Difference Functions of Dependence Spaces

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Z. Pawlak introduced his notion of information systems (sometimes called knowledge representation systems) in the early 1980's. Information concerning properties of objects is the basic knowledge included in information systems, and it is given in terms of attributes and values of attributes. For example, we may express statements concerning the color of objects if the information system includes the attribute "color" and a set of values of this attribute consisting of "green", "yellow", etc. In general, an information system is determined by specifying a set U of objects, a set A of attributes, and an indexed set $\{V_a\}_{a \in A}$ of value sets of attributes. Each attribute is a map $a\colon U \to V_a$, which assigns to every object a value of the attribute a.

In an information system $\mathcal{S} = (U, A, \{V_a\}_{a \in A})$ each subset of attributes B $(\subseteq A)$ defines an indiscernibility relation $I(B)$. This relation is an equivalence on the object set U such that its equivalence classes consist of objects which have the same values for all attributes in B. An attribute set C $(\subseteq A)$ is a reduct of B, if C is a minimal subset of B which defines the same indiscernibility relation as B. Hence, the reducts of an attribute set are its minimal subsets defining the same partition of objects. The reduction problem means that we want to enumerate all reducts of a given subset.

Reduction problem can be studied in a more simpler algebraic structure called dependence space. Here we characterize the reducts by the means of dense families of dependence spaces. Dense families are important since they contain enough information about the structure of dependence spaces. Dependence spaces induced by indiscernibility relations are also studied. We show how we can determine dense families of dependence spaces induced by indiscernibility relations by applying indiscernibility matrices.

We present an algorithm for finding the reducts of any given subset of a dependence space. The algorithm is based on the notion of difference function which connects the reduction problem to the general problem of identifying the set of all minimal Boolean vectors satisfying an isotone Boolean function.