Positive Periodic Solutions of Delayed Nicholson's Blowflies Model with a Linear Harvesting Term*

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Abstract: This paper is concerned with a class of Nicholson's blowflies model with a linear harvesting term. By applying the method of coincidence degree, some criteria are established for the existence and uniqueness of positive periodic solutions of the model. Moreover, an example is employed to illustrate the main results.

Keywords: Nicholson's blowflies model; positive periodic solutions; coincidence degree; existence and uniqueness; harvesting term.

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1. Introduction

In [1], Gurney et al. proposed the following nonlinear autonomous delay equation

$$N'(t) = -\delta N(t) + pN(t-\tau)e^{-aN(t-\tau)}, \delta, p, \tau, a \in (0,\ +\infty) \tag{1.1}$$

to describe the population of the Australian sheep-blowfly and to agree with the experimental data obtained in [2]. Here, $N(t)$ is the size of the population at time t, p is the maximum per capita daily egg production, $\frac{1}{a}$ is the size at which the population reproduces at its maximum rate, δ is the per capita daily adult death rate, and τ is the generation time. Since this equation explains Nicholson's data of blowfly more accurately, the model and its

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modifications have been now refereed to as the Nicholson's Blowflies Model. In the past forty years, the theory of the Nicholson's blowflies model has made a remarkable progress with main results scattered in numerous research papers, see, for example, [3−8]. In particular, there have been extensive results on the problem of the existence of positive periodic solutions for Nicholson's blowflies equation in the literature. We refer the reader to [9−11] and the references cited therein.

Recently, assuming that a harvesting function is a function of the delayed estimate of the true population, L. Berezansky et al. [12] proposed the Nicholson's blowflies model with a linear harvesting term:

$$N'(t) = -\delta N(t) + pN(t-\tau)e^{-aN(t-\tau)} - HN(t-\sigma),\ \ \delta, p, \tau, a, H, \sigma \in (0,\ +\infty). \qquad (1.2)$$

Moreover, L. Berezansky et al. [12] pointed out an open problem: How about the dynamic behaviors of the Nicholson's blowflies model with a linear harvesting term.

The main purpose of this paper is to give the conditions for the existence and uniqueness of positive periodic solutions for Nicholson's blowflies models with a linear harvesting term. Since the coefficients and delays in differential equations of population and ecology problems are usually time-varying in the real world, so we'll consider the delayed Nicholson's blowflies models with a linear harvesting term:

$$N'(t) = -\delta(t)N(t) + p(t)N(t-\tau(t))e^{-a(t)N(t-\tau(t))} - H(t)N(t-\tau(t)), \qquad (1.3)$$

where $\delta, p, a \in C(R, (0, \infty))$ and $\tau, H \in C(R, [0, \infty))$ are T-periodic functions.

Throughout this paper, given a bounded continuous function g defined on R, let g^+ and g^- be defined as

$$g^- = \inf_{t\in R} g(t), \quad g^+ = \sup_{t\in R} g(t). \qquad (1.4)$$

Then, we denote

$$A = 2\int_0^T \delta(t)dt, \quad f(t) = \frac{p(t)}{a(t)\delta(t)e}, \quad h(t) = \frac{p(t)}{\delta(t) + H(t)}. \qquad (1.5)$$

For the sake of convenience, we choose a constant κ such that

$$0 < \kappa < 1, \frac{1-\kappa}{e^{\kappa}} = \frac{1}{e^2}, \qquad (1.6)$$

which implies that

$$\sup_{x\geq\kappa} |\frac{1-x}{e^x}| = \frac{1}{e^2}. \qquad (1.7)$$

The remaining part of this paper is organized as follows. In section 2, we shall derive new sufficient conditions for checking the existence and uniqueness of positive periodic solutions of model (1.3). In Section 3, we shall give an example and a remark to illustrate our results obtained in the previous sections.

2. Existence and Uniqueness of Positive Periodic Solutions

The following continuation theorem of coincidence degree is crucial in the arguments of our main results.

