ASSESSMENT OF THE LEVEL OF DEVELOPMENT

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Part. I.

Economic development is a central problem of our age. Problems or sets of problems connected with the level of economic development of the territorial differences of the rate of development often arise. Representatives of many branches of science, economists, statisticians and geographers try to solve them. Analyzing of the problems connected with economic development is especially popular among theoreticians. The national and the international literature of the subject is very rich. The theoretical and methodological problems of the research of the subject is the theme of scientific programs and international conferences.

Economic development is such a complex process that one researcher or research group or even one discipline cannot undertake an all-round analysis of it. Our study is also confined to only a relatively narrow field of it, the assessment of the level of development of individual economic regions.

The foreign literature of the subject is very extensive. Among its cultivators are BRADISTILOV, JAN KAZIMOUR, M. K. BANNETT, and H. H. HARMAN.

As a consequence of the dynamic economic development following our liberation the subject has attracted increased interest of the researchers. It has become a social requirement, — especially since the second half of the 1960' s — to relieve or research the structural imbalance in certain areas. (Especially outstanding in this field is the work of M. BARABÁS, GY. BARTA, I. BARTKE, GY. BORA, K. NAGY Mrs. DUX, T. GERŐ, M. VISSI Mrs. HALMI, I. HUSZÁR, L. LACKÓ, L. KLONKAI, J. KÓRÓDI, L. KŐSZEGI, Mrs. L' KŐSZEGI, V. KULCSÁR, G. MÁRTON. J. RIMLER, GY. SZILÁGYI, GY. WIRTH, and Mrs. ZALAI.

The different authors worked out, modified, or applied methods to certain areal units. According to these areal units, however, countries, administrative units (counties, districts, towns, villages) and regions representing different levels (macro-, meso-, sub- and microregions) require differentiated methods. Each method gives reliable results only when applied to an areal unit of a certain level. E. g. an accepted indicator in comparisons between countries is the national income calculated for a single inhabitant. Although the result obtained in this way must be received with caution (because it may be distorted by calculation into the currency of different countries, by different interpretations of the concepts, etc.), on account of its simplicity it is doubtless the best indicator among those used. There are attempts at a comparison of the levels of economic development of the different countries based on a system consisting of natural indicators (e. g. the method of Jánossy employing 16+8 indicators); however, these are complicated and are gaining popularity only with difficulty.

Determination of the state of development of the parts of a country is an even more difficult task than the comparison between countries.

