Universal Characterisation of Non-Transitive Preferences

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Some decision support systems use preference relation and their final result is usually non transitive (for example the results of the French school: ELECTRE, PROMETHEE) Transitivity can be obtained by using the incomparability measure together with a cutting procedure.

The question of transitivity and non-transitivity appears on preference based decision structures. Using the classical utility approach the transitivity is always valid.

In multicriteria decision making there are two main research area. On one hand the preference approach will achieve the transitivity, because it gives a consistency and on the other hand the utility will establish intransitivity, because the human decision making has this property.

In classical utilities we order only one numerical value to an alternative and this is certainly transitive. But in the real life, there are many non-transitive structures, so the utilities need extension.

This was a long process (which has some crucial points, for example: Alois paradox, and the answers how the models can keep the linearity) in which the results was a new kind of utility concept. P. Fishburn introduced the skew-symmetric bilinear functional. In this model the non-transitive decision has an exact functional representation.

Our main result:

Introduction of the concept of the k-cyclically of preferences was considered in the structures of two-valued lotteries: to win \$m with probability p, and \$0 with probability $1-p$, denoted this lottery by $[m, p(m)]$. The k-cyclicity in this structure means (the preference relation is denoted by >):

$$[m,p(m)] > [m+1,p(m+1)] > ... > [m+k,p(m+k)] > [m,p(m)]$$
$$[m,p(m)] > [m+1,p(m+1)] > ... > [m+2k,p(m+2k)] > [m,p(m)]$$

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$$[m,p(m)] > [m+1,p(m+1)] > ... > [m+lk,p(m+lk)] > [m,p(m)]$$

In this structure we take the preference representation of P. Fishburn with skew-symmetric bilinear functional, and we suppose, that this functional can describe with real-valued function: $\varphi(x,y) = h(x-y), x \geq y$.

We can summarise our main result in the following theorem:

Theorem: For every $k, n \in N$, where $2k \leq n$ and for every $\varepsilon > 0$, there exists $j \in N$, such, that the preference $>$ is k-cyclic on the interval $[j, n]$ and there exist k-cyclic preference function $h(m)$, for every k in the form $h(m) = v(m)g(m)$ where $g(m)$ is the positive solution of: $F(m+i)g(m) - g(m+i)F(m) = g(i)$ $1 \leq m, i, m+i \leq n$ functional equation, on the set $1, 2, ..., n$, when $F(m) = \frac{f(m)}{v(m)}$, for positive solution $v(m)$ of the functional inequality system:

$v(i) \leq v(m)v(m+i)$ if $i \neq rk$, and

$v(rk) > v(m)v(m+rk)$

where $r \in N, r \leq [\frac{f(n)}{2k}]$, and $j \leq m, m+i, rk \leq n$.