GLOBAL EXISTENCE OF SOLUTIONS IN INVARIANT REGIONS FOR REACTION-DIFFUSION SYSTEMS WITH A BALANCE LAW AND A FULL MATRIX OF DIFFUSION COEFFICIENTS.

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Abstract. In this paper we generalize a result obtained in [15] concerning uniform boundedness and so global existence of solutions for reaction-diffusion systems with a general full matrix of diffusion coefficients satisfying a balance law. Our techniques are based on invariant regions and Lyapunov functional methods. The nonlinearity of the reaction term which we take positive in an invariant region has been supposed to be polynomial or of weak exponential growth..

1. INTRODUCTION

We consider the following reaction-diffusion system

$$\frac{\partial u}{\partial t} - a\Delta u - b\Delta v = -\sigma f(u,v) \quad \text{in } \mathbb{R}^+ \times \Omega, \tag{1.1}$$

$$\frac{\partial v}{\partial t} - c\Delta u - d\Delta v = \rho f(u,v) \quad \text{in } \mathbb{R}^+ \times \Omega, \tag{1.2}$$

with the boundary conditions

$$\frac{\partial u}{\partial \eta} = \frac{\partial v}{\partial \eta} = 0 \quad \textit{on } \mathbb{R}^+ \times \partial\Omega, \tag{1.3}$$

and the initial data

$$u(0,x) = u_0(x), \qquad v(0,x) = v_0(x) \qquad \textit{in } \Omega, \tag{1.4}$$

where Ω is an open bounded domain of class $\mathbb{C}^1$ in $\mathbb{R}^n$, with boundary $\partial\Omega$, and $\frac{\partial}{\partial \eta}$ denotes the outward normal derivative on $\partial\Omega$, σ and ρ are positive constants. The constants $a,\ b,\ c$ and d are supposed to be positive and $\big((b+c)^2 \leq 4ad\big)$ which reflects the parabolicity of the system and implies at the same time that the matrix of diffusion

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

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is positive definite; that is the eigenvalues λ_1 and λ_2 ($\lambda_1 < \lambda_2$) of its transposed are positive. The initial data are assumed to be in the following region

$$\Sigma = \begin{cases} \left\{(u_0, v_0) \in IR^2 \text{ such that } \mu_2 v_0 \le u_0 \le \mu_1 v_0\right\}, \text{ when } -\mu_2 \le \dfrac{\sigma}{\rho} \\ \\ \left\{(u_0, v_0) \in IR^2 \text{ such that } \dfrac{1}{\mu_2} u_0 \le v_0 \le \dfrac{1}{\mu_1} u_0\right\} \text{ when } -\mu_2 > \dfrac{\sigma}{\rho}. \end{cases}$$

Where

$$\mu_1 \equiv \frac{\underline{a} - \lambda_1}{c} > 0 > \mu_2 \equiv \frac{\underline{a} - \lambda_2}{c}, \; i = 1, 2$$

and

$$\underline{a} = \min\{a, d\}$$

The function $f(r, s)$ is continuously differentiable, nonnegative on Σ with

$$\begin{cases} f(\mu_2 s, s) = 0, \text{ for all } s \ge 0, \text{ when } -\mu_2 \le \dfrac{\sigma}{\rho} \\ \text{and} \\ f(r, \frac{1}{\mu_1} r) = 0, \text{ for all } r \ge 0, \quad \text{when } -\mu_2 > \dfrac{\sigma}{\rho}, \end{cases} \tag{1.5}$$

and

$$\begin{cases} \lim\limits_{s \to +\infty} \left[\frac{log(1+f(r,s))}{s}\right] = 0, \text{ for any } r \ge 0, \text{when } -\mu_2 < \dfrac{\sigma}{\rho}, \\ \text{and} \\ \lim\limits_{r \to +\infty} \left[\frac{log(1+f(r,s))}{r}\right] = 0, \text{ for all } s \ge 0, \text{when } -\mu_2 > \dfrac{\sigma}{\rho}. \end{cases} \tag{1.6}$$

