ON THE INTERACTION OF AN ULTRASHORT LIGHT PULSE WITH A THIN RESONANT MEDIUM

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An optically thin, two level atomic system interacts with a pulse of resonant electromagnetic field. The transmitted and reflected waves are investigated for the case of exact resonance. The amplitudes and the phases of the secondary fields strongly depend on the area and on the width of the exciting pulse.

The resonant interaction of an ultrashort light pulse with a two level atomic system gives rise to several interesting pheomena [1]. Among them, however, only little attention has been paid to the reflected wave.

If the pulse is ultrashort and relaxation effects can be ignored, then the interaction of the two level system with the light field can be described by the optical Bloch equations. Using the rotating wave approximation one has [1]

$$\dot{u} = -\Delta v, \quad \dot{v} = \Delta u + \frac{p}{\hbar} E w, \quad \dot{w} = -\frac{p}{\hbar} E v. \tag{1}$$

Here u, v and w are the components of the Bloch vector, E is the slowly varying amplitude of the field acting on the atoms, p is the transition dipole moment beeing parallel to the linearly polarized field, and Δ is the detuning of the resonance frequency from the carrier frequency of the field.

If the medium is optically thin, then system (1) is usually solved for u, v and w regarding E as a given function of time, namely the incoming field. The effective field strength however consists of two parts. One is the external exciting field and the other is the field originating from the radiating dipoles themselves. The effects of this second field has been investigated in [2] and [3] for the case of superradiation when there is no external excitation but the system is in the upper unstable state.

We restrict our considerations to the case of exact resonance $\Delta = 0$. In this case the effective field is:

$$E = E_{ex} + \frac{2\pi n \omega_0 p}{c} v \tag{2}$$

where E_{ex} is the external excitation and the second term is the secondary field originating from the thin medium with a surface dipole density n. Substituting this expression into (1) with $\Delta=0$, and going over to a time scale $\tau_R=\hbar c/2\pi\omega_0 p^2 n$ we have

$$\dot{v} = (\mathscr{E}+v)w, \quad \dot{w} = -(\mathscr{E}+v)v \tag{3}$$

where $\mathscr{E}=\frac{p}{\hbar}E_{ex}\tau_R$ is the dimensionless amplitude of the external field.

Introducing the Bloch angle with $v=-\sin\theta$, $w=-\cos\theta$ we get the single equation

$$\dot{\theta}+\sin\theta = \mathscr{E} \tag{4}$$

which has been obtained in [4] in another way. We note that τ_R is the superfluorescence time [3] and in the case of a solid material it falls into the nanosecond range.

The reflected wave is proportional to $v=-\sin\theta$ while the transmitted wave is the sum of the incoming and the forward scattered wave $\mathscr{E}+v=\dot{\theta}$, which is identical with the effective field. Eq. (4) has been solved analytically in [4] for a square pulse, and approximate considerations were made for exciting pulses of smooth envelope.

Here we shall present the results of numerical solutions of eq. (4) for smooth pulses of the form

$$\mathscr{E} = \mathscr{E}_0 \operatorname{sech}\frac{t-t_0}{\tau}, \quad \mathscr{E}_0 = \frac{A}{\tau\pi},$$

where A is the pulse area. We have performed calculations for Gaussian and Lorentzian pulses as well but except for one special case, which will be discussed below, there are no qualitative differences in the evolution of the process.

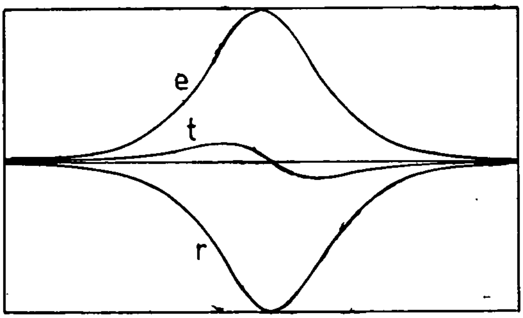

Fig. 1. $A=\pi/2$, $\tau=4$

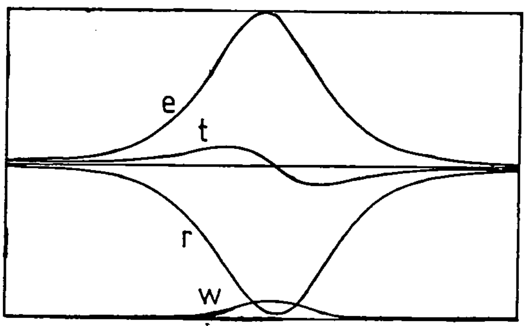

Fig. 2. $A=2\pi$, $\tau=4$

In Figs. 1-8 are shown the time dependence of the inversion (w) and of the exciting (e), transmitted (t) and reflected (r) waves.

