

Positive Solutions for Systems of n th Order Three-point Nonlocal Boundary Value Problems

J. Henderson¹ and S. K. Ntouyas²

¹ Department of Mathematics, Baylor University
Waco, Texas 76798-7328 USA
e-mail: Johnny_Henderson@baylor.edu

² Department of Mathematics, University of Ioannina
451 10 Ioannina, Greece
email: sntouyas@uoi.gr

Abstract

Intervals of the parameter λ are determined for which there exist positive solutions for the system of nonlinear differential equations, $u^{(n)} + \lambda a(t)f(v) = 0$, $v^{(n)} + \lambda b(t)g(u) = 0$, for $0 < t < 1$, and satisfying three-point nonlocal boundary conditions, $u(0) = 0, u'(0) = 0, \dots, u^{(n-2)}(0) = 0$, $u(1) = \alpha u(\eta), v(0) = 0, v'(0) = 0, \dots, v^{(n-2)}(0) = 0$, $v(1) = \alpha v(\eta)$. A Guo-Krasnosel'skii fixed point theorem is applied.

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1 Introduction

We are concerned with determining intervals of the parameter λ (eigenvalues) for which there exist positive solutions for the system of differential equations,

$$\begin{aligned}u^{(n)} + \lambda a(t)f(v) &= 0, & 0 < t < 1, \\v^{(n)} + \lambda b(t)g(u) &= 0, & 0 < t < 1,\end{aligned}\tag{1}$$

satisfying the three-point nonlocal boundary conditions,

$$\begin{aligned}u(0) = 0, u'(0) = 0, \dots, u^{(n-2)}(0) = 0, & \quad u(1) = \alpha u(\eta), \\v(0) = 0, v'(0) = 0, \dots, v^{(n-2)}(0) = 0, & \quad v(1) = \alpha v(\eta),\end{aligned}\tag{2}$$

where $0 < \eta < 1, 0 < \alpha\eta^{n-1} < 1$ and

- (A) $f, g \in C([0, \infty), [0, \infty))$,
- (B) $a, b \in C([0, 1], [0, \infty))$, and each does not vanish identically on any subinterval,
- (C) All of $f_0 := \lim_{x \rightarrow 0^+} \frac{f(x)}{x}$, $g_0 := \lim_{x \rightarrow 0^+} \frac{g(x)}{x}$, $f_\infty := \lim_{x \rightarrow \infty} \frac{f(x)}{x}$ and $g_\infty := \lim_{x \rightarrow \infty} \frac{g(x)}{x}$ exist as real numbers.

There is currently a great deal of interest in positive solutions for several types of boundary value problems. While some of the interest has focused on theoretical questions [5, 9, 13, 26], an equal amount of interest has been devoted to applications for which only positive solutions have meaning [1, 8, 17, 18]. While most of the above studies have dealt with scalar problems, some recent work has addressed questions of positive solutions for systems of boundary value problems [3, 12, 14, 15, 16, 19, 22, 25, 27, 30]. In addition, some studies have been directed toward positive solutions for nonlocal boundary value problems; see, for example, [4, 6, 10, 17, 18, 19, 21, 22, 20, 24, 26, 28, 29, 30].

Additional attention has been directed toward extensions to higher order problems, such as in [2, 4, 7, 8, 11, 23, 29]. Recently Benchohra *et al.* [3] and Henderson and Ntouyas [12] studied the existence of positive solutions of systems of nonlinear eigenvalue problems. Here we extend these results to eigenvalue problems for systems of higher order three-point nonlocal boundary value problems.

The main tool in this paper is an application of the Guo-Krasnosel'skii fixed point theorem for operators leaving a Banach space cone invariant [9]. A Green's function plays a fundamental role in defining an appropriate operator on a suitable cone.

2 Some preliminaries

In this section, we state some preliminary lemmas and the well-known Guo-Krasnosel'skii fixed point theorem.

Lemma 2.1 [4] *Let $0 < \eta < 1, 0 < \alpha\eta^{n-1} < 1$; then for any $u \in C[0, 1]$ the following boundary value problem*

$$u^{(n)}(t) = 0, \quad 0 < t < 1 \tag{3}$$

$$u(0) = 0, u'(0) = 0, \dots, u^{(n-2)}(0) = 0, \quad u(1) = \alpha u(\eta), \tag{4}$$

has a unique solution

$$u(t) = \int_0^1 k(t, s)u^{(n)}(s)ds$$

where $k(t, s) : [0, 1] \times [0, 1] \rightarrow \mathbb{R}^+$ is defined by

$$k(t, s) = \begin{cases} \frac{a(\eta, s)t^{n-1}}{(n-1)!}, & 0 \leq t \leq s \leq 1, \\ \frac{a(\eta, s)t^{n-1} + (t-s)^{n-1}}{(n-1)!}, & 0 \leq s \leq t \leq 1, \end{cases} \quad (5)$$

and

$$a(\eta, s) = \begin{cases} -\frac{(1-s)^{n-1}}{1-\alpha\eta^{n-1}}, & \eta \leq s, \\ -\frac{(1-s)^{n-1} - (\eta-s)^{n-1}}{1-\alpha\eta^{n-1}}, & s \leq \eta. \end{cases}$$

