Adaptive Multisection in Derivative Free Interval Methods for Global Optimization[2]

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A new adaptive multisection technique in interval methods for global optimization is investigated, and numerical tests demonstrate that the efficiency of the underlying global optimization method can be improved substantially. The heuristic rule is based on experiences that suggest the subdivision of the current subinterval into a larger number of pieces only if it is located in the neighbourhood of a minimizer point. An estimator of the proximity of a subinterval to the region of attraction to a minimizer point is utilized.

This paper investigates adaptive multisection variants of a branch-and-bound algorithm [3, 4] for solving the box constrained global optimization problem [5]:

$$\min_{x \in X} f(x), \tag{1}$$

where the n-dimensional interval $X \subseteq \mathbb{R}^n$ is the search region, and $f(x) : X \subset \mathbb{R}^n \to \mathbb{R}$ is the objective function. The global minimum value of f is denoted by f^*, and the set of global minimizer points of f on X by X^*. That is,

$$f^* = \min_{x \in X} f(x) \qquad \text{and} \qquad X^* = \{x^* \mid f(x^*) = f^*\}.$$

The paper discussed a new adaptive decision rule for multisection algorithms in interval global optimization. According to the theoretical results, the suggested algorithmic variant can save in the best case up to one half of the inclusion function evaluations, while in the worst case, it may require an arbitrary large times more function evaluations than standard bisection. The underlying $p\,\tilde{f}$ [1, 2] algorithm parameter is not necessarily reliable: in extreme cases the sequence of actual subintervals can be distracted for an arbitrary long time, and also the dynamic multisection rule can suggest inefficient subdivision types.

An extensive numerical study was made that was based on the exact, a priori known global minimum value, on an approximation of it, and on the easily available upper bound $\tilde{f}$ of it. These new adaptive multisection methods improved the number of interval (by about one third) and real function evaluations (by around two third), and therefore also the execution time substantially (by up to a half). The memory complexity proved also to be smaller when larger accuracy was requested (by up to one half). The new adaptive multisection strategies seem to be indispensable, and can improve both the computational and the memory complexity substantially. The introduced adaptive multisection technique is appropriate for both differentiable and non-differentiable functions. Nevertheless, for differentiable functions our proposal could be improved further by applying it together with sophisticated accelerating devices and other algorithmic changes.

References

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