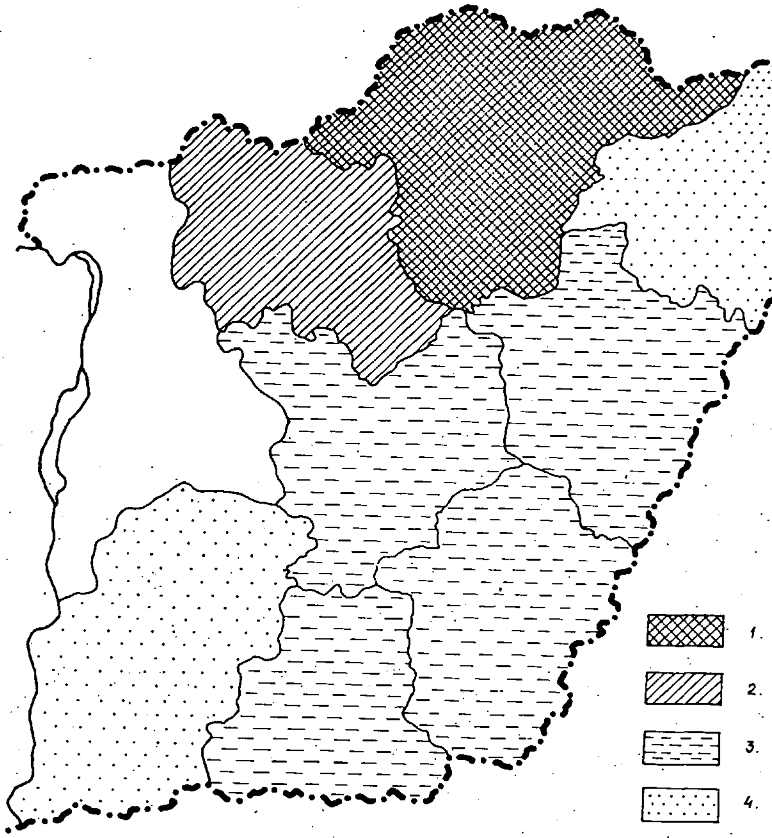


Figure 1. Industrial development of subregions of East Hungary
1. developed
2. medium developed
3. underdeveloped
4. very poorly developed

The problem of areal units

In our work we tried to elaborate a method that would reflect the state of development of the economy and would not only classify the areal units but also express their quality quantitatively. (By subregions we understand the third level from above of the hypothetical classification of regions worked out by the Department of Economic Geography of JATE and published in the Geographical Communications in 1969.)

We consider the areas chosen by us for basic units as suitable to give distortion-free indicators and true pictures. Areas of such size (on the average 5000 km^2 in East Hungary) may be regarded as "homogeneous" in respect of social production; in respect of the spatial distribution of the forces of production they form areal production complexes.

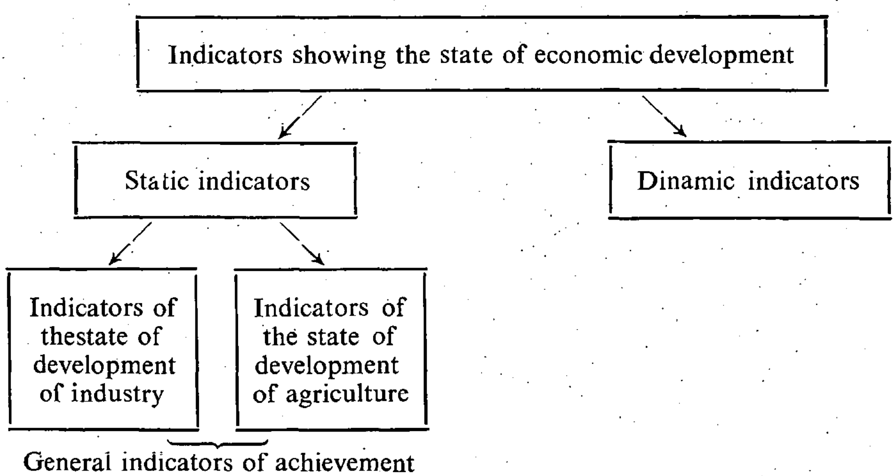
The requirements of the indicators

In the course of the analysis of the different areas there arises the need for a complex indicator that in itself, as if by summary, would express the level of development of a given areal unit. In our work we used the method employed also by many other researchers which was that with suitable grouping a system of indicators was constructed from the various natural indicators reflecting the different degrees of economic development. We think that the synthetic index thus formed approximates reality sufficiently.

A very important and difficult task is the proper selection of the economic indicators. The indicators must be connected with the economic development as closely as possible. At different stages of development different indicators express the level of development; therefore the system of indicators must also change in time and space. At the present stage of development of our economy the most highly industrialized areas are also the most highly developed ones. However, because any production complex of the branches of economy can, in principle, develop an economically advanced area, we think that the areal units established by us must be examined both from the point of view of industry and from the point of view of agriculture. We endeavored to form groups of indicators composed of approximately identical numbers of indicators. The indicators showing the state of development of industry and agriculture are without exception of static character. Therefore we have formed another, dynamic group of indicators from which the territorial differences of the rate of development appears. With the help of this the trends of development and the dynamism of our areas can be clearly seen and on the basis of these the perspectives of our areas can be clearly seen and on the basis of these the perspectives of our areas can be outlined.

