Training of Artificial Neural Networks by Linear Mappings

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The different types of artificial neural networks are widely used in computer science. One of the special fields of applications is the computer graphics and geometry, where an interesting problem is the analytical description and handling of linear mappings. The purpose of this presentation is to evaluate the learning ability of the back-propagation method in linear mapping recognition. For better understanding let us consider a well-known affine transformation of the plane given by the equations

$$\tilde{x} = a_{11}x + a_{12}y + t_1 \quad \tilde{y} = a_{21}x + a_{22}y + t_2$$

where (x, y) and $(\tilde{x}, \tilde{y})$ are corresponding points. If we consider a classical two-layered perceptron with two input neurons and two output neurons, the weights and the thresholds can be given by

$$w_{ij} = a_{ij}, \quad \theta_i = t_i \quad (i, j = 1, 2)$$

where w_{ij} is the weight to the connection between the input node i and the output node j, while θ_i is the threshold connected to the output node i. This neural network produces the same transformation described above. How many pairs of points do we need to train this neural network starting with random weights? What can we say in terms of other kind of transformations, e.g. projective mappings, degenerated mapping of the space to the plane? These were the main questions of our project, where the latest results of the analytical description of central axonometric mappings were also applied.

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