4)$$

or, considering the splitting,  $d(C_{3v}) = a + 2e$  (4)

of the fivefold  $d$ -orbital and the forms

$$a = d_{z^2}, \quad (5.1)$$

$$e_1^1 = \sqrt{\frac{1}{3}} (\sqrt{2} d_{x^2-y^2} + d_{xz}), \quad (5.2)$$

$$e_1^2 = \sqrt{\frac{1}{3}} (\sqrt{2} d_{xy} + d_{yz}), \quad (5.3)$$

$$e_2^1 = \sqrt{\frac{1}{3}} (d_{x^2-y^2} - \sqrt{2} d_{xz}), \quad (5.4)$$

$$e_2^2 = \sqrt{\frac{1}{3}} (d_{xy} - \sqrt{2} d_{yz}), \quad (5.5)$$

of the real  $d$ -orbitals<sup>3</sup> in trigonal field, the energy matrix elements<sup>4</sup> of the real  $d$ -functions are

$$\langle a | V(C_{3v}) | a \rangle = 2D_2^0(3) + 6D_4^0(3), \quad (6.1)$$

$$\langle e_1^1 | V(C_{3v}) | e_1^1 \rangle = \langle e_1^2 | V(C_{3v}) | e_1^2 \rangle = -D_2^0(3) - \frac{2}{3} D_4^0(3) - \frac{2\sqrt{2}}{3} D_4^3(3), \quad (6.2)$$

$$\langle e_2^1 | V(C_{3v}) | e_2^1 \rangle = \langle e_2^2 | V(C_{3v}) | e_2^2 \rangle = -\frac{7}{3} D_4^0(3) + \frac{2\sqrt{2}}{3} D_4^3(3), \quad (6.3)$$

$$\langle e_1^1 | V(C_{3v}) | e_2^1 \rangle = \langle e_1^2 | V(C_{3v}) | e_2^2 \rangle = -\sqrt{2} D_2^0(3) + \frac{5\sqrt{2}}{3} D_4^0(3) + \frac{1}{3} D_4^3(3). \quad (6.4)$$

The meaning of the quantities<sup>5</sup> occurring in the expressions (3) and (6) are as follows (for simplicity  $R_A$  taken equal to  $R_B$ ):

$$D_2^0(3) = \frac{6}{7} \varrho \left[ 1 + \frac{3}{2} (3 \cos^2 \alpha - 1) \right] Dq, \quad (7.1)$$

$$D_4^0(3) = \frac{2}{7} \left[ 1 + \frac{3}{8} (35 \cos^4 \alpha - 30 \cos^2 \alpha + 3) \right] Dq, \quad (7.2)$$

$$D_4^3(3) = \frac{15}{2} \sin^3 \alpha \cos \alpha Dq, \quad (7.3)$$

where

$$Dq = \frac{1}{6} R_4(r), \quad (8)$$

and

$$\varrho = \frac{R_2(r)}{R_4(r)}. \quad (9)$$

The explanations of other symbols are given in former papers (see [1—4, 7, 9, 10, 13]). Applying the strong field approximation to the  $d^2(d^8)$  and  $d^3(d^7)$  configurations appearing most frequently in practice, according to the general procedure [10] of the ligand field method [11], the energy matrix elements of the multi-electron states can be obtained as the sums of integrals composed of the operator (1) and of either wave functions of types described in [12] or the product eigenfunctions of a recursion method [13] and of integrals of the same functions and the operator (8) of [1]. Thus the energy matrices for  $d^2$  are:

<sup>3</sup> The subscripts of  $e$  in the suffix of the functions indicate the two  $e$ -orbitals in (4) and the superscripts of  $e$  distinguish one function from the other of the degenerate pairs of functions.