The scales are fixed by the maximum of the exciting pulse, namely its position is at $t_0=5.5\tau$, its height is $\mathscr{E}_0$, while for the inversion the vertical scale goes from -1 to $+1$. Initially the two level system is in its ground state, $\theta=0$, $w=-1$. It is seen in all cases, that at the beginning of the excitation θ is increasing in time, the transmitted wave is positive, which means that it is in same phase as the incoming field. At the same time $-\sin\theta$ is negative, *i.e.* the reflected wave is in the opposite phase. The subsequent evolution of the process however depends on both the area of the incoming pulse and its steepness.

When the area is small, *e.g.* for a $\pi/2$ pulse, and the pulse width is large, *e.g.* $\tau = 4$, θ increases only slowly, $\dot{\theta}$ remains small, so that $\sin\theta = \mathscr{E}$, and the reflected wave will be nearly equal to the exciting wave (Fig. 1). The transmission of the medium turns to be small, the reflexion will dominate. The weak transmitted wave exhibits two small peaks at symmetric positions to the maximum of the exciting pulse. Between these peaks the transmitted wave changes its phase though the system remains close to its ground state, $w \approx -1$ during the whole process. Figs. 2 and 3 show the effects of 2π and 4π secant hyperbolic pulses with $\tau = 4$, respectively. In these cases the evolving of the fields is similar to that of $\pi/2$ pulses but with a more pronounced increase in the inversion. Note however that the behaviour of the Bloch angle is far from what is expected when neglecting the self field.

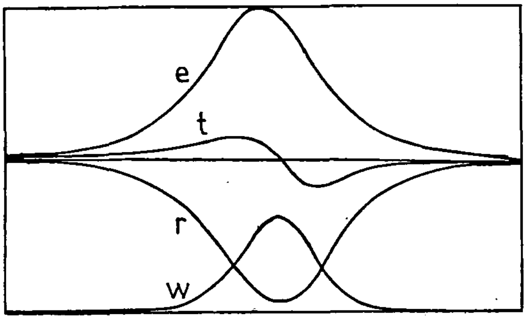

Fig. 3. $A = 4\pi$, $\tau = 4$

A $\pi/2$ pulse of a much smaller width, $\tau = 0.25$ shows a somewhat different character: the solution becomes asymmetric. The first peak of the transmitted amplitude is stronger and gets closer to the maximum of the exciting pulse, while the second peak in the opposite phase becomes smaller. The reflected wave is getting weaker too and its maximum is delayed (Fig. 4).

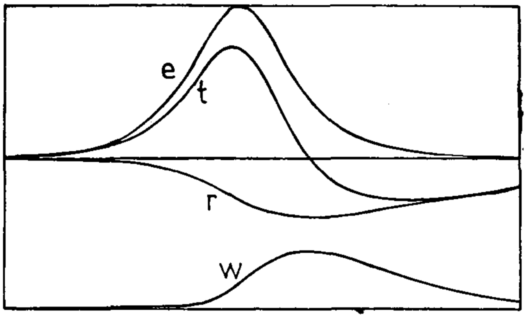

Fig. 4. $A = \pi/2$, $\tau = 0.25$

Now if we have a short pulse of area π the picture is changing further (Fig. 5). The maximum of the amplitude of the transmitted wave takes place at the centre of the excitation, the reflected wave has a long smooth tail of small amplitude and after the decay of the incoming pulse the transmitted wave goes over into the same form as the reflected wave. This part of the transmission can be regarded as a forward scattered wave, which has to be the same as the backward wave. The inversion grows over zero, and the pulse evolving after the excitation can be regarded as superradiation from a not fully inverted state. This process is strengthened by the tail of the excitation.