Lemma 2.2 [4] Let $0 < \alpha^{n-1} < 1$. Let u satisfy $u^{(n)}(t) \leq 0, 0 < t < 1$, with the nonlocal conditions (2). Then

$$\inf_{t \in [\eta, 1]} u(t) \geq \gamma \|u\|,$$

where $\gamma = \min \left\{ \alpha\eta^{n-1}, \frac{\alpha(1-\eta)}{1-\alpha\eta}, \eta^{n-1} \right\}$.

Define $\theta(s) = \max_{t \in [0, 1]} |k(t, s)|$. From Lemma 1.2 in [4], we know that

$$|k(t, s)| \geq \gamma\theta(s), \quad t \in [\eta, 1], \quad s \in [0, 1]. \quad (6)$$

By simple calculation we have (see [11])

$$\theta(s) = \max_{t \in [0, 1]} |k(t, s)| \leq \frac{(1-s)^{n-1}}{(1-\alpha\eta^{n-1})(n-1)!}, \quad s \in (0, 1). \quad (7)$$

We note that a pair $(u(t), v(t))$ is a solution of eigenvalue problem (1), (2) if, and only if,

$$u(t) = -\lambda \int_0^1 k(t, s)a(s)f \left(-\lambda \int_0^1 k(s, r)b(r)g(u(r))dr \right) ds, \quad 0 \leq t \leq 1, \quad (8)$$

where

$$v(t) = -\lambda \int_0^1 k(t, s)b(s)g(u(s))ds, \quad 0 \leq t \leq 1.$$

Values of λ for which there are positive solutions (positive with respect to a cone) of (1), (2) will be determined via applications of the following fixed point theorem.

Theorem 2.1 *Let \mathcal{B} be a Banach space, and let $\mathcal{P} \subset \mathcal{B}$ be a cone in \mathcal{B} . Assume Ω_1 and Ω_2 are open subsets of \mathcal{B} with $0 \in \Omega_1 \subset \overline{\Omega}_1 \subset \Omega_2$, and let*

$$T : \mathcal{P} \cap (\overline{\Omega}_2 \setminus \Omega_1) \rightarrow \mathcal{P}$$

be a completely continuous operator such that, either

- (i) $\|Tu\| \leq \|u\|, u \in \mathcal{P} \cap \partial\Omega_1$, and $\|Tu\| \geq \|u\|, u \in \mathcal{P} \cap \partial\Omega_2$, or
- (ii) $\|Tu\| \geq \|u\|, u \in \mathcal{P} \cap \partial\Omega_1$, and $\|Tu\| \leq \|u\|, u \in \mathcal{P} \cap \partial\Omega_2$.

Then T has a fixed point in $\mathcal{P} \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

3 Positive solutions in a cone

In this section, we apply Theorem 2.1 to obtain solutions in a cone (that is, positive solutions) of (1), (2). For our construction, let $\mathcal{B} = C[0, 1]$ with supremum norm, $\|\cdot\|$, and define a cone $\mathcal{P} \subset \mathcal{B}$ by

$$\mathcal{P} = \left\{ x \in \mathcal{B} \mid x(t) \geq 0 \text{ on } [0, 1], \text{ and } \min_{t \in [\eta, 1]} x(t) \geq \gamma \|x\| \right\}.$$

For our first result, define positive numbers L_1 and L_2 by

$$L_1 := \max \left\{ \left[\gamma^2 \int_{\eta}^1 \theta(r)a(r)f_{\infty} dr \right]^{-1}, \left[\gamma^2 \int_{\eta}^1 \theta(r)a(r)g_{\infty} dr \right]^{-1} \right\},$$

and

$$L_2 := \min \left\{ \left[\int_0^1 \theta(r)a(r)f_0 dr \right]^{-1}, \left[\int_0^1 \theta(r)b(r)g_0 dr \right]^{-1} \right\}.$$

Theorem 3.1 *Assume conditions (A), (B) and (C) are satisfied. Then, for each λ satisfying*

$$L_1 < \lambda < L_2, \tag{9}$$

there exists a pair (u, v) satisfying (1), (2) such that $u(x) > 0$ and $v(x) > 0$ on $(0, 1)$.