Finally — as if by control — we introduced a group of indicators named general indicators of achievement. In this group we have collected indicators which with more or less distortion express the state of development of a given area.

Survey of the system of indicators



I

Indicators of the state of development of industry

1. The corrected national income per inhabitant (Ft per head).
2. Fixed assets of industry: The gross value of fixed assets per inhabitant (Ft per head).
3. The value of current assets per unit of current assets in industry (Ft/Ft).
4. The propelling power supply of industry: The capacity of power machines and electric motors per head in industry (kW per head).
5. Electric energy consumption per worker (kW per head).
6. Mechanization of industrial production:
   The number of places of work beside machines in industry (places per piece).
7. The ratio of those employed in industry. The number of industrial wage earners per 1000 people employed (wage earners per head).
8. The concentration of industry: The number of workers working at industrial establishments employing more than 500 persons per 10 000 inhabitants (persons employed per head).

II

Indicators of the state of development of agriculture

1. The corrected national income from agriculture per inhabitant.
2. The fixed assets of agriculture:
   The value of fixed assets per inhabitant in the agricultural large-scale cooperatives.
3. Total traction power per 100 ha of agricultural area in traction units (tractors per ha).
4. The utilization of artificial fertilizers per ha of plowland (kg per ha).
5. The importance of livestock farming:
   Number of animals per inhabitant (number per inhabitant).
6. The importance of livestock farming; Milk production per inhabitant (l per head)
7. The ratio of intensive plant cultivation:
   The garden, orchard, vinyard, and vegetable-growing plowland area per active agricultural wage earner (ha per head).
8. The ratio of irrigated area in % of the total agricultural area.
9. Buying up per inhabitant in agriculture (Ft per head).

III

General indicators of achievement

1. The national income per active wage earner (Ft per head).
2. Consumption of the national income per inhabitant (Ft per head).
3. The total value of fixed assets and current assets per inhabitant (Ft per head).
4. The number of those employed per 1000 persons in the population constituting the manpower resource (persons employed per head).
5. The number of city dwellers per 1000 inhabitants.

IV

The indicators of dynamic development

1. The total investments per inhabitant in the last five years (Ft per head).
2. The ratio of machine stocks younger than five years to the total machine stocks (%).
3. Intensive plant cultivation and the size of irrigated areas as compared to the level five years earlier (%).
4. The number of new diploma holders per 1000 inhabitants (in 5 years) (heads per head).
5. The number of flats built per 1000 inhabitants in the last 5 years (flats per head).

The method of investigation

The mathematical method of investigation is the factor analysis. The factor analysis is a branch of statistical analyses with several variables which is a very widely applicable mathematical statistical method. The aim of the factor analysis is to produce simple hypothetical variables, factors strating from the set of variables observed which reproduce the data observed fairly accurately and explain them in a sense. Whille the statistical methods with several variables generally examine essentially given hypotheses, the aim of the factor analysis is just the search for a hypothesis or she making of it. The factor analysis tries to set up a model which is as simple as possible, with well interpretable values and real correspondences.

The factor analysis model

Let a number m of random variables $Y_1, Y_2, \ldots, Y_m$ be given. It is suitable to work with standardized variables. That is instead of the original variables Y_i we work with the standardized variables

$$Z_i = \frac{Y_i - M(Y_i)}{D(Y_i)} \qquad (i = 1, 2, \ldots, m),$$

where $M(Y_i)$ is the expectation of Y_i, and $D(Y_i)$ is its standard deviation. Therefore

$$M(Z_i) = 0, \quad D(Z_i) = 1 \qquad (i = 1, 2, \ldots, m).$$

The factor analysis starts from the hypothesis that the Z_i variables are the functions of further hypothetical variables; they can be ritten as the linear function of the so-called factors;