The limit (1.6) is not only valid for functions f polynomially bounded but it also valid for functions with exponential growth as $f(u, v) = ue^{\sqrt{v}}$ or $f(u, v) = ue^{\varepsilon v}$ (see remark 3.3). For the diagonal case (i.e. when $b = c = 0$) and when $\sigma = \rho = 1$, N. Alikakos [1] established global existence and L^∞-bounds of solutions for positive initial data for $f(u, v) = uv^\beta$ and $1 < \beta < \frac{(n+2)}{n}$ and K. Masuda [17] showed that solutions to this system exist globally for every $\beta > 1$ and converge to a constant vector as $t \to +\infty$. A. Haraux and A. Youkana [6] have generalized the method of K.Masuda to nonlinearities $uF(v)$ satisfying (1.6). Recently S. Kouachi and A. Youkana [14] have generalized the method of A. Haraux and A. Youkana to the triangular case, i.e. when $b = 0$ and the limit (1.6) is a small number strictly positive, hypothesis that is in fact, weaker than the last one. This article is a continuation of [15] where $a = d$. In that article the calculations was relatively simple since the system can be regarded as a perturbation of the simple and trivial case where $b = c = 0$; for which nonnegative solutions exist globally in time.

The components $u(t, x)$ and $v(t, x)$ represent either chemical concentrations or biological population densities and system (1.1)-(1.2) is a mathematical model describing various chemical and biological phenomena (see E. L. Cussler [2], P. L. Garcia-Ybarra and P. Clavin [4], S. R. De Groot and P. Mazur [5], J. Jorne [9], J.

S. Kirkaldy [13], A. I. Lee and J. M. Hill [16] and J. Savchik, b. Changs and H. Rabitz[19].

It is well known that, to establish a global existence of unique solutions for (1.1)-(1.3), usual techniques based on Lyapunov functionals which need invariant regions(see M. Kirane and S. Kouachi [11], [12] and S. Kouachi and A. Youkana [14]) are not directly applicable. For this purpose we construct invariant regions.

2. EXISTENCE.

2.1. **Local existence.** The usual norms in spaces $\mathbb{L}^p(\Omega)$, $\mathbb{L}^\infty(\Omega)$ and $\mathbb{C}\left(\overline{\Omega}\right)$ are respectively denoted by :

$$\|u\|_p^p = \frac{1}{|\Omega|}\int_\Omega |u(x)|^p \, dx,$$

$$\|u\|_\infty = \max_{x\in\Omega} |u(x)| \, .$$

For any initial data in $\mathbb{C}\left(\overline{\Omega}\right)$ or $\mathbb{L}^p(\Omega)$, $p \in (1, +\infty)$; local existence and uniqueness of solutions to the initial value problem (1)-(4) follow from the basic existence theory for abstract semilinear differential equations (see A. Friedman [3], D. Henry [7] and Pazy [18]). The solutions are classical on $]0, T^*[$, where T^* denotes the eventual blowing-up time in $\mathbb{L}^\infty(\Omega)$.

2.2. **Invariant regions.**

Proposition 1. *Suppose that the function f is nonnegative on the region Σ and that the condition (1.5) is satisfied, then for any (u_0, v_0) in Σ the solution $(u(t,.), v(t,.))$ of the problem (1.1)-(1.4) remains in Σ for any time and there exists a positive constant M such that*

$$\left\{ \begin{array}{c} \|u - \mu_2 v\|_\infty \leq M, \textit{ when } -\mu_2 < \dfrac{\sigma}{\rho} \\ \textit{and} \\ \|u - \mu_1 v\|_\infty \leq M, \textit{ when } -\mu_2 > \dfrac{\sigma}{\rho}. \end{array} \right. \tag{2.1}$$

Proof. One starts with the case where $-\mu_2 < \dfrac{\sigma}{\rho}$:

Multiplying equation (1.2) one time through by μ_1 and subtracting equation (1.1) and another time by $-\mu_2$ and adding equation (1.1) we get

$$\frac{\partial(\mu_1 v - u)}{\partial t} - \Delta\left[(\mu_1 c - a)u + (\mu_1 d - b)v\right] = (\rho\mu_1 + \sigma)f$$

$$\frac{\partial(-\mu_2 v + u)}{\partial t} - \Delta\left[(-\mu_2 c + a)u + (-\mu_2 d + b)v\right] = -(\rho\mu_2 + \sigma)f.$$

