The Effects of the Boxing Method for Interval Subdivision Algorithms[3]

A.E. Csallner, D. Ratz and R. Klatte

The bound-constrained global optimization problem which is the scope of this work can be defined in general as $\min_{x \in X} f(x)$ where X is a — possibly multidimensional — interval. Note, that a great class of real-life problems are covered by the bound-constrained global optimization problem.

The original problem can be solved with verified accuracy with the aid of interval subdivision methods (see, e.g., [1]). These algorithms are based on the well-known branch-and-bound principle. Thus, a search tree is built where the whole search region — the interval X — is the root and the particular levels consist of subintervals which are partitions of their parents in the tree. Those branches that cannot be pruned have to be stored for later treatment. Thus, it is desirable to exclude as many subintervals from further investigation as possible.

The methods bounding the search tree are called accelerating devices. One of the most effective of these is the interval Newton step. It can, however, only be applied if f is once continuously differentiable. On the other hand, its time complexity is relatively high with regard to other accelerating devices like the cut-off test or the monotonicity test. Therefor it should only be deployed if there are no other possibilities to effectively bound the search tree. A new method ([2]) called the *boxing method* makes the decision not to carry out the interval Newton step if a good bound for the cut-off test has been found in the midpoint of an interval *Y*, but to split Y into several subboxes leaving a small interval containing the midpoint of Y.

The oral presentation discusses the effects of the boxing method and gives some possibilities to apply this idea to various interval subdivision algorithms.

References

[1] Ratschek H. and Rokne J. (1993), *Interval Methods*, In: Handbook of Global Optimization, Horst R. and Pardalos P.M. (eds.), Kluwer, Dordrecht, pp. 751–828.

[2] Wiethoff A. (1997), *Verifizierte globale Optimierung auf Parallelrechnern*, PhD Thesis, Universität Karlsruhe.

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