$$\begin{aligned} Z_1 &= a_{11}K_1 + a_{12}K_2 + \ldots + a_{1r}K_r + b_1U_1 \\ Z_2 &= a_{21}K_1 + a_{22}K_2 + \ldots + a_{2r}K_r + b_2U_2 \\ Z_m &= a_{m1}K_1 + a_{m2}K_2 + \ldots + a_{mr}K_r + b_m U_m, \end{aligned}$$

where $K_1, K_2, \ldots, K_r, U_2, \ldots, U_m$ are the so-called *factors*, and $a_{11}, a_{12}, \ldots, a_{mr}$; $b_1, b_2, \ldots$ $\ldots b_m$ are the *factor loadings*.

If the loading of a factor differs essentially from O in the case of at least two variables, it is called *common factor*. It this condition is satisfied for all variables, then we have a *general factor*. If the factor loading differs from O only for one variable, it is called *unique factor*. In our notation $K_1, K_2, \ldots, K_r$ are common factors, $U_1, U_2, \ldots, U_m$ are unique factors, and r is the number of common factors.

The factors analysis model in matrix form can be expressed as follows:

$$Z = A \cdot f$$

where

$$z = (z_1, z_2, \ldots, z_m)^*$$

is the column vector of the standardized variables,

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{1r} & b_1 & 0 \ldots 0 \\ a_{21} & a_{22} & a_{2r} & 0 & b_2 \ldots 0 \\ a_{m1} & a_{m2} & a_{mr} & 0 & 0 \ldots b_m \end{pmatrix}$$

the matrix of the factor loadingd, and

$$f = (K_1, K_2, \ldots, K_r, u_1, u_2, \ldots, u_m)^*$$

the column vector of the factors.

The factor analysis model showing the common and the unique factors can be written in this form:

$$z = A_k f_k + A_u f_u,$$

where

$$A_k = \begin{pmatrix} a_{11} & a_{12} \ldots a_{1r} \\ a_{21} & a_{22} \ldots a_{2r} \\ \vdots & \\ a_{m1} & a_{m2} \ldots a_{mr} \end{pmatrix}$$

is the matrix of the factor loadings of the common factors,

$$A_u = \begin{pmatrix} b_1 & 0 \ldots 0 \\ 0 & b_2 \ldots 0 \\ 0 & 0 \ldots b_m \end{pmatrix}$$

is the diagonal matrix of the factor loadings of the unique factors,

$$f_k = (K_1, K_2, \ldots, K_r)^*$$

is the column vector of the common factors.

Summarizing:

$$A = [A_k, A_u] \quad \text{and} \quad f = [f_k, f_u].^*$$

The matrix A is also called *factor pattern*.

In the following we always suppose that the factors are standardized random variables:

$$M(K_i) = 0, \quad D(K_i) = 1 \quad (i = 1, 2, \ldots, r),$$

$$M(U_j) = 0, \quad D(U_j) = 1 \quad (j = 1, 2, \ldots, m),$$

furthermore, that the unique factors are always uncorrelated with each other and with the common factors:

$$R(U_j, U_l) = 0 \quad \text{if} \quad j \neq l \, (j, l = 1, 2, \ldots, m),$$

$$R(K_i, U_j) = 0 \qquad (i = 1, 2, \ldots, r; \; j = 1, 2, \ldots, m),$$

where $R(K, U)$ denote the correlation coefficient of the variables K and U.

From these assumtions it follows that the $(r+m)\times(r+m)$ correlation matrix of the factors is the following:

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{1r} & 0 & 0 & 0 \\ c_{21} & c_{22} & c_{2r} & 0 & 0 & 0 \\ c_{r1} & c_{r2} & c_{rr} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where $c_{ij} = R(K_i, K_j) \qquad (i, j = 1, 2, \ldots, r)$.