<sup>4</sup> Throughout this paper the interaction between the states  $e_1$  and  $e_2$  has been neglected. This has no serious effect on the energies, especially at angles close to that of the regular tetrahedron and  $\varrho=1$ . For a more precise treatment, the solutions of the one-electron energy matrix containing the off-diagonal elements  $\langle e_1 | V(C_{3v}) | e_2 \rangle$  should be used to calculate the matrix elements of multi-electron states at given angles.

<sup>5</sup> The subscripts 2 and 4 of  $D$  refer to the degree  $l$ , the superscripts 0 and 3 to the order  $m$  of the associated Legendre polynomial  $P_l^m(\cos \vartheta)$  involved in the operator (1) and the adjoining number 3 in parentheses refers to the (trigonal) symmetry. In the expressions (10) and (11) the latter are omitted.

${}^3E$	$(a)(e_1)$	$(a)(e_2)$	$(e_1)(e_2)$
$(a)(e_1)$	$3B + D_2^0 + \frac{16}{3}D_4^0 - \frac{2\sqrt{2}}{3}D_4^3$	$3\sqrt{2}B$	$3\sqrt{2}B$
$(a)(e_2)$		$6B + 2D_2^0 + \frac{11}{3}D_4^0 + \frac{2\sqrt{2}}{3}D_4^3$	$6B$
$(e_1)(e_2)$			$6B - D_2^0 - 3D_4^0$

(10.1)

${}^3A_1$	$(e_1)(e_2)$
$(e_1)(e_2)$	$-D_2^0 - 3D_4^0$

(10.2)

${}^3A_2$	$(e_1)^2$	$(e_2)^2$	$(e_1)(e_2)$
$(e_1)^2$	$3B - 2D_2^0 - \frac{4}{3}D_4^0 - \frac{4\sqrt{2}}{3}D_4^3$	0	$-6B$
$(e_2)^2$		$-\frac{14}{3}D_4^0 + \frac{4\sqrt{2}}{3}D_4^3$	0
$(e_1)(e_2)$			$12B - D_2^0 - 3D_4^0$

(10.3)

${}^1A_1$	$(a)^2$	$(e_1)^2$	$(e_2)^2$	$(e_1)(e_2)$
$(a)^2$	$12B + 3C + 4D_2^0 + 12D_4^0$	$\sqrt{2}(3B + C)$	$\sqrt{2}(2B + C)$	$-2\sqrt{2}B$
$(e_1)^2$		$15B + 4C - 2D_2^0 - \frac{4}{3}D_4^0 - \frac{4\sqrt{2}}{3}D_4^3$	$4B + 2C$	$2B$
$(e_2)^2$			$12B + 2C - \frac{14}{3}D_4^0 + \frac{4\sqrt{2}}{3}D_4^3$	0
$(e_1)(e_2)$				$8B + 2C - D_2^0 - 3D_4^0$

(10.4)

${}^1A_2$	$(e_1)(e_2)$
$(e_1)(e_2)$	$12B + 2C - D_2^0 - 3D_4^0$

(10.5)

${}^2E$	$(e_1)^2$	$(e_2)^2$	$(a)(e_1)$	$(a)(e_2)$	$(e_1)(e_2)$
$(e_1)^2$	$9B+2C-2D_2^0-$ $\frac{4}{3}D_4^0-\frac{4\sqrt{2}}{3}D_4^2$	$2B$	$0$	$-2B$	$-2B$
$(e_2)^2$	$8B+2C-\frac{14}{3}D_4^0+\frac{4\sqrt{2}}{3}D_4^2$		$2\sqrt{2}B$	$0$	$2B$
$(a)(e_1)$			$9B+2C+D_2^0+$ $\frac{16}{3}D_4^0-\frac{2\sqrt{2}}{3}D_4^2$	$\sqrt{2}B$	$\sqrt{2}B$
$(a)(e_2)$			$10B+2C+2D_2^0+$ $+\frac{11}{3}D_4^0+\frac{2\sqrt{2}}{3}D_4^2$		$-2B$
$(e_1)(e_2)$					$10B+2C-D_2^0-3D_4^0$ (10.6)

and for  $d^2$ :