Lemma 2.1 (Continuation Theorem) [13]**.** Let X and Z be two Banach spaces. Suppose that $L : D(L) \subset X \to Z$ is a Fredholm operator with index zero and $N : X \to Z$ is L -compact on $\overline{\Omega}$, where Ω is an open subset of X. Moreover, assume that all the following conditions are satisfied:

(1) $Lx \neq \lambda Nx$, for all $x \in \partial\Omega \cap D(L), \lambda \in (0, 1)$;

(2) $Nx \notin ImL$, for all $x \in \partial\Omega \cap KerL$;

(3) The Brouwer degree

$$\deg\{QN, \Omega \cap KerL, 0\} \neq 0.$$

Then equation $Lx = Nx$ has at least one solution in $D(L) \cap \overline{\Omega}$.

Theorem 2.1. Set

$$p^- e^{-a^+ f^+} - H^+ \geq 0. \tag{2.1}$$

Then (1.3) has a positive T-periodic solution.

Proof. Set $N(t) = e^{x(t)}$, then (1.3) can be rewritten as

$$\begin{aligned} x'(t) &= -\delta(t) + p(t)e^{x(t-\tau(t))-x(t)-a(t)e^{x(t-\tau(t))}} - H(t)e^{x(t-\tau(t))-x(t)} \\ &:= \Delta(x, t). \end{aligned} \tag{2.2}$$

Then, to prove Theorem 2.1, it suffices to show that equation (2.2) has at least one T-periodic solution. Let $X = Z = \{x \in C(R, R) : x(t + T) = x(t) \text{ for all } t \in R\}$ be Banach spaces equipped with the norm $||\cdot||$, where $||x|| = \max\limits_{t\in[0,T]} |x(t)|$. For any $x \in X$, because of periodicity, it is easy to see that $\Delta(x, \cdot) \in C(R, R)$ is T-periodic, Let

$$L : D(L) = \{x \in X : x \in C^1(R, R)\} \ni x \longmapsto x' \in Z,$$

$$P : X \ni x \longmapsto \frac{1}{T}\int_0^T x(s)ds \in X,$$

$$Q : Z \ni z \longmapsto \frac{1}{T}\int_0^T z(s)ds \in Z,$$
$$N : X \ni x \longmapsto \Delta(x, \cdot) \in Z.$$

Clearly,

$$ImL = \{x | x \in Z, \int_0^T x(s)ds = 0\}, KerL = R, ImP = KerL, KerQ = ImL.$$

It follows that the operator L is a Fredholm operator with index zero. Set $L_P = L|_{D(L)\cap KerP}$, then L_P has continuous inverse L_P^{-1} defined by

$$L_P^{-1} : ImL \to D(L) \cap KerP, L_P^{-1}y(t) = -\frac{1}{T}\int_0^T\int_0^t y(s)dsdt + \int_0^t y(s)ds. \tag{2.3}$$

To apply Lemma 2.1, we first claim that N is L-compact on $\overline{\Omega}$, where Ω is a bounded open subset of X. From (2.3), it follows that

$$\begin{aligned} QNx &= \frac{1}{T}\int_0^T Nx(t)dt \\ &= \frac{1}{T}\int_0^T [-\delta(t) + p(t)e^{x(t-\tau(t))-x(t)-a(t)e^{x(t-\tau(t))}} - H(t)e^{x(t-\tau(t))-x(t)}]dt, \quad (2.4) \\ L_P^{-1}(I-Q)Nx &= \int_0^t Nx(s)ds - \frac{t}{T}\int_0^T Nx(s)ds - \frac{1}{T}\int_0^T\int_0^t Nx(s)dsdt \\ &\quad + \frac{1}{T}\int_0^T\int_0^t QNx(s)dsdt. \quad (2.5) \end{aligned}$$

Obviously, QN and $L_P^{-1}(I - Q)N$ are continuous. It is not difficult to show that $L_P^{-1}(I - Q)N(\overline{\Omega})$ is compact for any open bounded set $\Omega \subset X$ by using the Arzela-Ascoli theorem. Moreover, $QN(\overline{\Omega})$ is clearly bounded. Thus, N is $L-$compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

Considering the operator equation $Lx = \lambda Nx, \lambda \in (0, 1)$, we have

$$x'(t) = \lambda\Delta(x, t). \tag{2.6}$$

Assume that $x \in X$ is a solution of (2.6) for some $\lambda \in (0, 1)$. Then, there exist $\xi, \eta \in [0, T]$ such that

$$x(\xi) = \min_{t\in[0,T]} x(t),\ x(\eta) = \max_{t\in[0,T]} x(t),\ x'(\xi) = x'(\eta) = 0. \tag{2.7}$$