Then if we assume without loss that $a \leq d$, using the fact that λ_1 and λ_2 are the eingenvalues of A^t which implies that

$$\mu_i c - a = -\lambda_i \text{ and } \mu_i d - b = \lambda_i \mu_i, \ i = 1, 2,$$

we get

$$\frac{\partial w}{\partial t} - \lambda_2 \Delta w = -(\rho\mu_2 + \sigma)F(w,z) \tag{2.2}$$

$$\frac{\partial z}{\partial t} - \lambda_1 \Delta z = (\rho\mu_1 + \sigma)F(w,z) \tag{2.3}$$

with the boundary conditions

$$\frac{\partial w}{\partial \eta} = \frac{\partial z}{\partial \eta} = 0 \qquad \text{on }]0,T^*[\times \partial\Omega\ , \tag{2.4}$$

and the initial data

$$w(0,x) = w_0(x), \qquad z(0,x) = z_0(x) \qquad \textit{in } \Omega, \tag{2.5}$$

where

$$w(t,x) = (u - \mu_2 v)(t,x) \text{ and } z(t,x) = (-u + \mu_1 v)(t,x) \tag{2.6}$$

for any (t,x) *in* $]0,T^*[\times\Omega$, and

$$F(w,z) = f(u,v).$$

Now, it suffices to prove that the region

$$\left\{(w_0,z_0) \in IR^2 \text{ such that } w_0 \geq 0, z_0 \geq 0\right\} = IR^+ \times IR^+,$$

is invariant for system (2.2)-(2.3) and $w(t,x)$ is uniformly bounded in $]0,T^*[\times\Omega$. Since, from (1.5), $F(0,z) = f(\mu_2 v, v) = 0$ for all $z \geq 0$ and all $v \geq 0$, then $w(t,x) \geq 0$ for all $(t,x) \in]0,T^*[\times\Omega$, thanks to the invariant region's method (see Smoller [20]) and because $F(w,z) \geq 0$ for all (w,z) in $IR^+ \times IR^+$ and $z_0(x) \geq 0$ in Ω, we can deduce by the same method applied to equation (2.3), that

$$z(t,x) = (-u + \mu_1 v)(t,x) \geq 0 \text{ in }]0,T^*[\times\Omega;$$

then Σ is an invariant region for the system (1.1)-(1.3).
At the end, to show that $w(t,x)$ is uniformly bounded on $]0,T^*[\times\Omega$, since $-\mu_2 < \dfrac{\sigma}{\rho}$, it is sufficient to apply the maximum's principle directly to equation (2.2).

For the case $-\mu_2 > \dfrac{\sigma}{\rho}$, the same reasoning with equations

$$\frac{\partial w}{\partial t} - \lambda_1 \Delta w = -(\rho\mu_1 + \sigma)F(w,z) \tag{2.2$'$}$$

$$\frac{\partial z}{\partial t} - \lambda_2 \Delta z = -(\rho\mu_2 + \sigma)F(w,z), \tag{2.3$'$}$$

with the same boundary condition (2.4) implies the invariance of $IR^+ \times IR^+$ and the uniform boundedness of $w(t,x)$ on $]0,T^*[\times\Omega$, where in this case we take

$$w(t,x) = (u - \mu_1 v)(t,x) \text{ and } z(t,x) = (u - \mu_2 v)(t,x), \tag{2.6$'$}$$

for all (t,x) in $]0,T^*[\times\Omega$. ∎

Once, invariant regions are constructed, one can apply Lyapunov technique and establish global existence of unique solutions for (1.1)-(1.4).

2.3. **Global existence.** As the determinant of the linear algebraic system (2.6) or $(2.6)^{'}$, with regard to variables u and v, is different from zero, then to prove global existence of solutions of problem (1.1)-(1.4) reduced to proving it for problem (2.2)-(2.5). For this purpose, it is well known that (see Henry [7]) it suffices to derive a uniform estimate of $\|F(w,z)\|_p$ on $[0,T^*[$ for some $p>n/2$.