${}^4A_1$	$(e_1)(e_2)(a)$	$D_2^0+3D_4^0$	$(e_1)^2(a)$	$(e_2)^2(a)$	$(e_1)(e_2)(a)$
$(e_1)(e_2)(a)$			$\frac{14}{3}D_4^0-\frac{4\sqrt{2}}{3}D_4^2$	$0$	$0$
$(e_2)^2(a)$				$3B+2D_2^0+\frac{4}{3}D_4^0+\frac{4\sqrt{2}}{3}D_4^2$	$-6B$
$(e_1)(e_2)(a)$					$12B+D_2^0+3D_4^0$ (11.2)

  

${}^4E$	$(e_1)^2(e_2)$	$(e_2)^2(e_1)$	$(e_1)(e_2)(a)$
$(e_1)^2(e_2)$	$6B-2D_2^0-\frac{11}{3}D_4^0-\frac{2\sqrt{2}}{3}D_4^2$	$3\sqrt{2}B$	$6B$
$(e_2)^2(e_1)$	$3B-D_2^0-\frac{16}{3}D_4^0+\frac{2\sqrt{2}}{3}D_4^2$		$3\sqrt{2}B$
$(e_1)(e_2)(a)$			$6B+D_2^0+3D_4^0$ (11.3)

${}^2A_1$	$(e_1)^2(a)$	$(e_2)^2(a)$	$(e_1)^2(e_2)$	$(e_2)^2(e_1)$	$(e_1)(e_2)(a)$	$(e_1)(e_2)(a)$
$(e_1)^2(a)$	$15B+5C+\frac{14}{3}D_1^2-\frac{4\sqrt{2}}{3}D_2^2$	$2(2B+C)$	$5\sqrt{2}B$	$-2B$	$0$	$\sqrt{3}B$
$(e_2)^2(a)$	$21B+5C+2D_2^2+\frac{4}{3}D_1^2+\frac{4\sqrt{2}}{3}D_2^2$		$\sqrt{2}B$	$-10B$	$0$	$-\sqrt{3}B$
$(e_1)^2(e_2)$			$\frac{14B+3C-2D_2^2}{3}-\frac{11}{3}D_1^2-\frac{2\sqrt{2}}{3}D_2^2$	$\sqrt{2}B$	$0$	$-2\sqrt{3}B$
$(e_2)^2(e_1)$			$\frac{13B+3C-D_2^2}{3}-\frac{16}{3}D_1^2+\frac{2\sqrt{2}}{3}D_2^2$		$0$	$\sqrt{6}B$
$(e_1)(e_2)(a)$					$\frac{21}{2}B+3C+D_2^2+3D_1^2$	$0$
$(e_1)(e_2)(a)$						$\frac{15}{2}B+3C+D_2^2+3D_1^2$

(11.4)

${}^2A_g$	$(e_1)^2(a)$	$(e_2)^2(a)$	$(e_1)^2(e_2)$	$(e_2)^2(e_1)$	$(e_1)(e_2)(a)$	$(e_1)(e_2)(a)$
$(e_1)^2(a)$	$9B+3C+\frac{14}{3}D_2^0-\frac{4\sqrt{2}}{3}D_4^2$	0	$-\sqrt{6}B$	$2\sqrt{3}B$	$\sqrt{3}B$	$B$
$(e_2)^2(a)$	$9B+3C+2D_2^0+\frac{4}{3}D_4^0+\frac{4\sqrt{2}}{3}D_4^2$		$-\sqrt{6}B$	$2\sqrt{3}B$	$-\sqrt{3}B$	$B$
$(e_1)^2(e_2)$			$\frac{14B+3C-2D_2^0}{3}-\frac{2\sqrt{2}}{3}D_4^2$	$-\sqrt{2}B$	$-4B$	$2\sqrt{3}B$
$(e_2)^2(e_1)$			$\frac{13B+3C-D_2^0}{3}-\frac{16}{3}D_4^0+\frac{2\sqrt{2}}{3}D_4^2$		$2\sqrt{2}B$	$\sqrt{6}B$
$(e_1)(e_2)(a)$					$\frac{29}{2}B+3C+D_2^0+3D_4^0$	$-\frac{\sqrt{3}}{2}B$
$(e_1)(e_2)(a)$						$\frac{39}{2}B+3C+D_2^0+D_4^0$