In view of the fact that $\sup\limits_{u\geq 0} ue^{-u} = \frac{1}{e}$, using (2.6) and (2.7), we obtain

$$\begin{aligned}
\delta(\eta) &\leq \delta(\eta) + H(\eta)e^{x(\eta-\tau(\eta))-x(\eta)} \\
&= p(\eta)e^{x(\eta-\tau(\eta))-x(\eta)-a(\eta)e^{x(\eta-\tau(\eta))}} \\
&= p(\eta)\frac{a(\eta)e^{x(\eta-\tau(\eta))}e^{-a(\eta)e^{x(\eta-\tau(\eta))}}}{a(\eta)e^{x(\eta)}} \\
&\leq \frac{p(\eta)}{a(\eta)e}\frac{1}{e^{x(\eta)}},
\end{aligned}$$

which implies that

$$x(t) \leq x(\eta) < \ln(\frac{p(\eta)}{a(\eta)\delta(\eta)e}) \leq \ln f^{+} := H_1, \quad \text{for all} \quad t \in R. \tag{2.8}$$

According to $p^{-}e^{-a^{+}f^{+}} - H^{+} \geq 0$, we have

$$\begin{aligned}
& p(t)e^{x(t-\tau(t))-x(t)-a(t)e^{x(t-\tau(t))}} - H(t)e^{x(t-\tau(t))-x(t)} \\
= \ & e^{x(t-\tau(t))-x(t)}[p(t)e^{-a(t)e^{x(t-\tau(t))}} - H(t)] \\
\geq \ & e^{x(t-\tau(t))-x(t)}[p^{-}e^{-a^{+}e^{x(\eta)}} - H^{+}] \\
\geq \ & e^{x(t-\tau(t))-x(t)}[p^{-}e^{-a^{+}f^{+}} - H^{+}] \\
\geq \ & 0.
\end{aligned}$$

Thus,

$$\begin{aligned}
& \int_0^T |p(t)e^{x(t-\tau(t))-x(t)-a(t)e^{x(t-\tau(t))}} - H(t)e^{x(t-\tau(t))-x(t)}|dt \\
= \ & \int_0^T [p(t)e^{x(t-\tau(t))-x(t)-a(t)e^{x(t-\tau(t))}} - H(t)e^{x(t-\tau(t))-x(t)}]dt \\
= \ & \int_0^T \delta(t)dt. \qquad (2.9)
\end{aligned}$$

It follows from (2.6) and (2.9) that

$$\begin{aligned}
\int_0^T |x'(t)|dt &\leq \lambda\int_0^T |p(t)e^{x(t-\tau(t))-x(t)-a(t)e^{x(t-\tau(t))}} - H(t)e^{x(t-\tau(t))-x(t)}|dt \\
&\quad +\lambda\int_0^T |\delta(t)|dt \\
&< 2\int_0^T \delta(t)dt = A. \qquad (2.10)
\end{aligned}$$

Since

$$p(\xi)e^{-a(\xi)e^{x(\xi-\tau(\xi))}} - H(\xi) \geq p^{-}e^{-a^{+}f^{+}} - H^{+} \geq 0,$$

again from (2.6) and (2.7), we obtain

$$\begin{aligned}\delta(\xi) &= e^{x(\xi-\tau(\xi))-x(\xi)}[p(\xi)e^{-a(\xi)e^{x(\xi-\tau(\xi))}} - H(\xi)] \\ &\geq p(\xi)e^{-a(\xi)e^{x(\xi-\tau(\xi))}} - H(\xi),\end{aligned}$$

which yields

$$x(\xi-\tau(\xi)) \geq \ln(\frac{1}{a(\xi)}\ln\frac{p(\xi)}{\delta(\xi)+H(\xi)}) \geq \ln(\frac{1}{a^+}\ln h^-). \tag{2.11}$$

Let $\xi-\tau(\xi) = \tilde{n}T+\tilde{\gamma}$, where $\tilde{\gamma}\in[0,T]$ and $\tilde{n}$ is an integer. Then (2.10) and (2.11) imply that

$$x(t) \geq x(\tilde{\gamma}) - \int_0^T |x'(t)|dt \geq \ln(\frac{1}{a^+}\ln h^-) - A := H_2, \quad \text{for all} \quad t\in R. \tag{2.12}$$

Let

$$K = 1+\max\{|H_1|,|H_2|,|\ln(\frac{1}{a^+}\ln(\frac{\int_0^T p(t)dt}{\int_0^T(\delta(t)+H(t))dt}))|,|\ln(\frac{1}{a^-}\ln(\frac{\int_0^T p(t)dt}{\int_0^T(\delta(t)+H(t))dt}))|\}, \tag{2.13}$$

and define $\Omega = \{x\in X : ||x|| < K\}$. Then (2.8) and (2.12) imply that there is no $\lambda\in(0,1)$ and $x\in\partial\Omega$ such that $Lx=\lambda Nx$.