The main result and in some sense the heart of the paper is:

Theorem 2. *Let $(w(t,.),z(t,.))$ be any solution of system (2.2)-(2.3) (respectively $(2.2)^{'}-(2.3)^{'}$) with initial data in $IR^+\times IR^+$ and boundary conditions (2.4), then the functional*

$$t\longrightarrow L(t)=\int_\Omega (M-w(t,x))^{-\gamma}\exp\beta z(t,x)dx \tag{2.7}$$

is nonincreasing on $[0,T^*[$, for all positive constants β and γ such that

$$\beta\mu M<\gamma<\frac{4\lambda_1\lambda_2}{(\lambda_1-\lambda_2)^2}, \tag{2.8}$$

and all M satisfying

$$\|w_0\|_\infty<M, \tag{2.9}$$

where $\mu=\dfrac{(\rho\mu_1+\sigma)}{(\rho\mu_2+\sigma)}$ (respectively $\mu=-\dfrac{(\rho\mu_2+\sigma)}{(\rho\mu_1+\sigma)}$) and where $w(t,x)$ and $z(t,x)$ are given by (2.6) (respectively $(2.6)^{'}$).

Proof. Let's demonstrate the theorem in the case $-\mu_2<\dfrac{\sigma}{\rho}$.

Differentiating L with respect to t yields:

$$\dot{L}(t)=\int_\Omega\left[\gamma\left((M\text{-}w)^{\text{-}\gamma\text{-}1}e^{\beta z}\right)\frac{\partial w}{\partial t}+\left(\beta(M\text{-}w)^{\text{-}\gamma}e^{\beta z}\right)\frac{\partial z}{\partial t}\right]dx$$

$$=\int_\Omega\left(\gamma(M\text{-}w)^{\text{-}\gamma\text{-}1}e^{\beta z}\right)\left(\lambda_2\Delta w-(\rho\mu_2+\sigma)F(w,z)\right)dx+$$
$$\int_\Omega\left(\beta(M\text{-}w)^{\text{-}\gamma}e^{\beta z}\right)\left(\lambda_1\Delta z+(\rho\mu_1+\sigma)F(w,z)\right)dx$$

$$= \int_\Omega \left[\gamma\lambda_2(M\text{-}w)^{\text{-}\gamma\text{-}1}e^{\beta z}\Delta w + \beta\lambda_1(M\text{-}w)^{\text{-}\gamma}e^{\beta z}\Delta z\right]dx$$

$$+ \int_\Omega \left[(\rho\mu_1+\sigma)\beta(M\text{-}w)^{\text{-}\gamma} - (\rho\mu_2+\sigma)\gamma(M\text{-}w)^{\text{-}\gamma\text{-}1}\right]e^{\beta z}F(w,z)dx$$

$$= I + J.$$

By simple use of Green's formula, we get

$$I = \text{-}\int_\Omega T(\nabla w, \nabla z)(M-w)^{\text{-}\gamma\text{-}2}e^{\beta z}dx,$$

where

$$\begin{aligned} T(\nabla w, \nabla z) &= \lambda_2\gamma(\gamma+1)\left|\nabla w\right|^2 + \\ &\quad \beta(M-w)(\lambda_1+\lambda_2)\gamma\nabla w\nabla z + \\ &\quad \lambda_1\beta^2(M-w)^2\left|\nabla z\right|^2. \end{aligned}$$

The discriminant of T is given by:

$$\begin{aligned} D &= \left[((\lambda_1+\lambda_2)\gamma)^2 - 4\lambda_2\lambda_1\gamma(\gamma+1)\right]\beta^2(M-w)^2 \\ &= \left((\lambda_1-\lambda_2)^2\gamma^2 - 4\lambda_1\lambda_2\gamma\right)\beta^2(M-w)^2. \end{aligned}$$

$D < 0$, if

$$\gamma > 0 \text{ and } (\lambda_1-\lambda_2)^2\gamma - 4\lambda_1\lambda_2 < 0.$$

Theses last two inequalities can be written as follows:

$$0 < \gamma < \frac{4\lambda_1\lambda_2}{(\lambda_1-\lambda_2)^2}.$$

Using the following inequality

$$\xi_1 x^2 + \xi_2 xy + \xi_3 y^2 \le \text{-}\frac{\left(\xi_2^2\text{-}4\xi_1\xi_3\right)}{2}\left[\frac{y^2}{4\xi_1} + \frac{x^2}{4\xi_3}\right] \quad \text{for all } (x,y) \in \mathbb{R}^2,$$

where ξ_1 and ξ_3 are two negative constants and $\xi_2 \in \mathbb{R}$, we can show that