A simple calculation shows that

$$1 = D^2(Z_i) = M(Z_i^2) = M\left[\left(\sum_{p=1}^{r} a_{ip}K_p + b_iU_i\right)^2\right] = \sum_{p=1}^{r}\sum_{q=1}^{r} a_{ip}a_{iq}M(K_pK_q) + b_i^2 =$$

$$= \sum_{p=1}^{r}\sum_{q=1}^{r} a_{ip}c_{pq}a_{iq} + b_i^2,$$

where $c_{pq} = R(K_p, K_q) = M(K_p \cdot K_q)$. Thus we get

$$1 = a_i^* C a_i$$

where

$$a_i^* = (a_{i1}, a_{i2}, \ldots, a_{ir}, 0, 0, \ldots, \overbrace{b_i}^{r+1}, \ldots 0)$$

is the i' th row vector of the matrix A, while a_i is the same written in the form of a column vector $(i = 1, 2, \ldots, m)$.

Separating lhe common and the unipue factors, the above formula may also be written:

$$1 = a_{k,i}^* C_k a_{k,i} + b_i^2,$$

where

$$C_k = \begin{pmatrix} c_{11} & c_{12} \ldots c_{1r} \\ c_{21} & c_{22} \ldots c_{2r} \\ c_{r1} & c_{r2} \ldots c_{rr} \end{pmatrix}$$

is the correlation matrix of the common factors,

$$a_{k,i}^* = (a_{i1}, a_{i2}, \ldots, a_{ir})$$

the i'th row vector of the matrix A_k, while $a_{k,i}$ is the same in the form of column vector $(i = 1, 2, \ldots, m)$. Written in a short form:

$$C = \begin{bmatrix} C_k & O \\ O & E_m \end{bmatrix};$$

where E_m is the mxm indentity matrix and O is the zero matrix.

If the pairs of common factors are uncorrelated among themselves, then

$$C_k = E_r \quad \text{and} \quad C = E_{r+m}.$$

Then the above formula obtained for the variance $D^2(Z_i)$ of the variable Z_i becomes essentially simpler:

$$1 = a_i^* a_i = a_{k,i}^* a_{k,i} + b_i^2 = a_{i1}^2 + a_{i2}^2 + \ldots + a_{ir}^2 + b_i^2 = h_i^2 + b_i^2 \;\; (i = 1, 2, \ldots, m),$$

where the *communality*

$$h_i^2 = a_{i1}^2 + a_{i2}^2 + \ldots + a_{ir}^2$$

can be interpreted as that part of the variance of the variable Z_i which can be explained by the common factors together, while b_i^2 is that part of the variance of the variable Z_i which can be explained by the unique factor and which is generally termed the *uniqueness* of the given variable ($i = 1,2, \ldots, m$).

On the basis of the equality

$$h_i^2 + b_i^2 = 1 \qquad (i = 1, 2, \ldots, m),$$

it is enough to determine the common factor loadings in the course of the solution.

In some cases the *factors structure*

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{1r} & b_1 & 0 \ldots 0 \\ s_{21} & s_{22} & s_{2r} & 0 & b_2 \ldots 0 \\ \vdots & & & & \\ s_{m1} & s_{m2} & s_{mr} & 0 & 0 \ldots b_m \end{pmatrix}$$

which contains the correlation coefficients of each Z_i variables with the common K_j and unique U_k factors ($i=1,2,\ldots,m$; $j=1,2,\ldots,r$, $k=1,2,\ldots,m$) plays an important role.