Proof. Let λ be as in (9). And let $\epsilon > 0$ be chosen such that

$$\max \left\{ \left[\gamma^2 \int_{\eta}^1 \theta(r)a(r)(f_{\infty} - \epsilon) dr \right]^{-1}, \left[\gamma^2 \int_{\eta}^1 \theta(r)a(r)(g_{\infty} - \epsilon) dr \right]^{-1} \right\} \leq \lambda$$

and

$$\lambda \leq \min \left\{ \left[\int_0^1 \theta(r)a(r)(f_0 + \epsilon)dr \right]^{-1}, \left[\int_0^1 \theta(r)b(r)(g_0 + \epsilon)dr \right]^{-1} \right\}.$$

Define an integral operator $T : \mathcal{P} \rightarrow \mathcal{B}$ by

$$Tu(t) := -\lambda \int_0^1 k(t,s)a(s)f \left(-\lambda \int_0^1 k(s,r)b(r)g(u(r))dr \right) ds, \quad u \in \mathcal{P}. \quad (10)$$

We seek suitable fixed points of T in the cone \mathcal{P} .

By Lemma 2.2, $T\mathcal{P} \subset \mathcal{P}$. In addition, standard arguments show that T is completely continuous.

Now, from the definitions of f_0 and g_0 , there exists an $H_1 > 0$ such that

$$f(x) \leq (f_0 + \epsilon)x \text{ and } g(x) \leq (g_0 + \epsilon)x, \quad 0 < x \leq H_1.$$

Let $u \in \mathcal{P}$ with $\|u\| = H_1$. We first have from (7) and choice of ϵ ,

$$\begin{aligned} -\lambda \int_0^1 k(s,r)b(r)g(u(r))dr &\leq \lambda \int_0^1 \theta(r)b(r)g(u(r))dr \\ &\leq \lambda \int_0^1 \theta(r)b(r)(g_0 + \epsilon)u(r)dr \\ &\leq \lambda \int_0^1 \theta(r)b(r)dr(g_0 + \epsilon)\|u\| \\ &\leq \|u\| \\ &= H_1. \end{aligned}$$

As a consequence, we next have from (7), and choice of ϵ ,

$$\begin{aligned} Tu(t) &= -\lambda \int_0^1 k(t,s)a(s)f \left(-\lambda \int_0^1 k(s,r)b(r)g(u(r))dr \right) ds \\ &\leq \lambda \int_0^1 \theta(s)a(s)f \left(-\lambda \int_0^1 k(s,r)b(r)g(u(r))dr \right) ds \\ &\leq \lambda \int_0^1 \theta(s)a(s)(f_0 + \epsilon) \left[-\lambda \int_0^1 k(s,r)b(r)g(u(r))dr \right] ds \\ &\leq \lambda \int_0^1 \theta(s)a(s)(f_0 + \epsilon)H_1 ds \\ &\leq H_1 \\ &= \|u\|. \end{aligned}$$

So, $\|Tu\| \leq \|u\|$. If we set

$$\Omega_1 = \{x \in \mathcal{B} \mid \|x\| < H_1\},$$

then

$$\|Tu\| \leq \|u\|, \text{ for } u \in \mathcal{P} \cap \partial\Omega_1. \quad (11)$$

Next, from the definitions of f_∞ and g_∞ , there exists $\overline{H}_2 > 0$ such that

$$f(x) \geq (f_\infty - \epsilon)x \text{ and } g(x) \geq (g_\infty - \epsilon)x, \quad x \geq \overline{H}_2.$$

Let

$$H_2 = \max \left\{ 2H_1, \frac{\overline{H}_2}{\gamma} \right\}.$$

Let $u \in \mathcal{P}$ and $\|u\| = H_2$. Then,

$$\min_{t \in [\eta, 1]} u(t) \geq \gamma \|u\| \geq \overline{H}_2.$$

Consequently, from (8) and choice of ϵ ,

$$\begin{aligned} -\lambda \int_0^1 k(s, r)b(r)g(u(r))dr &\geq \lambda\gamma \int_\eta^1 \theta(r)b(r)g(u(r))dr \\ &\geq \lambda\gamma \int_\eta^1 \theta(r)b(r)g(u(r))dr \\ &\geq \lambda\gamma \int_\eta^1 \theta(r)b(r)(g_\infty - \epsilon)u(r)dr \\ &\geq \lambda\gamma \int_\eta^1 \theta(r)b(r)(g_\infty - \epsilon)dr\gamma \|u\| \\ &\geq \|u\| \\ &= H_2. \end{aligned}$$