(11.5)





${}^2E$	...	$(e_1)^2(e_2)$	$(e_2)^2(e_1)$	$(e_2)^2(e_1)$	$(e_2)^2(e_1)$	$(e_1)^2(e_2)$	$(e_1)^2(e_2)$	$(e_1)(e_2)(a)$	$(e_1)(e_2)(a)$
$(e_1)^2$		$4\sqrt{2}B$	$\sqrt{2}(2B+C)$	0	$-2B$	0	0	0	0
$(e_2)^2$		$-2B$	0	0	0	0	$-\sqrt{2}B$	$-\sqrt{2}B$	$\sqrt{6}B$
$(a)^2(e_1)$		$\sqrt{2}B$	$\sqrt{2}(2B+C)$	0	0	0	0	$-B$	$-3\sqrt{3}B$
$(a)^2(e_2)$		0	$B$	$\sqrt{3}B$	$\sqrt{2}B$	$\sqrt{2}B$	$-B$	$-B$	$3\sqrt{3}B$
$(e_1)^2(a)$		0	0	0	$-2B$	0	$B$	$3B$	$\sqrt{3}B$
$(e_2)^2(a)$		$\sqrt{2}B$	$-5\sqrt{2}B$	$\sqrt{6}B$	0	$\sqrt{6}B$	$B$	$B$	$-1\sqrt{3}B$
$(e_1)^2(e_2)$		$-5\sqrt{2}B$	$-\frac{\sqrt{2}}{2}B$	$-\frac{\sqrt{6}}{2}B$	$-B$	$-\frac{\sqrt{6}}{2}B$	$\frac{\sqrt{2}}{2}B$	$\frac{\sqrt{2}}{2}B$	$-1\sqrt{6}B$
$(e_1)^2(e_2)$		$-1\sqrt{6}B$	$-\frac{3\sqrt{6}}{2}B$	$\frac{3\sqrt{2}}{2}B$	$3\sqrt{2}B$	$\frac{3\sqrt{2}}{2}B$	$2\sqrt{3}B$	$2\sqrt{3}B$	$-4B$
$\vdots$									(11.6b) (cont.)
${}^2E$	...	$(e_1)^2(e_2)$	$(e_2)^2(e_1)$	$(e_2)^2(e_1)$	$(e_2)^2(e_1)$	$(e_2)^2(e_1)$	$(e_1)^2(e_2)$	$(e_1)(e_2)(a)$	$(e_1)(e_2)(a)$
$\vdots$									
$(e_1)^2(e_2)$		$14B+3C-2D_2^0-$ $-\frac{11}{3}D_4^0-\frac{2\sqrt{2}}{3}D_4^2$	$B$	$-1\sqrt{3}B$	0	0	0	0	0
$(e_2)^2(e_1)$		$21B+5C-D_2^0-$ $-\frac{16}{3}D_4^0+\frac{2\sqrt{2}}{3}D_4^2$		0	$-5\sqrt{2}B$		$-\frac{3\sqrt{2}B}{2}$	$-\frac{\sqrt{6}B}{2}$	$-\frac{\sqrt{6}B}{2}$
$(e_2)^2(e_1)$		$9B+3C-D_2^0-$ $-\frac{16}{3}D_4^0+\frac{2\sqrt{2}}{3}D_4^2$			$\sqrt{6}B$		0	0	0
$(e_2)^2(e_1)$		$13B+3C-D_2^0-$ $-\frac{16}{3}D_4^0+\frac{2\sqrt{2}}{3}D_4^2$					0	0	0
$(e_1)(e_2)(a)$		$\frac{25}{2}B+3C+D_2^0+3D_4^0$					$-\sqrt{3}B$		
$(e_1)(e_2)(a)$		$\frac{27}{2}B+3C+D_2^0+3D_4^0$							(11.6c) (cont.)