Otherwise

$$S = [S_k, S_u],$$

where

$$S_k = \begin{pmatrix} s_{11} & s_{12} \ldots s_{1r} \\ s_{21} & s_{22} \ldots s_{2r} \\ \vdots & \\ s_{m1} & s_{m2} \ldots s_{mr} \end{pmatrix}, \qquad S_u = A_u.$$

Here s_{ij} is the correlation coefficient among the variable Z_i and the common factor K_j. ($i=1,2,\ldots,m$ $j=1,2,\ldots,r$) These correlation coefficients can be determined on the basis of the model in the following way:

$$s_{ij} = R(Z_i, K_j) = M(Z_i K_j) = M\left[\left(\sum_{p=1}^{r} a_{ip} K_p + b_i U_i\right) K_j\right] =$$

$$= \sum_{p=1}^{r} a_{ip} M(K_p \cdot K_j) = \sum_{p=1}^{r} a_{ip} c_{pj} \qquad (i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, r).$$

Thus the following connection exists between the factor structure and the factor pattern:

$$S = A \cdot C,$$

or what is equivalent to this,

$$S_k = A_k \cdot C_k \quad \text{and} \quad S_u = A_u.$$

Hence it can easily be seen that in the case of uncorrelated pairs of common factors

$$S = A,$$

because in this case $C = E_{r+m}$. Therefore, in the case when the solution consists of pairs of uncorrelated common factors, it is sufficient to determine only the factor pattern A. However, when correlated common factors are also allowed in the model, then the solution must contain both the factor pattern and the factor structure.

On the basis of the model of factor analysis there is an opportunity not only for the composition of the variance of the different variables, but it is also possible to determine the correlation coefficients among the variables. Thus the fidelity of the model of factor analysis can also be determined.

Let

$$R = \begin{pmatrix} r_{11} & r_{12} \dots r_m \\ r_{21} & r_{22} \dots r_{2m} \\ \vdots & \\ r_{m1} & r_{m2} \dots r_{mm} \end{pmatrix}$$

be the correlation matrix of the variables $Z_1, Z_2, \dots, Zm$, where $r_{ik} = R(Z_i, Z_j)$ $(i, j = 1, 2, \dots, m)$. A simple calculation shows that

$$\begin{aligned} r_{ij} &= R(Z_i, Z_j) = M(Z_i \cdot Z_j) = \\ &= M\left[\left(\sum_{p=1}^{r} a_{ip} K_p + b_i U_i\right)\left(\sum_{q=1}^{r} a_{jq} K_q + b_j U_j\right)\right] = \\ &= \sum_{p=1}^{r} \sum_{q=1}^{r} a_{ip} a_{jq} M(K_p \cdot K_q) + b_i b_j M(U_i \cdot U_j) = \\ &= \sum_{p=1}^{r} \sum_{q=1}^{r} a_{ip} c_{pq} a_{jq} + \delta_{ij} b_i^2, \end{aligned}$$

where $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$ $(i, j = 1, 2, \dots, m)$.

Writing this in matrix form we get

$$R = A_k C_k A_k^* + A_u A_u^* = ACA^*,$$

where A^* is the transpose of the matrix A.

If the pairs of common factors are uncorrelated among themselves then $C = E_{r+m}$. Therefore in this case

$$R = AA^* = A_k A_k^* + A_u A_u^* = A_k A_k^* + A_u^2.$$

We call the matrix $R_h = R - A_u^2$ *reduced correlation matrix*. The reduced correlation matrix R_h differs from R in that just the h_i^2 communalities stand in its diagonal.

Of course, when the factors are uncorrelated, the above formula simplifies to the following expression for the reduced correlation matrix:

$$R_h = A_k A_k^*.$$

This equation has been called "the fundamental factor theorem" by Thurstone.

Finally we mention H. H. Harman' s excellent book, Modern Factor Analysis, The University of Chicago Press, 1960, which deals in exhaustive detail with the method of factor analysis. Besides this, the description of the method of factor analysis, together with economic applications can be found in Judit Rimler' s paper "Investigation of economic development and the factors' analysis, Közgazdasági Szemle, 1970, pp. 913—926 and László Vita's paper "The possibilities of the economic application of factor analysis, Szigma, 1970, pp. 127—152.

V.