And so, we have from (8) and choice of ϵ ,

$$\begin{aligned} Tu(\eta) &\geq \lambda\gamma \int_\eta^1 \theta(s)a(s)f \left(-\lambda \int_\eta^1 k(s, r)b(r)g(u(r))dr \right) ds \\ &\geq \lambda\gamma \int_\eta^1 \theta(s)a(s)(f_\infty - \epsilon) \left[-\lambda \int_\eta^1 k(s, r)b(r)g(u(r))dr \right] ds \\ &\geq \lambda\gamma \int_\eta^1 \theta(s)a(s)(f_\infty - \epsilon)H_2 ds \end{aligned}$$

$$\begin{aligned}
&\geq \lambda\gamma^2 \int_{\eta}^1 \theta(s)a(s)(f_{\infty} - \epsilon)H_2 ds \\
&\geq H_2 \\
&= \|u\|.
\end{aligned}$$

Hence, $\|Tu\| \geq \|u\|$. So, if we set

$$\Omega_2 = \{x \in \mathcal{B} \mid \|x\| < H_2\},$$

then

$$\|Tu\| \geq \|u\|, \text{ for } u \in \mathcal{P} \cap \partial\Omega_2. \tag{12}$$

Applying Theorem 2.1 to (11) and (12), we obtain that T has a fixed point $u \in \mathcal{P} \cap (\overline{\Omega}_2 \setminus \Omega_1)$. As such, and with v defined by

$$v(t) = -\lambda \int_0^1 k(t, s)b(s)g(u(s))ds,$$

the pair (u, v) is a desired solution of (1), (2) for the given λ . The proof is complete. \square

Prior to our next result, we introduce another hypothesis.

(D) $g(0) = 0$ and f is an increasing function.

We now define positive numbers L_3 and L_4 by

$$L_3 := \max \left\{ \left[\gamma^2 \int_{\eta}^1 \theta(r)a(r)f_0 dr \right]^{-1}, \left[\gamma^2 \int_{\eta}^1 \theta(r)a(r)g_0 dr \right]^{-1} \right\},$$

and

$$L_4 := \min \left\{ \left[\int_0^1 \theta(r)a(r)f_{\infty} dr \right]^{-1}, \left[\int_0^1 \theta(r)b(r)g_{\infty} dr \right]^{-1} \right\}.$$

Theorem 3.2 *Assume conditions (A)–(D) are satisfied. Then, for each λ satisfying*

$$L_3 < \lambda < L_4, \tag{13}$$

there exists a pair (u, v) satisfying (1), (2) such that $u(x) > 0$ and $v(x) > 0$ on $(0, 1)$.

Proof. Let λ be as in (13). And let $\epsilon > 0$ be chosen such that

$$\max \left\{ \left[\gamma^2 \int_{\eta}^1 \theta(r)a(r)(f_0 - \epsilon) dr \right]^{-1}, \left[\gamma^2 \int_{\eta}^1 \theta(r)a(r)(g_0 - \epsilon) dr \right]^{-1} \right\} \leq \lambda$$

and

$$\lambda \leq \min \left\{ \left[\int_0^1 \theta(r)a(r)(f_\infty + \epsilon)dr \right]^{-1}, \left[\int_0^1 \theta(r)b(r)(g_\infty + \epsilon)dr \right]^{-1} \right\}.$$

Let T be the cone preserving, completely continuous operator that was defined by (10).

From the definitions of f_0 and g_0 , there exists $H_1 > 0$ such that

$$f(x) \geq (f_0 - \epsilon)x \text{ and } g(x) \geq (g_0 - \epsilon)x, \quad 0 < x \leq H_1.$$

Now $g(0) = 0$ and so there exists $0 < H_2 < H_1$ such that

$$\lambda g(x) \leq \frac{H_1}{\int_0^1 \theta(r)b(r)dr}, \quad 0 \leq x \leq H_2.$$

Choose $u \in \mathcal{P}$ with $\|u\| = H_2$. Then

$$\begin{aligned} -\lambda \int_0^1 k(s,r)b(r)g(u(r))dr &\leq \lambda \int_0^1 \theta(r)b(r)g(u(r))dr \\ &\leq \lambda \int_0^1 \theta(r)b(r)g(u(r))dr \\ &\leq \frac{\int_0^1 \theta(r)b(r)H_1dr}{\int_0^1 \theta(r)b(s)ds} \\ &\leq H_1. \end{aligned}$$