The expressions (10) and (11) can easily be adapted to the cases of  $d^8$  and  $d^7$  by leaving the matrix elements of the Coulomb interactions (containing the linear combinations of Racah's  $B$  and  $C$  parameters) unchanged and by reversing the signs of the ligand field energies given by linear combinations of  $D_2^0$ ,  $D_4^0$  and  $D_4^2$  with respect to the cases  $d^2$  and  $d^3$ . The theoretical results obtained can be used for the explanation of the physico-chemical properties connected with the electronic energies and with their changes in trigonally distorted tetrahedral complexes of  $MA_3B$  composition.

#### References

- [1] Bán, M. I.: Acta Phys. et Chem. Szeged **18**, 45 (1972).
- [2] Bán, M. I.: Acta Phys. et Chem. Szeged **18**, 185 (1972).
- [3] Bán, M. I.: Acta Phys. et Chem. Szeged **19**, 57 (1973).
- [4] Bán, M. I.: Acta Phys. et Chem. Szeged **19**, 245 (1973).
- [5] Bán, M.: Paper presented at the VI. Colloquy on Coordination Chemistry, Eger (Hungary), 1971.  
Bán, M., J. Császár: Paper presented at the VIII. Colloquy on Coordination Chemistry, Sopron, (Hungary), 1973.  
Bán, M., M. Révész: Paper presented at the IX. Colloquy on Coordination Chemistry, Esztergom (Hungary), 1974.
- [6] Bán, M. I.: Paper presented at the International Conference on Molecular Spectroscopy, Wrocław, (Poland), 1972.
- [7] Bán, M. I., J. Császár, M. Hegyháti: J. Mol. Structure **19**, 454 (1973).
- [8] Bán, M. I., M. Révész: Acta Phys. et Chem. Szeged **20**, 393 (1974).
- [9] Bán, M. I.: Unpublished results.  
Révész, M.: Diploma Thesis, Szeged, 1973.
- [10] See e.g. Császár, J., M. I. Bán: Optikai szinkép, ligandumtér elmélet, komplexszerkezet (Optical Spectra, Ligand Field Theory, Structure of Coordination Compounds, in Hungarian). Akadémiai Kiadó, Budapest, 1972.
- [11] See e.g. Ballhausen, C. J.: Introduction to Ligand Field Theory, McGraw Hill, New York, 1962.
- [12] Gilde, F., M. I. Bán: Acta Phys. Hung. **12**, 13 (1960).  
Bán, M. I., F. Gilde: Coord. Chem. Revs. **2**, 137 (1967).  
Bán, M. I., Gy. Dömötör, F. Gilde: Acta Phys. Hung. **25**, 57 (1968).
- [13] Bán, M. I.: To be published. Presented in part at the First International Congress of Quantum Chemistry, Menton (France), 1973.

#### МОДЕЛЬ УГЛОВОЙ ЗАВИСИМОСТИ ЛИГАНДНЫХ ПОЛЕЙ $d^2$ И $d^3$ ЭЛЕКТРОННЫХ КОНФИГУРАЦИЙ, I

Теоретическое рассмотрение тригональной деформации тетраэдрических комплексов

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Используя приближение сильного поля в тригонально деформированной тетраэдрической системе, рассчитали энергетические состояния матрицов энергии, происходящих из расщепления  $d^2$  и  $d^3$  электронных конфигураций, как функции кулоновских взаимодействий, параметров лигандных полей и угла деформации.