Determination of the state of industrial development of subregions by the method of factor analysis

In our days the economic political endeavor to reduce the differences in state of economic development of different areal units (economic regions of different levels) and in the standard of living of their popolation is becoming ever stronger. The literature of the subject has now made it reasonably clear that the state of economic development of an area is determined by the whole of its production sphere. However, we have already mentioned in the above that in the present state of the development of our economy industry plays the decisive role among the branches of production. Therefore our investigation covered first of all industry.

In the introductory part (in the chapter on principles and methods) we could ronsider only the following five of the above-described eight indicators chosen for assessing the level of development of industry:

1. The gross value of fixed assets in industry per inhabitant.
2. The power machine and electric motor capacity in industry per inhabitant.
3. The consumption of electric energy per worker.
4. The number of industrial wage arners per 1000 persons employed.
5. The number of workers in industrial establishments employing more than 500 perssons per 10 000 inhabitants.

The following eight subregions were considered in the investigation: the areas of Bács, Békés, Borsod, Csongrád, Hajdú, Heves, Szabolcs and Szolnok counties.

Table 1 shows the values of the different indicators concerning the different subregions:

Table 1

indicators \ subregions	Bács	Békés	Borsod	Csongrád
Indicator 1	10.230	15.840	54.560	21.730
Indicator 2	0.160	0.220	1.550	0.340
Indicator 3	2.690	3.640	20.520	3.970
Indicator 4	0.250	0.390	0.520	0.330
Indicator 5	27.010	27.370	102.680	53.770

subregions / indicators	Hajdú	Heves	Szabolcs	Szolnok
Indicator 1	15.080	44.630	8.130	17.690
Indicator 2	0.180	0.930	0.090	0.320
Indicator 3	4.160	7.470	2.680	4.530
Indicator 4	0.370	0.320	0.240	0.290
Indicator 5	32.670	61.470	21.920	37.000

If we consider the different indicators as random variables, then each row of Table 1 is an eight-element sample for the variable concerned. After standardization Table 2 contains the matrix of the standardized values.

Table 2

subregions / indicators	Bács	Békés	Borsod	Csongrád
Indicator 1	−0.786	−0.453	1.842	−0.104
Indicator 2	−0.617	−0.499	2.119	−0.263
Indicator 3	−0.588	−0.429	2.395	−0.374
Indicator 4	−0.984	+0.568	2.010	−0.097
Indicator 5	−0.687	−0.674	2.128	+0.308

subregions / indicators	Hajdú	Heves	Szabolcs	Szolnok
Indicator 1	−0.498	1.254	−0.910	−0.343
Indicator 2	−0.578	0.898	−0.755	−0.302
Indicator 3	−0.342	0.211	−0.590	−0.280
Indicator 4	+0.346	−0.207	−1.095	−0.540
Indicator 5	−0.476	0.594	−0.876	−0.315

The majority of indicators (variables) considered are characterized by strong interdependence. In the correlation matrix the value of the smallest correlation coefficient is +0,711, which means that stronger than average positive correlation exists among any two indicators examined by us. The correlation matrix of the indicators is as follows:

Indicator 1	1.000				
Indicator 2	0.979	1.000			
Indicator 3	0.883	0.953	1.000		
Indicator 4	0.711	0.745	0.832	1.000	
Indicator 5	0.946	0.965	0.938	0.769	1.000

On the basis of the correlation matrix the following values were found for the communalities:

indicators	communalities
Indicator 1	0.9966
Indicator 2	0.9985
Indicator 3	0.9951
Indicator 4	0.9158
Indicator 5	0.9489

The communality belonging to the different indicators can be interpreted as that part of the variance of the variable concerned which can be explained by the common factors together.

Replacing the units in the diagonal of the correlation matrix with communalities we obtain the so-called reduced correlation matrix:

Indicator 1	0.996				
Indicator 2	0.979	0.998			
Indicator 3	0.883	0.953	0.995		
Indi ator 4	0.711	0.745	0.832	0.915	
Indicator 5	0.946	0.965	0.938	0.769	0.948

Starting from the reduced correlation matrix we determined the factor pattern using the method of factor analysis. We found four common factors the matrix of whose factor loadings are shown in Table 3.

