Construction of Recursive Algorithms for Polarity Matrices Calculation in Polynomial Logical Function Representation

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Compact representation of switching functions is not only the matter of notation convenience, but highly relates to the analysis and synthesis of these functions. Both analysis and synthesis procedures, as well as final realizations, can be greatly simplified by choosing appropriate representations of switching functions.

In the case of Reed-Muller (RM) expressions, the problem to determine the most compact representation reads as determination of optimal polarity for switching variables. By choosing between the positive or negative literals for each variable, but not both at the same time, the Fixed polarity RM (FPRM) expressions are defined [5].

In a FPRM, the number of products, or equivalently, the number of non-zero coefficients may be considerably reduced by choosing different polarities for the variables. The FPRM with the minimum number of products is taken as the optimal FPRM for f. If there are two FPRMs with the same number of products, the one with the smaller number of literals in the products is taken.

There is no method to determine apriori the polarities of variables for a given function f. In practice, it is necessary to generate all the FPRMs and chose the optimal one. That can be efficiently done by generating the polarity matrices $\mathbf{P}_{\mathrm{RM}}$ whose rows are RM-coefficients for the given f with different polarities of variables. The efficiency of generation of $\mathbf{P}_{\mathrm{RM}}$ is based upon its recursive structure originating in the Kronecker product representation of the RM-transform matrix.

Polynomial representations of Multiple-valued (MV) functions are very interesting with advent of multiple-valued circuit technology, in particular recent experience with current-mode circuits that are very attractive for implementation of MV functions. Specially, the realization of 4-valued corresponding circuit is very efficient. The problem of compact representations is even harder in the case of MV functions. Galois field (GF) expressions are generalization of RM-expressions to MV case [7]. Optimization of GF-expressions can be studied and solved in a way similar to that used for RM expressions. In particular,efficient methods for generation of polarity matrices $\mathbf{P}_{\mathrm{GF}}$ for GF-expressions of ternary functions are reported in [6], while the correspond methods for quaternary functions are reported in [3], and further elaborated in [1], [2], [4].

Reed-Muller-Fourier expressions are an alternative extension of RM expressions to MV case [8]. It has been shown that RMF expressions require on the average smaller number of products than GF expressions to represent a given function f[9]. The optimization of RMF expressions is performed in the same way as in the GF-expressions by choosing different polarities for the variables. As in the case of RM and GF expressions, there are no methods to determine apriori the polarity for the variables in a given f to get the RMF expression with the minimum number of products. For that reason, the efficient calculation of polarity matrices is a very important task. An analyse of present recursive methods for calculation of polarity matrix for some particular expressions shows that recursive approaches are more efficient than others methods. Therefore, the construction of recursive relations for polarity matrix calculation for various expressions is a very interesting problem.

In this paper, we uniformly consider the coefficients in various expressions for logic functions as spectral coefficients in particular spectral transforms. We show that polarity matrix can be generated as convolution of f with columns of related transform matrix. The recursive properties of the polarity matrix result from properties of the convolution matrix. We give a unique method to construct recursive procedures for the polarity matrices calculation for any Kronecker product based expression of MV functions.

This method involves existing methods as particular cases and permits various generalizations. For illustration, we derive two recursive algorithms for calculation of fixed polarity Reed-Muller-Fourier expressions for vour-valued functions.

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