$$I \le \text{-}\int_\Omega \left(\mathbf{m_1}\left|\nabla w\right|^2 + \mathbf{m}_2\left|\nabla z\right|^2\right)(M\text{-}w)^{\text{-}\gamma\text{-}2}e^{\beta z}dx,$$

where the positive constants $\mathbf{m}_1$ and $\mathbf{m}_2$ are given by:

$$\mathbf{m}_1 = \frac{\left(4\lambda_1\lambda_2\text{-}\gamma\left(\lambda_1\text{-}\lambda_2\right)^2\right)\gamma}{8\lambda_1}$$

$$\mathbf{m}_2 = \frac{\left(4\lambda_1\lambda_2\text{-}\gamma\left(\lambda_1\text{-}\lambda_2\right)^2\right)\beta^2(M-\|w_0\|_\infty)^2}{8\lambda_2(\gamma+1)}.$$

For the second integral, we have

$$J = \int_\Omega \left(\beta M(\rho\mu_1+\sigma) - \gamma(\rho\mu_2+\sigma)\right)(M\text{-}w)^{\text{-}\gamma\text{-}1} e^{\beta z} F(w,z)dx,$$

if we choose

$$\beta < \frac{(\rho\mu_2+\sigma)\gamma}{(\rho\mu_1+\sigma)M},$$

then

$$J \leq -C(\beta,\gamma,\mu_1,\mu_2,M)\int_\Omega (M-w)^{\text{-}\gamma\text{-}1} e^{\beta z} F(w,z)dx,$$

where $C(\beta,\gamma,\mu_1,\mu_2,M)$ is a positive constant.
Hence,

$$\begin{aligned} \dot{L}(t) \quad \leq \quad & \text{-}\int_\Omega \left(\mathbf{m_1}\left|\nabla w\right|^2 + \mathbf{m}_2\left|\nabla z\right|^2\right)(M\text{-}w)^{\text{-}\gamma\text{-}2} e^{\beta z} dx \\ & \text{-}C(\beta,\gamma,\mu_1,\mu_2,M)\int_\Omega (M\text{-}w)^{\text{-}\gamma\text{-}1} e^{\beta z} F(w,z)dx \leq 0. \end{aligned}$$

Concerning the case $-\mu_2 > \dfrac{\sigma}{\rho}$, the same reasoning with equations $(2.2)^{'}$ and $(2.3)^{'}$ implies that the functional given by (2.7) is nonincreasing on $[0, T^*[$, for all positive constants β, γ and M satisfying $(2.8)^{'}$ and (2.9).

Theorem 2.2 is completely proved. ∎

Corollary 3. *Suppose that the function $f(r,s)$ is continuously differentiable, nonnegative on ⊁ and satisfying conditions (1.5) and (1.6) .Then all solutions of (1.1)-(1.3) with initial data in Σ are global in time and uniformly bounded on $(0,+\infty)\times\Omega$.*

Proof. Let us take $-\mu_2 < \dfrac{\sigma}{\rho}$, as it has been mentioned in the beginning of section 1.3; it suffices to derive a uniform estimate of $\|F(w,z)\|_p$ on $[0, T^*[$ for some $p > n/2$. Since, for u and v in Σ, $w \geq 0$ and $z \geq 0$, and as $w+z = (\mu_1-\mu_2)v$ with w uniformly bounded on $[0, T^*[\times \Omega$ by M and $\mu_1 > \mu_2$, then (1.6) is equivalent to

$$\lim_{s\to+\infty}\left[\frac{log\left(1+F(r,s)\right)}{s}\right] = 0, \text{ for all } r \geq 0.$$

As F is continuous on $\mathbb{R}^+ \times \mathbb{R}^+$, then

$$\lim_{s\to+\infty}\left[\frac{log\,(1+F(r,s))}{s}\right]=0,$$

uniformly for $r\in[0,M]$ and we can choose positive constants α and C such that:

$$1+F(r,s)\leq Ce^{\alpha s},\quad \text{for all } s\geq 0 \text{ and for all } r\in[0,M]\,, \tag{2.10}$$

and

$$\alpha<\frac{8\lambda_1\lambda_2(\rho\mu_2+\sigma)}{n(\lambda_1-\lambda_2)^2(\rho\mu_1+\sigma)\,\|w_0\|_\infty},$$

then we can choose $p>n/2$ such that

$$p\alpha<\frac{4\lambda_1\lambda_2(\rho\mu_2+\sigma)}{(\lambda_1-\lambda_2)^2(\rho\mu_1+\sigma)\,\|w_0\|_\infty}. \tag{2.11}$$