When $x\in\partial\Omega\cap KerL = \partial\Omega\cap R$, $x=\pm K$. Then

$$QN(-K) > 0 \quad \text{and} \quad QN(K) < 0. \tag{2.14}$$

Otherwise, if $QN(-K)\leq 0$, it follows from (2.4) that

$$\int_0^T \delta(t)dt \geq \int_0^T p(t)e^{-a(t)e^{-K}}dt - \int_0^T H(t)dt \geq \int_0^T p(t)e^{-a^+e^{-K}}dt - \int_0^T H(t)dt,$$

which yields

$$-K \geq \ln(\frac{1}{a^+}\ln(\frac{\int_0^T p(t)dt}{\int_0^T(\delta(t)+H(t))dt})) \geq -|\ln(\frac{1}{a^+}\ln(\frac{\int_0^T p(t)dt}{\int_0^T(\delta(t)+H(t))dt}))|.$$

This is a contradiction the choice of K. Thus, $QN(-K)>0$.

If $QN(K)\geq 0$, it follows from (2.4) that

$$\int_0^T \delta(t)dt \leq \int_0^T p(t)e^{-a(t)e^{K}}dt - \int_0^T H(t)dt \leq \int_0^T p(t)e^{-a^-e^{K}}dt - \int_0^T H(t)dt.$$

Consequently,

$$K \leq \ln(\frac{1}{a^-}\ln(\frac{\int_0^T p(t)dt}{\int_0^T(\delta(t)+H(t))dt})) \leq |\ln(\frac{1}{a^-}\ln(\frac{\int_0^T p(t)dt}{\int_0^T(\delta(t)+H(t))dt}))|,$$

This is a contradiction to the choice of K. Thus, $QN(K) < 0$.

Furthermore, define continuous function $\bar{H}(x,\mu)$ by setting

$$\bar{H}(x,\mu) = -(1-\mu)x + \mu\frac{1}{T}\int_0^T [-\delta(t) + p(t)e^{-a(t)e^x} - H(t)]dt.$$

It follows from (2.14) that $x\bar{H}(x,\mu) \neq 0$ for all $x \in \partial\Omega \cap kerL$. Hence, using the homotopy invariance theorem, we obtain

$$\begin{aligned}\deg\{QN, \Omega \cap kerL, 0\} &= \deg\{\tfrac{1}{T}\textstyle\int_0^T[-\delta(t) + p(t)e^{-a(t)e^x} - H(t)]dt, \Omega \cap kerL, 0\} \\ &= \deg\{-x, \Omega \cap kerL, 0\} \neq 0.\end{aligned}$$

In view of all the discussions above, we conclude from Lemma 2.1 that Theorem 2.1 is proved.

Corollary 2.1. Let (2.1) hold, and

$$\ln(\frac{1}{a^+}\ln h^-) - A := H_2 \geq \ln\frac{\kappa}{a^-}. \tag{2.15}$$

If $N(t)$ is a positive T-periodic solution of (1.3), then,

$$N(t) \geq \frac{\kappa}{a^-} \quad \text{for all} \quad t \in R. \tag{2.16}$$

Proof. If $N(t)$ is a positive T-periodic solution of (1.3). Let $x(t) = \ln N(t)$, then $x(t)$ is a T-periodic solution of (2.2). Applying the similar mathematical analysis techniques as in the proof of Theorem 2.1, we can obtain

$$x(t) \geq \ln(\frac{1}{a^+}\ln h^-) - A := H_2 \geq \ln\frac{\kappa}{a^-} \quad \text{for all} \quad t \in R,$$

which implies that

$$N(t) = e^{x(t)} \geq \frac{\kappa}{a^-} \quad \text{for all} \quad t \in R.$$

This completes the proof.