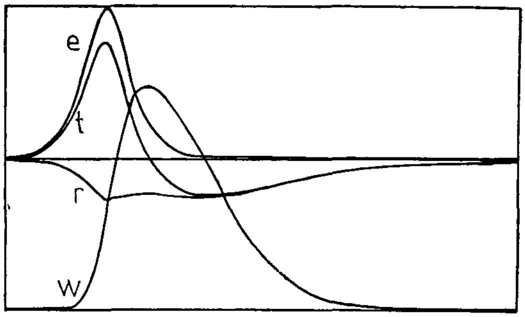

Fig. 5. $A = \pi$, $\tau = 0.25$

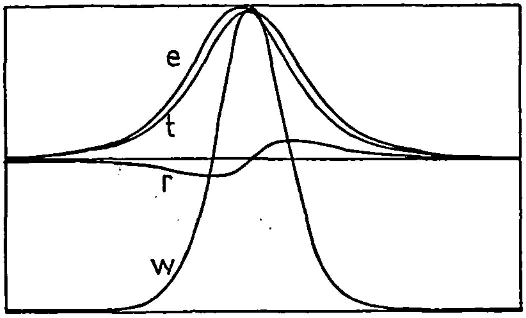

Fig. 6. $A=2\pi$, $\tau=0.25$

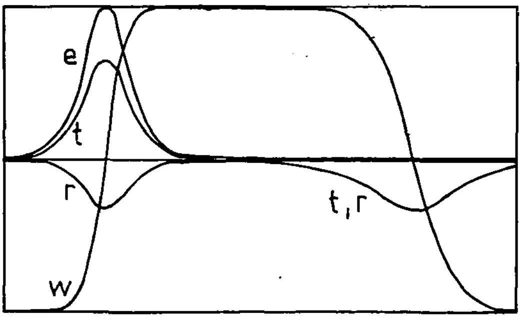

Fig. 7. $A=1.5\pi$ $\tau=0.5$

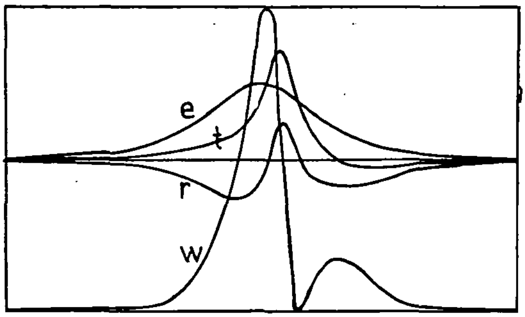

Fig. 8. $A=4\pi$ $\tau=2$

For short 2π pulses the situation will be the opposite to the weak and long excitation case. Now the transmitted wave will be in the same phase during the whole process, and its single maximum will somewhat delayed. The weak reflected wave is double peaked and changes its phase at the centre of the incoming pulse, while the inversion behaves as usual, the Bloch vector is turned around by 2π. (Fig. 6).

There is an interesting limiting case, when an analytical solution for Eq. (4) can be obtained. If $\mathcal{E}=\frac{\tau+1}{\tau}$ sech $\frac{t-t_0}{\tau}$ then $\theta=2$ arctg $e^{(t-t_0)/\tau}$ is a solution which corresponds to the initial condition $\theta(0)=$ $=2\,\mathrm{arctg}\,e^{-t_0/\tau}$. The numerical results show however that essentially the same solution can be obtained if $\theta(0)=0$ (Fig. 7). In the numerical procedure the exciting pulse was set up with $t_0=5.5\tau$. In this case θ tends to π as the excitation decays (with other forms of excitations this could be achieved only by cutting them off when θ gets equal to π). The area of the exciting pulse is $(\tau+1)\pi$ which shows that in order to reach complete inversion one must have a pulse area larger than π, and this is the more so, the wider is the incoming pulse. The reflected and the transmitted waves are in opposite phase both of them having the same form as the exciting pulse, and the ratio of their amplitudes turns to be τ, *i.e.* independent of time.

Of course the $\theta=\pi$ value is not stable, the system will radiate spontaneously according to the sech $(t-t_1)$ law in both directions after a small perturbation. The characteristic time of this superradiating pulse is τ_R not depending on the excitation.

For pulses of area larger than 2π also a different behaviour can be obtained for different widths. For a 4π pulse and $\tau=4$ see Fig. 3. For $\tau=2$ however the inversion may increase once again after having decayed to -1. Accordingly the reflected wave exhibits further oscillations. What is more interesting the transmitted pulse may be higher than the excitation, while it is narrower than the incoming pulse, thus showing up the effect of pulse compression (Fig. 8.)

For still shorter pulses the situation will be close to the case when the self field is neglected, though in all cases this self field turns back the system into its ground state.

These examples clearly show that the behaviour of the transmitted and reflected waves are very different for wide and short pulses of the same area. Beside the variations of the amplitudes of the secondary waves the changes in the phases also deserve attention. We note that measurements of similar phase jumps for choherent amplification, predicted at first time in [5] have been reported only very recently [6]. The results of our calculations show, at least in principle, the possibility to control the amplitude and phase of a coherent light pulse.

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О ВЗАИМОДЕЙСТВИИ УЛЬТРАКОРОТКОГО СВЕТОВОГО ИМПУЛЬСА С ТОНКИМ РЕЗОНАНСНЫМ СЛОЕМ

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Оптически тонкая двухуровневая атомная система взаимодействует с резонансным электромагнитным импульсом. Исследуются пропущенная и отраженная волны в случае точного резонанса. Амплитуды и фазы во вторичных полях сильно зависят от площади и ширины возбуждающего импульса.