Set $\beta=p\alpha$, hence

$$\beta\,\|w_0\|_\infty<\frac{4\lambda_1\lambda_2(\rho\mu_2+\sigma)}{(\lambda_1-\lambda_2)^2(\rho\mu_1+\sigma)}, \tag{2.12}$$

thus we can choose γ and M such that (2.8) and (2.9) are satisfied. Using Theorem 2.2 we get,

$$e^{\beta z(t,.)}=\left(e^{\alpha z(t,.)}\right)^p\in\mathbb{L}^\infty\left([0,T^*[\,;\mathbb{L}^1(\Omega)\right),$$

therefore

$$e^{\alpha z(t,.)}\in\mathbb{L}^\infty\left([0,T^*[\,;\mathbb{L}^p(\Omega)\right),$$

and from (2.10) we deduce that

$$f(u(t,.),v(t,.))\equiv F(w(t,.),z(t,.))\in\mathbb{L}^\infty\left([0,T^*[\,;\mathbb{L}^p(\Omega)\right),\qquad \text{for some } p>n/2.$$

By the preliminary remarks, we conclude that the solution is global and uniformly bounded on $[0,+\infty[\,\times\Omega$.

For the case $-\mu_2>\dfrac{\sigma}{\rho}$, the same reasoning with w and z given by $(2.6)'$ and using the limit (1.6) we deduce the same result. ∎

3. Remarks and comments

Remark 1. *In the case when* $-\mu_2=\dfrac{\sigma}{\rho}$ *and initial data given in* Σ *(defined in the case when* $-\mu_2>\dfrac{\sigma}{\rho}$ *) we have global existence of solutions of problem (1.1)-(1.4) without any condition on the constants or on the growth of the function* f *to part* *its positivity and* $f(r, \frac{1}{\mu_1} r) = 0$*, for all* $r \geq 0$*. To verify this, it suffices to apply the maximum principle directly to equations (2.2)′ − (2.3)′.*

Remark 2. *when the condition* $f(r, \frac{1}{\mu_1} r) = 0$*, for all* $r \geq 0$ *is not satisfied, then by application of the comparison's principle to equation (2.3), blow up in finite time can occur in the case where* $-\mu_2 = \dfrac{\sigma}{\rho}$*, especially when the reaction term satisfies an inequality of the form:*

$$|f(u,v)| \geq C_1 |u|^{\alpha_1} + C_2 |v|^{\alpha_2},$$

where C_1*,* C_2*,* α_1 *and* α_2 *are positive constants such that*

$$C_1^2 + C_2^2 \neq 0, \ \alpha_1 \ > 1 \text{ and } \alpha_2 \ > 1.$$

Remark 3. *One showed the global existence for functions* $f(u,v)$ *of polynomial growth (condition 1.6), but our results remain applicable for functions of exponential growth (but small) while replacing the condition 1.6 by:*

$$\begin{cases} \lim\limits_{s\to+\infty} \left[\frac{log(1+f(r,s))}{s}\right] < \dfrac{8\lambda_1\lambda_2(\rho\mu_2+\sigma)}{n(\lambda_1-\lambda_2)^2(\rho\mu_1+\sigma)\left\|w_0\right\|_\infty}, \text{ for any } r \geq 0, \text{ when } -\mu_2 < \dfrac{\sigma}{\rho}, \\ \text{and} \\ \lim\limits_{r\to+\infty} \left[\frac{log(1+f(r,s))}{r}\right] < \dfrac{-8\lambda_1\lambda_2(\rho\mu_1+\sigma)}{n(\lambda_1-\lambda_2)^2(\rho\mu_2+\sigma)\left\|w_0\right\|_\infty}, \text{ for any } s \geq 0, \text{ when } -\mu_2 > \dfrac{\sigma}{\rho}. \end{cases}$$

Remark 4. *If* $\lambda_1 = \lambda_2$ *or* $-\mu_2 = \dfrac{\sigma}{\rho}$ *we have global existence for any exponential growth.*

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