Theorem 2.2. Let (2.1) and (2.15) hold. Moreover, assume that

$$-\delta^- + p^+\frac{1}{e^2} + H^+ < 0. \tag{2.17}$$

Then equation (1.3) has a unique positive T-periodic solution.

Proof. Assume that $N_1(t)$ and $N_2(t)$ are two positive T-periodic solutions of equation (1.3). Set $y(t) = N_1(t) - N_2(t)$, where $t \in R$. Then

$$y'(t) = -\delta(t)y(t) + p(t)[N_1(t-\tau(t))e^{-a(t)N_1(t-\tau(t))}$$

$$-N_2(t-\tau(t))e^{-a(t)N_2(t-\tau(t))}] - H(t)y(t-\tau(t)). \tag{2.18}$$

Define a continuous function $\Gamma(u)$ by setting

$$\Gamma(u) = -(\delta^- - u) + p^+ \frac{1}{e^2} e^{u\tau^+} + H^+ e^{u\tau^+}, u \in [0,1]. \tag{2.19}$$

Then, from (2.17), we have

$$\Gamma(0) = -\delta^- + p^+ \frac{1}{e^2} + H^+ < 0,$$

which implies that there exist two constants $\bar{\eta} > 0$ and $\lambda \in (0,1]$ such that

$$\Gamma(\lambda) = -(\delta^- - \lambda) + p^+ \frac{1}{e^2} e^{\lambda\tau^+} + H^+ e^{\lambda\tau^+} < -\bar{\eta} < 0. \tag{2.20}$$

We consider the Lyapunov functional

$$V(t) = |y(t)|e^{\lambda t}. \tag{2.21}$$

Calculating the upper right derivative of $V(t)$ along the solution $y(t)$ of (2.18), we have

$$\begin{aligned} D^+(V(t)) \leq\ & -\delta(t)|y(t)|e^{\lambda t} + p(t)|N_1(t-\tau(t))e^{-a(t)N_1(t-\tau(t))} \\ & -N_2(t-\tau(t))e^{-a(t)N_2(t-\tau(t))}|e^{\lambda t} + H(t)|y(t-\tau(t))|e^{\lambda t} + \lambda|y(t)|e^{\lambda t} \\ =\ & [(\lambda - \delta(t))|y(t)| + p(t)|N_1(t-\tau(t))e^{-a(t)N_1(t-\tau(t))} \\ & -N_2(t-\tau(t))e^{-a(t)N_2(t-\tau(t))}| + H(t)|y(t-\tau(t))|]e^{\lambda t}, \text{ for all } t \in R. \end{aligned} \tag{2.22}$$

For any fixed $t_0 \in R$, we claim that

$$V(t) = |y(t)|e^{\lambda t} < e^{\lambda t_0}(\max_{t\in[0,\ T]} |N_1(t) - N_2(t)| + 1) := M \text{ for all } t > t_0. \tag{2.23}$$

Contrarily, there must exist $t_* > t_0$ such that

$$V(t_*) = M \quad \text{and} \quad V(t) < M \text{ for all } \quad t < t_*, \tag{2.24}$$

which implies that

$$V(t_*) - M = 0 \quad \text{and} \quad V(t) - M < 0 \text{ for all } t < t_*. \tag{2.25}$$

By Corollary 2.1, we get

$$N_i(t) \geq \frac{\kappa}{a^-} \text{ for all } t \in R, \ i = 1, 2. \tag{2.26}$$

From (1.7), (2.22), (2.25) and the inequality

$$\begin{aligned}|se^{-s}-te^{-t}| &= |\frac{1-(s+\theta(t-s))}{e^{s+\theta(t-s)}}||s-t| \\ &\le \frac{1}{e^2}|s-t| \ \text{ where } \ s,t\in[\kappa,+\infty), 0<\theta<1,\end{aligned} \tag{2.27}$$