Then, by (8) and (D)

$$\begin{aligned} Tu(\eta) &\geq \lambda\gamma \int_\eta^1 \theta(s)a(s)f \left(\lambda\gamma \int_\eta^1 \theta(r)b(r)g(u(r))dr \right) ds \\ &\geq \lambda\gamma \int_\eta^1 \theta(s)a(s)(f_0 - \epsilon)\lambda\gamma \int_\eta^1 \theta(r)b(r)g(u(r))drds \\ &\geq \lambda\gamma \int_\eta^1 \theta(s)a(s)(f_0 - \epsilon)\lambda\gamma^2 \int_\eta^1 \theta(r)b(r)(g_0 - \epsilon)\|u\|drds \\ &\geq \lambda\gamma \int_\eta^1 \theta(s)a(s)(f_0 - \epsilon)\|u\|ds \\ &\geq \lambda\gamma^2 \int_\eta^1 \theta(s)a(s)(f_0 - \epsilon)\|u\|ds \\ &\geq \|u\|. \end{aligned}$$

So, $\|Tu\| \geq \|u\|$. If we put

$$\Omega_1 = \{x \in \mathcal{B} \mid \|x\| < H_2\},$$

then

$$\|Tu\| \geq \|u\|, \text{ for } u \in \mathcal{P} \cap \partial\Omega_1. \quad (14)$$

Next, by definitions of f_∞ and g_∞ , there exists \overline{H}_1 such that

$$f(x) \leq (f_\infty + \epsilon)x \text{ and } g(x) \leq (g_\infty + \epsilon)x, \quad x \geq \overline{H}_1.$$

There are two cases, (a) g is bounded, and (b) g is unbounded.

For case (a), suppose $N > 0$ is such that $g(x) \leq N$ for all $0 < x < \infty$. Then, for $u \in \mathcal{P}$

$$-\lambda \int_0^1 k(s, r)b(r)g(u(r))dr \leq N\lambda \int_0^1 \theta(r)b(r)dr.$$

Let

$$M = \max \left\{ f(x) \mid 0 \leq x \leq N\lambda \int_0^1 \theta(r)b(r)dr \right\},$$

and let

$$H_3 > \max \left\{ 2H_2, M\lambda \int_0^1 \theta(s)a(s)ds \right\}.$$

Then, for $u \in \mathcal{P}$ with $\|u\| = H_3$,

$$\begin{aligned} Tu(t) &\leq \lambda \int_0^1 \theta(s)a(s)Mds \\ &\leq H_3 \\ &= \|u\|, \end{aligned}$$

so that $\|Tu\| \leq \|u\|$. If

$$\Omega_2 = \{x \in \mathcal{B} \mid \|x\| < H_3\},$$

then

$$\|Tu\| \leq \|u\|, \text{ for } u \in \mathcal{P} \cap \partial\Omega_2. \quad (15)$$

For case (b), there exists $H_3 > \max\{2H_2, \overline{H}_1\}$ such that $g(x) \leq g(H_3)$, for $0 < x \leq H_3$. Similarly, there exists $H_4 > \max\left\{H_3, \lambda \int_0^1 \theta(r)b(r)g(H_3)dr\right\}$ such that $f(x) \leq f(H_4)$, for $0 < x \leq H_4$. Choosing $u \in \mathcal{P}$ with $\|u\| = H_4$, we have by (D) that

$$Tu(t) \leq \lambda \int_0^1 \theta(s)a(s)f \left(\lambda \int_0^1 \theta(r)b(r)g(H_3)dr \right) ds$$

$$\begin{aligned}
&\leq \lambda \int_0^1 \theta(s)a(s)f(H_4)ds \\
&\leq \lambda \int_0^1 \theta(s)a(s)ds(f_\infty + \epsilon)H_4 \\
&\leq H_4 \\
&= \|u\|,
\end{aligned}$$

and so $\|Tu\| \leq \|u\|$. For this case, if we let

$$\Omega_2 = \{x \in \mathcal{B} \mid \|x\| < H_4\},$$

then

$$\|Tu\| \leq \|u\|, \text{ for } u \in \mathcal{P} \cap \partial\Omega_2. \quad (16)$$

In either of the cases, application of part (ii) of Theorem 2.1 yields a fixed point u of T belonging to $\mathcal{P} \cap (\overline{\Omega_2} \setminus \Omega_1)$, which in turn yields a pair (u, v) satisfying (1), (2) for the chosen value of λ . The proof is complete. \square

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