Table 3

factors / indicators	K_1	K_2	K_3	K_4
Indicator 1	0.985	0.230	0.155	0.009
Indicator 2	0.983	0.162	−0.031	0.037
Indicator 3	0.937	−0.084	−0.198	0.007
Indicator 4	0.388	—0.455	0.102	0.005
Indicator 5	0.966	0.083	−0.010	−0.059

The factor loading at the crossing of the K_1 column and the first row shows for example the degree of the correlation among the first factor and indicator 1. The value of the correlation coefficient in question is 0.958 which is indicative of a strong positive correlation. In case the sign of the factor loading is negative, the correlation among the factor and the variable is negative.

Table 4 shows the part of the variance of the five standardized variables considered and explained by the different factors.

Table 4.

Serial numbers of factors	The variance explained by the factor in percentage of the combined variance of all variables	
	The value of the factors	Cumulated sums
1	89.6	89.6
2	6.0	95.6
3	1.5	97.1
4	1.0	98.1

As the first factor explains 89.6% of the combined variance of all the variables the first factor K_1 can rightly be regarded as a synthetic indicator of the state of development of industry which comprises a considerable part of the information represented by the five indicators considered in the investigation. In the present study we did not deal with the analysis of the remaining three factors.

Table 5 shows in what measure the first factor, K_1, contributes to the variance of the variables (indicators).

Table 5

Indicator	Communality	Uniqueness
	%	
Indicator 1	91.8	8.2
Indicator 2	96.8	3.2
Indicator 3	94.7	5.3
Indicator 4	69.4	30.5
Indicator 5	93.5	6.5

According to Table 5, with the exception of the fourth indicator, at least 91.8% of the variance of the other indicators is explained already by the first factor, which justifies again our dealing only with the first factor.

After this we determined the value of the first factor for each area unit considered and on the basis of this classified our subregions according to their state of industrial development. The results are summarized in Table 6.

Table 6

Subregion	Value of K_1	$K_1 + 1{,}826$
Bács	−0.826	1.000
Szabolcs	−0.783	1.043
Békés	−0.416	1.410
Szolnok	−0.336	1.490
Hajdú	−0.260	1.566
Csongrád	−0.240	1.586
Heves	+0.663	2.489
Borsod	+2.200	4.026

It should be noted that we shifted the values the K_1 factor by 1.826 to bring the level of the industrially least developed subregion of Bács to +1.000 and to make thereby comparison easier.

Finally, on the basis of the factors analysis we counted back the correlation coefficients among the different variables and obtained the so-called reproduced reduced correlation matrix:

Indicator 1	0.995				
Indicator 2	0.975	0.996			
Indicator 3	0.882	0.950	0.993		
Indicator 4	0.709	0.742	0.829	0.913	
Indicator 5	0.943	0.962	0.935	0.766	0.945

On this basis the fidelity of the factor analysis can also be assessed, because the difference between the reduced correlation matrix and the reproduced reduced correlation matrix obtained on the basis of the model is practically the zero matrix:

Indicator 1	0.000				
Indicator 2	0.003	0.001			
Indicator 3	0.000	0.003	0.001		
Indicator 4	0.002	0.002	0.002	0.002	
Indicator 5	0.002	0.002	0.003	0.002	0.003

The above table shows that the factors explain well the correlation of the different variables (indicators) among themselves. At the same time this proves the correctness of the basis hypothesis of the factor analysis model and its applicability in our study of the state of development of industry.

Finally we attempted to classify our subregions according to categories of levels of development. Establishing four grades we obtained the following results:

Borsod-Abaúj-Zemplén county	developed
Heves county	medium developed
Csongrád county Hajdú-Bihar county Szolnok county Békés county	underdeveloped
Szabolcs-Szatmár county Bács-Kiskun county	very poorly developed