we obtain

$$\begin{aligned}0 &\le D^+(V(t_*)-M) \\ &= D^+(V(t_*)) \\ &\le [(\lambda-\delta(t_*))|y(t_*)|+p(t_*)|N_1(t_*-\tau(t_*))e^{-a(t_*)N_1(t_*-\tau(t_*))} \\ &\quad -N_2(t_*-\tau(t_*))e^{-a(t_*)N_2(t_*-\tau(t_*))}|+H(t_*)|y(t_*-\tau(t_*))|]e^{\lambda t_*} \\ &= [(\lambda-\delta(t_*))|y(t_*)|+\frac{p(t_*)}{a(t_*)}|a(t_*)N_1(t_*-\tau(t_*))e^{-a(t_*)N_1(t_*-\tau(t_*))} \\ &\quad -a(t_*)N_2(t_*-\tau(t_*))e^{-a(t_*)N_2(t_*-\tau(t_*))}|+H(t_*)|y(t_*-\tau(t_*))|]e^{\lambda t_*} \\ &\le (\lambda-\delta(t_*))|y(t_*)|e^{\lambda t_*}+p(t_*)\frac{1}{e^2}|y(t_*-\tau(t_*))|e^{\lambda(t_*-\tau(t_*))}e^{\lambda\tau(t_*)} \\ &\quad +H(t_*)|y(t_*-\tau(t_*))|e^{\lambda(t_*-\tau(t_*))}e^{\lambda\tau(t_*)} \\ &\le [(\lambda-\delta^-)+p^+\frac{1}{e^2}e^{\lambda\tau^+}+H^+e^{\lambda\tau^+}]M.\end{aligned} \tag{2.28}$$

Thus,

$$0\le(\lambda-\delta^-)+p^+\frac{1}{e^2}e^{\lambda\tau^+}+H^+e^{\lambda\tau^+},$$

which contradicts with (2.20). Hence, (2.23) holds. It follows that

$$|y(t)|<Me^{-\lambda t} \ \text{ for all } \ t>t_0. \tag{2.29}$$

In view of (2.29) and the periodicity of $y(t)$, we have

$$y(t)=N_1(t)-N_2(t)=0 \text{ for } \ \text{all} \ \ t\in R.$$

This completes the proof.

3. An Example

In this section we present an example to illustrate our results.

Example 3.1. Consider the Nicholson's blowflies model with a linear harvesting term:

$$\begin{aligned}N'(t) &= -\tfrac{1}{1000000}(1.000001+0.000001\cos t)N(t) \\ &+ \tfrac{e^{1.9999}}{1000000}(0.999999+0.000001\sin t)N(t-1-10^{-4}|\sin t|)e^{-N(t-1-10^{-4}|\sin t|)} \\ &- \tfrac{1}{10000000}(1-e^{-0.0001})|\cos t|N(t-1-10^{-4}|\sin t|).\end{aligned} \tag{3.1}$$

Obviously,

$$\delta(t) = \frac{1}{1000000}(1.000001 + 0.000001\cos t),$$
$$p(t) = \frac{e^{1.9999}}{1000000}(0.999999 + 0.000001\sin t),$$
$$H(t) = \frac{1}{10000000}(1 - e^{-0.0001})|\cos t|,$$
$$\tau(t) = 1 + 0.00001|\sin t|, a(t) \equiv 1,$$
$$A = 2\int_0^{2\pi} \delta(t)dt = 4.000004\pi \times 10^{-6},$$
$$f(t) = \frac{p(t)}{a(t)\delta(t)e} \leq e^{0.9999},$$
$$h(t) = \frac{p(t)}{\delta(t) + H(t)} \geq \frac{0.999998e^{1.9999}}{1.100002},$$
$$p(t) \leq \frac{e^{1.9999}}{1000000}.$$

Then

$$p^{-}e^{-a^{+}f^{+}} - H^{+} \geq \frac{0.999998e^{1.9999}e^{-e^{0.9999}}}{1000000} - \frac{1}{10000000} > 0,$$
$$\ln(\frac{1}{a^{+}}\ln h^{-}) - A \geq \ln(\ln(\frac{0.999998e^{1.9999}}{1.100002})) - 4.000004\pi \times 10^{-6} > 0 > \ln\kappa,$$
$$-\delta^{-} + p^{+}\frac{1}{e^{2}} + H^{+} < -\frac{1}{1000000} + \frac{e^{-0.0001}}{1000000} + \frac{1}{10000000}(1 - e^{-0.0001}) < 0.$$

This implies that the Nicholson's blowflies model (3.1) satisfies (2.1), (2.15) and (2.17) . Hence, from Theorems 2.2 , equation (3.1) has a unique positive 2π-periodic solution.

Remark 3.1. Equation (3.1) is a form of Nicholson's blowflies models with a linear harvesting term. Therefore, all the results in [8-12] and the references therein are inapplicable for proving the existence and uniqueness of positive 2π-periodic solution of equation (3.1). This implies that the results of this paper are essentially new.

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